International Journal of Computer Applications (0975 – 8887)
Volume 98– No.17, July 2014

Image Denoising by Two-Pass of Total Variation Filter

Dao Nam Anh
Electric Power University
235 Hoang Quoc Viet road,
Hanoi, Vietnam

a. b. c.

Fig. 1. Denoise keeping edges by TV denoising and TV inpainting.

ABSTRACT
Total variation based methods are widely applied for image enhancement and particularly for de-noising. The majority of these is designed for a specific noise model. The alternative total variation based approach proposed here can deal with multiple noise models via two-pass iterative algorithm basing on total variation. The first pass is designed for draft denoising and to detect noise region. The second pass restores the noise region by total variation based inpainting. Experiments on Salt & Pepper, Gaussian, Speckle, Poisson, and Impulse noise models demonstrate the effectiveness of the proposed method.

General Terms
Image Processing

Keywords
denoising, inpainting, restoration, keeping edge, filtering, total variation

1. INTRODUCTION
Images are viewed as realization of optical and digital processes where noise from transmission errors or external factors can be added. Most widely used noise models are Salt & Pepper, Gaussian, Speckle, Poisson, Impulse. Removing these different noises is necessary to achieve quality for still images and videos. Image denoising methods have been improved recently in accuracy and performance for noise variance. However limitations and effects are found in some denoising methods: blurring fine image structure or introducing artifacts.

Total variation (TV) concept of Rudin, Osher and Fatemi (ROF) [1] presented an effective algorithm for denoising image but keeping edges. Success of TV concept is confirmed by its further studies for deblurring [2, 3, 4], inpainting [5, 6, 7], interpolation [8], super-resolution [9], cartoon/texture decomposition [10]. This paper studies TV in recent works for image denoising and introduces a new way of its application with performance improvement. This is illustrated in fig.1, where 1a is input image with Poisson noise, 1b is denoised by the first pass of TV method and 1c is final denoised result after second pass with TV method in term of inpainting.

2. OUTLINE OF PAPER
The rest of the paper is organized as follows: in the next section - a brief review of related work and contributions. Major development of total variation will be presented. This is followed by description of two-pass algorithm of total variation for denoising gray image in section 4. Section 5 describes algorithm for color image. In section 6, experiments on a variety of image data set are discussed with conclusion.

3. CONTRIBUTIONS AND RELATED WORK
Let's denote image as a *M*-dimension function of space.

$$u:\Omega \to \Re^N, u(x) := (u_1(x),..u_M(x)),\ x\in\Omega\in\Re^2 \qquad (1)$$

$$u_i:\Omega\to\Re, i=1,..,M \qquad (2)$$

Suppose input image u includes pure signal v and additive noise n:

$$u(x)=v(x)+n(x) \qquad (3)$$

Reconstruction objective is to recover v from u. Total variation regularization method by ROF for denoising consists of two terms [1]:

$$F_{ROF}(v)=\int_\Omega \|v(x)\|dx+\frac{\lambda}{2}\int_\Omega |u(x)-v(x)|^2\,dx \qquad (4)$$

The first is a regularization term, the second is data-fidelity term in L2 norm. The model regards as the solution to a variation problem, to minimize $F_{ROF}(v)$ (4). The solution attends to diminish variation of v and keep v close to the input u.

The selection of noise model can have significant influence on outcomes, so the denoise algorithm should agree with the actual noise model in the image. Some previous denoising methods were addressed to specific noise models.

Le, Chartrand and Asaki [2] proposed an alternative TV data-fidelity term suitably adaptive for Poisson noise:

$$d_{Poisson}(v) = \int_\Omega u(x) - v(x)\log u(x)dx \qquad (5)$$

The method requires that the data-fidelity term $d_{Poisson}(u)$ for the input image and $d_{Poisson}(v)$ for the reconstructed image should be matched.

Alliney [11], Chan and Esedoglu [12] use L1-norm for Laplace noise in TV-L1 method:

$$F_{TV-L1}(v) = \int_\Omega |v(x)|dx + \tfrac{\lambda}{2}\int_\Omega |u(x) - v(x)|\, dx \qquad (6)$$

The data-fidelity in (6) uses the first differences with recursive median filters of appropriate window length in TV-L1. Split Bregman method is a technique for solving for L1-regularized optimization problems. Goldstein, Osher [13] and Getreuer [14] particularly proposed split Bregman method for solving ROF total variation regularization. TV in L1 norm for Laplace noise uses the same form of (6) and it is solved by the split Bregman. TV for Poisson noise model is formulated in the following L1 norm and also applicable by split Bregman [14]:

$$F_{Poisson}(v) = \int_\Omega \|v(x)\|dx + \tfrac{\lambda}{2}\int_\Omega (u(x) - v(x)\log(u(x)))dx \qquad (7)$$

Bioucas-Dias and Figueiredo [4] proposed total variation based restoration for images with the Speckle (Gamma) noise by the same TV regularizer formula (1) with split-Bregman method. Rodríguez, Rojas and Wohlberg [15] provided TV restoration for image with mixed Gaussian-Impulse noise by L1 and L2 norms.

Presented above denoising methods concern application of total variation for specific noise models. The method in this work attends to get an alternative version of total variation that can deal with many noise models. The method uses two-pass of TV regularization. The two-pass approach is similar to two-phase scheme of Chan, Ho and Nikolova in [16] that removing salt-and-pepper and impulse noise by an adaptive median filter. Other works in [17, 18, 19] proposed two-stage iterative method for removing random-valued impulse noise with adaptive center-weighted median filter.

Main contribution of the work is a new adaptive total variation based denoising method in two-pass approach that is effective for multiple noise models. The method is described in next section.

4. TWO-PASS FILTER

4.1 Denoising and Inpainting by Total Variation

Denote $\Psi, \Psi \subset \Omega$ is the region where noise is applied to image u in (1):

$$\Psi = \{x, x \in \Omega, n(x) \neq 0\} \qquad (8)$$

Noise model with Ψ now can be split into two regions: noise region Ψ and region without noise Ω / Ψ, where $u(x) = v(x)$:

$$u_\Omega(x) = \begin{cases} v(x), x \in \Omega/\Psi, as: n_{\Omega/\Psi}(x) = 0 \\ v(x) + n(x), x \in \Psi \end{cases} \qquad (9)$$

Or simply

$$u_\Omega(x) = v_{\Omega/\Psi}(x) + v_\Psi(x) + n_\Psi(x) \qquad (10)$$

The third operand in (10) presents effect of noise. Given $u_\Omega(x)$, the denoising problem is to reconstruct $v_{\Omega/\Psi}(x)$ and $v_\Psi(x)$. If the noise region Ψ is found then $v_{\Omega/\Psi}(x)$ is the same $u_{\Omega/\Psi}(x)$, that is given in $u_\Omega(x)$. The question is to recover $v_\Psi(x)$. In our approach, solution of finding $v_\Psi(x)$ can be seen from inpainting problem: for each $x \in \Psi$, $v(x)$ can be presented by a function of $v(y)$, $y \in neighbour(x) \subset \Omega / \Psi$.

Denoising process basing on detection of high gradient can remove noise but it blurs edges and fine image structure [20]. Total variation based method of ROF [1] was proposed initially for denoising image reserving edges. The method likes a minimization problem to find solution $u_{ROF}(x)$:

$$u_{ROF}(x) = \arg\min_{v \in BV(\Omega)} \|v(x)\|_{TV(\Omega)} + \tfrac{\lambda}{2}\int_\Omega (u(x) - v(x))^2 dx \qquad (11)$$

where BV is bounded variation image [14].

Function u in region of edges and region of noise usually have the same signal of high gradient. Difference of u and u_{ROF} can define region that has edges or noise:

$$\Phi = \{x, \|u_{ROF}(x) - u(x)\| > \tau\} \qquad (12)$$

Where τ is parameter to define level of high gradient.

Continuousness of pixels in Ψ can be used to distinct region of noise: pixels in edge are usually continuous:

$$continue_\Phi(x) = \begin{cases} true, if\_exist\_neighbor(x) \in \Phi \\ false, otherwise \end{cases} \qquad (13)$$

So noise region is

$$\Psi = \{x, x \in \Phi, continue(x) = false\} \qquad (14)$$

Inpainting Ψ may be viewed as denoising with a spatially-varying regularization strength $\lambda(x)$ [21]:

$$u_{Br}(x) = \arg\min_v \|v(x)\|_{TV(\Omega)} + \tfrac{\lambda(x)}{2}\int_\Omega (u(x) - v(x))^2 dx \qquad (15)$$

where

$$\lambda(x) = \begin{cases} 0, x \in \Psi \\ > 0, x \notin \Psi \end{cases} \qquad (16)$$

Iterative split Bregman algorithm by Goldstein and Osher [13] can be applied for inpainting for noise region Ψ in the second pass, where $u_\Psi(x)$ will be calculated by inpainting problem in form of constrained minimization problem:

$$\begin{cases} u_{Br}(x) = \arg\min_{v,w} \|v(x)\|_{TV(\Omega)} + \tfrac{\lambda(x)}{2}\int_\Omega (u(x) - v(x))^2 dx \\ subject\_to\_w = \nabla u \end{cases} \qquad (17)$$

here w is auxiliary vector field that is constrained to equal discrete derivative ∇u.

So, the proposed denoising method can be seen like two-pass of total variation. The first pass is to get $u_{ROF}(x)$ by (11), then $u_\Psi(x)$ is calculated by second pass by (17).

4.2 Denoising Gray Image by Two-Pass of Total Variation Filter

Basing on two-pass filter from 4.1, denoising gray image can be described in algorithm TV2P.

ALGORITHM TV2P
FOR DENOISING GRAY IMAGE

Given: Noise gray image $u(x)$ and parameter $\tau, \kappa \in [0,1]$.

The denoising algorithm takes the following steps:

Step 1: Run ROF TV denoising for input image $u(x)$, then get $u_{ROF}(x)$ by (11) and Φ by (12). See flow 1 in fig.2.

Step 2: Get noise region Ψ from (14). See flow 2 in fig.2.

Step 3: Run iterative ROF TV inpainting for

$$u^*(x) = \kappa u(x) + (1-\kappa) u_{ROF}(x) \tag{18}$$

and get mask Ψ by (17). See flow 3 in fig.2.

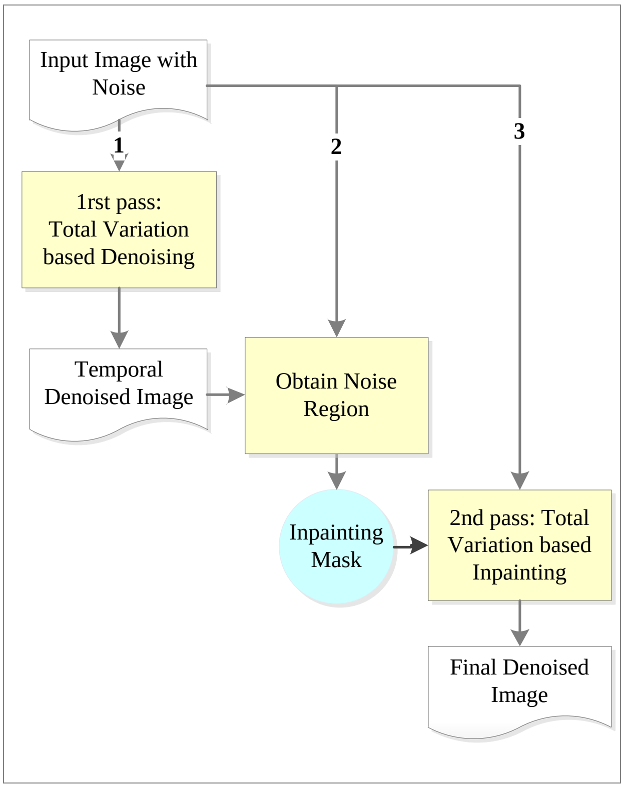


Fig. 2. Diagram of TV2P Denoising Algorithm

4.3 Denoising Color Image by Two-Pass of Total Variation Filter

Color image is presented by *3-D* function u:

$$u : \Omega \to \Re^3, u(x) := (u^1(x), u^2(x), u^3(x)), \tag{19}$$

Where channel $u^i : \Omega \to \Re, i = 1,2,3$

Given parameters τ and λ, two-pass TV filter for each channel $u^i, i = 1,2,3$ is described as follows:

$$u^i_{ROF}(x) = \arg\min_{v^i \in BV(\Omega)} \left\| v^i(x) \right\|_{TV(\Omega)} + \tfrac{\lambda}{2} \int_\Omega (u^i(x) - v^i(x))^2 dx \tag{20}$$

Having solution of (19), noise region Ψ can be found from

$$\Phi = \{x, \exists i, \left\| u^i_{ROF}(x) - u^i(x) \right\| > \tau\} \tag{21}$$

Continuousness of pixels (13) has its 3D form:

$$\Psi = \{x, x \in \Phi, \exists i, continue^i(x) = false\} \tag{22}$$

Here, $continue^i_\Phi(x) = \begin{cases} true, if\_exist\_neighbor^i(x) \in \Phi \\ false, otherwise \end{cases}$

Inpainting constrained minimization problem (17) in 3D form:

$$u^i_{Br}(x) = \arg\min{}^i_v \left\| v(x) \right\|_{TV(\Omega)} + \tfrac{\lambda(x)}{2} \int_\Omega (u^i(x) - v^i(x))^2 dx \tag{23}$$

ALGORITHM TV2P
FOR DENOISING COLOR IMAGE

Given: Noise color image $u(x)$ and parameter $\tau, \kappa \in [0,1]$.

The denoising algorithm takes the following steps:

Step 1: Run ROF TV denoising for input image $u(x)$, then get $u^i_{ROF}(x)$ by (20) and Φ by (21) for each $i, i = 1,2,3$.

Step 2: Get noise region Ψ from (22).

Step 3: Run iterative ROF TV inpainting for $u^*(x) = \kappa u(x) + (1-\kappa) u_{ROF}(x)$ and mask Ψ by (23).

5 IMPLEMENTATION

Algorithm TV2P is implemented for both gray and color images. Next section presents experimental results and some remarks.

5.1 TV2P Denoising Gray Image

Cameraman image with different noise models are tested for TV2P. Fig 3.a is input image with salt and pepper noise. Result of TV denoising of step 1 is fig.3b; step 3 gives mask in fig 3c. TV inpainting produces final denoising result fig 2d. It shows fine denoised image with better contrast, in comparison with single TV denoised result (fig.3c). Formulas is to define initial value for inpainting. Average value ($\kappa = .5$) is selected for examples in Fig 2, and Fig 3.

Fig 2e is image added Gaussian noise, fig 2i- Speckle noise, fig 2m – Poisson noise and fig.2q – Impulse noise. Results of single TV filter are in second column of fig2. Final results are in the fourth column, where noise is removed and edges are recovered better than results of single TV denoising in second column.

Experiments show TV2P keeps edges better than single TV for cameraman gray image dealing with Salt & Pepper, Gaussian, Speckle, Poisson and Impulse noise.

5.2 TV2P Denoising Color Image

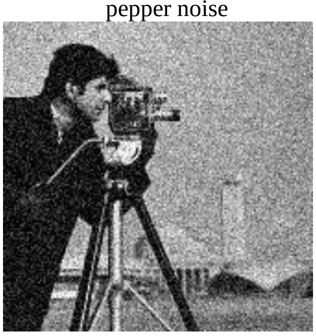
Color images from Berkeley Segmentation Dataset and Benchmark (BSDB) were selected for testing TV2P algorithm in fig.4. Five noise models were added to the images for the input of the TV2P algorithm, see the first column of fig 4. Denoising of the images by TV with Split Bregman gives results in the second column of fig 4. Step 2 of the algorithm produces mask in the third column. Finally, TV inpainting with split Bregman outcome images in the fourth column. Poisson noise and Impulse noise are more complex for denoising by TV and TV2P, thought TV2P gives better results than single TV for all noise models.


a. Input image with salt & pepper noise


b. Result of ROF denoising (a)

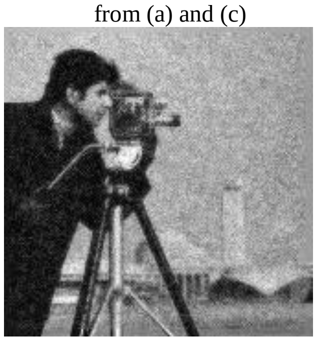

c. Inpainting mask for (a)


d. Result of ROF inpainting from (a) and (c)


e. Input image with Gaussian noise


f. Result of TV denoising (e)

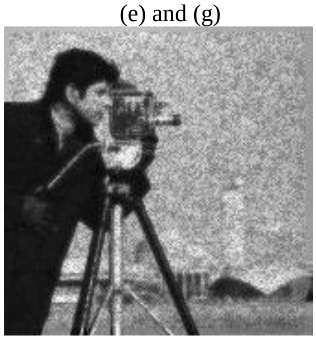

g. Inpainting mask for (e)


h. Result of TV inpainting from (e) and (g)


i. Input image with Speckle noise


j. Result of TV denoising (i)

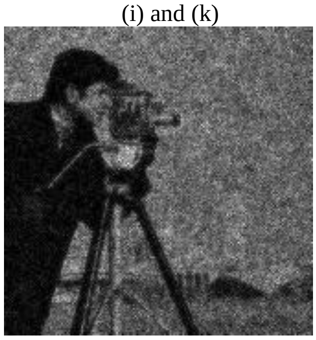

k. Inpainting mask for (i)

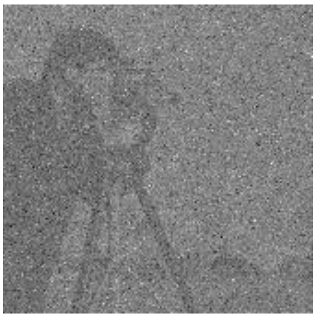

l. Result of TV inpainting from (i) and (k)

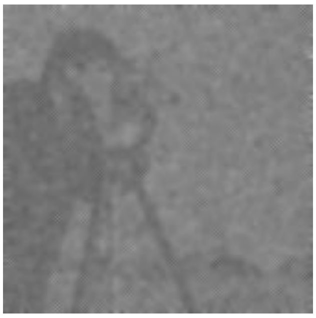

m. Input image with Poisson noise

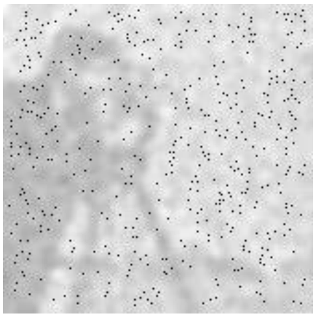

n. Result of TV denoising (m)


o. Inpainting mask for (m)


p. Result of TV inpainting from (m) and (o)


q. Input image with Impulse noise


r. Result of TV denoising (q)


s. Inpainting mask for (q)

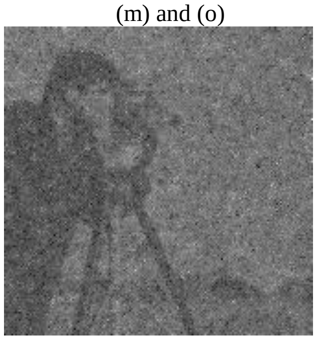
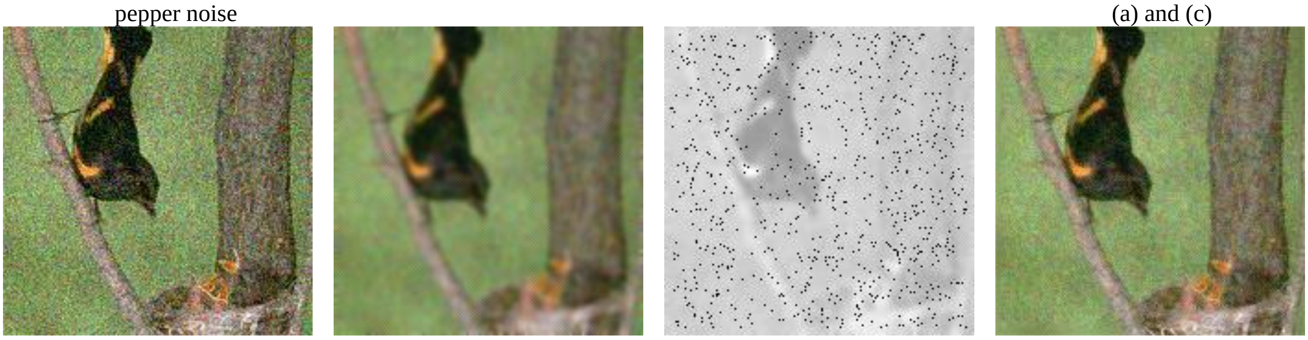
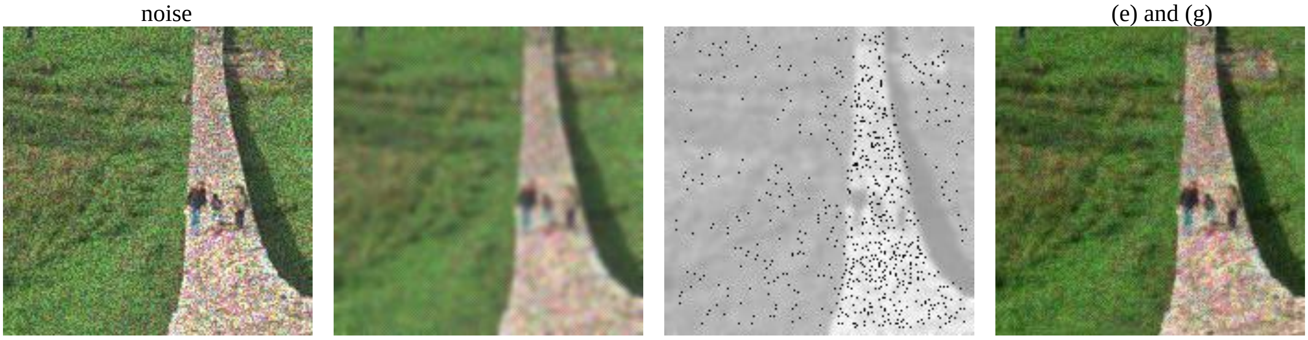

t. Result of TV inpainting from (q) and (s)

Fig. 3. Effect of TV and TV2P for "cameraman" image

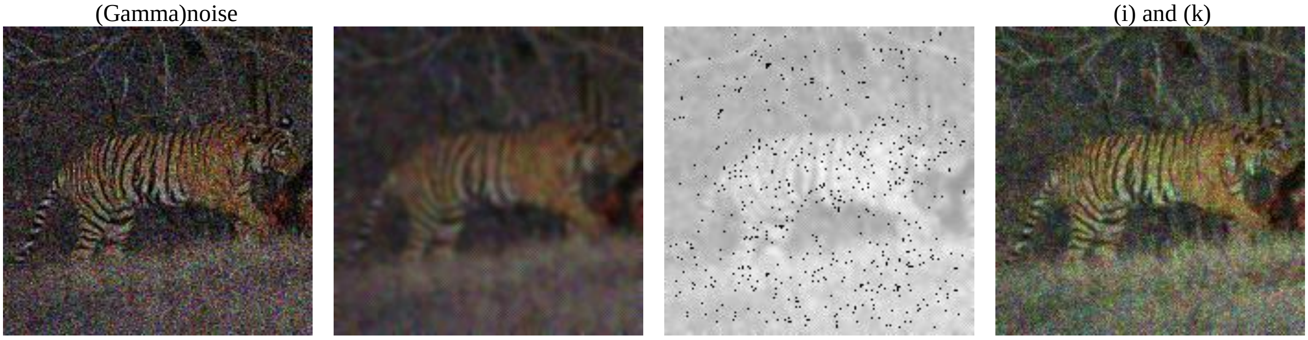
a. Input image with salt & pepper noise | b. Result of TV denoising (a) | c. Inpainting mask for (a) | d. Result of TV inpainting from (a) and (c)



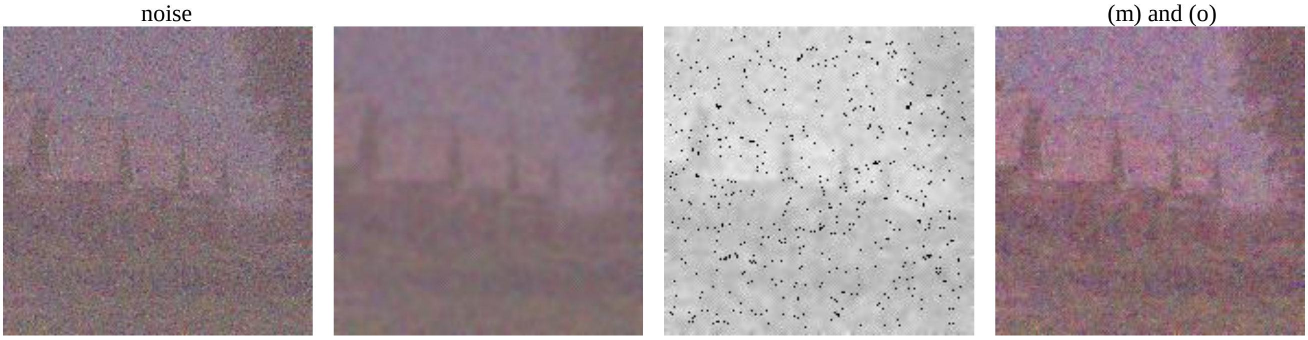
e. Input image with Gaussian noise | f. Result of TV denoising (e) | g. Inpainting mask for (e) | h. Result of TV inpainting from (e) and (g)



i. Input image with Speckle (Gamma)noise | j. Result of TV denoising (i) | k. Inpainting mask for (i) | l. Result of TV inpainting from (i) and (k)



m. Input image with Poisson noise | n. Result of TV denoising (m) | o. Inpainting mask for (m) | p. Result of TV inpainting from (m) and (o)



q. Input image with Impulse noise | r. Result of TV denoising (q) | s. Inpainting mask for (q) | t. Result of TV inpainting from (q) and (s)

Fig. 4. Effect of TV and TV2P for BSDB color images

6 DISCUSSION

The algorithm TV2D has parameter τ, its value effects on performance of the algorithm. Parameter τ is used in step 2 of the algorithm to separate noise of edge. Wrong value of τ could lead to over-noise or over-edges. However it can be managed manually with experiences. Parameter κ regulates smooth level in final result κ, output image is as smooth as big κ.

7 FUTURE WORK

There are a few models of pure noise and mixed noise. Finding general denoising method for multiple noise models is challenge. The algorithm TV2P was tested with five common noise models. It may need to check possibility of noise removal of the algorithm for other noise models.

8 CONCLUSION

In this paper an alternative total variation based denoising method is presented for gray and color image. The method is tested with Salt & Pepper, Gaussian, Speckle, Poisson, Impulse noise models. Experiments were taken on the noise models and demonstrated the effectiveness of the proposed method method but with higher computational cost. The novel contribution of the work is two-pass of TV approach for denoising problem. We regard the algorithm effective application in future and it would be desirable to extend the method to other noise models including mixed noise models.

9 ACKNOWLEDGMENT

Author would like to thank the Faculty of Information Technology, Electric Power University, especially Tao Ngo Quoc and Quynh Nguyen Huu for useful discussions during the development of the algorithm.

Special thanks Pascal Getreuer for his comprehensive papers and codes of TV denoising and TV inpainting at IPOL, with these implementation this work started. We wish to thank Berkeley Computer Vision group for the Berkeley Segmentation Dataset and Benchmark, its images were used in fig 2 and fig 3 in this article. Author would also like to thank the internal reviewers at EPU and the anonymous referees for their valuable comments.

The article was supported by the 2014 EPU project “e-Lib of Electric Power Industry Standards for EVN” and the results of the article are applied for denoising images in the e-Lib’s documents.

10 REFERENCES

[1] L. Rudin, S. Osher, and E. Fatemi, “Nonlinear total variation based noise removal algorithms”, Phys. D, 60 (1992), pp. 259–268.

[2] T. Le, R. Chartrand, T. Asaki. “A Variational Approach to Constructing Images Corrupted by Poisson Noise,” Journal of Mathematical Imaging and Vision, vol. 27(3), pp. 257–263, 2007.

[3] Aubert, G., Aujol, J.F., “A variational approach to remove multiplicative noise”. J.on Applied Mathematics 68(4), 925–946 (2008).

[4] Jose M. Bioucas-Dias, Mario A. T. Figueiredo, “Total Variation Restoration of Speckled Images Using a Split-Bregman Algorithm”, IEEE International Conference on Image Processing, 2009.

[5] S. Masnou and J. Morel. “Level-lines based Disocclusion”. In Proc. 5th IEEE Int. Conf. on Image Process., pages 259{263, Chicago, IL, 1998.

[6] M. Bertalmio, G. Sapiro, V. Caselles, and C. Ballester. “Image Inpainting”. In Proc. 27th Ann. Conf. on Comput. Graph. & Interactive Tech., pages 417-427, 2000.

[7] T. Chan and J. Shen. “Mathematical Models of Local Non-texture Inpaintings”. SIAM J. Appl. Math., 62:1019{1043, 2001.

[8] F. Guichard, F. Malgouyres, “Total Variation Based Interpolation”, Proceedings of Eusipco 1998.

[9] S. Derin Babacan, Rafael Molina, Aggelos K. Katsaggelos, “Total variation super resolution using a variational approach”, Image Processing, 2008. ICIP 2008. 15th IEEE International Conference.

[10] Stanley Osher, Andres Sole, And Luminita Vese, “Image Decomposition And Restoration Using Total Variation Minimization And The H−1 Form”, Multiscale Model. Simul. 2003 Society for Industrial and Applied Mathematics, Vol. 1, No. 3, pp. 349–370.

[11] S. Alliney, “A property of the minimum vectors of a regularizing functional defined by means of the absolute norm,” IEEE Transactions on Signal Processing, vol. 45, no. 4, pp. 913–917, 1997.

[12] T.F. Chan, S. Esedoglu. “Aspects of total variation regularized L1 function approximation.” SIAM Journal on Applied Mathematics, vol. 65, no. 5, pp. 1817–1837, 2005.

[13] T. Goldstein, S. Osher, “The Split Bregman Method for L1-Regularized Problems”, SIAM Journal on Imaging Sciences, vol. 2, no. 2, pp. 323-343, 2009.

[14] Pascal Getreuer, “Rudin-Osher-Fatemi Total Variation Denoising using Split Bregman”, Image Processing On Line, 2012. ISSN 2105-1232, 2012 IPOL.

[15] Paul Rodríguez, Renán Rojas and Brendt Wohlberg, "Mixed Gaussian-Impulse Noise Image Restoration via Total Variation", ICASSP, (Kyoto, Japan), pp. 1077-1080, Mar 2012.

[16] R. Chan, C.-W. Ho, and M. Nikolova. “Salt-and pepper noise removal by median-type noise detectors and detail-preserving regularization”. IEEE Transactions on Image Processing, vol. 14, no. 10, pp. 1479–1485, 2005.

[17] S. Leung and S. Osher, “Global Minimization of the Active Contour Model with TV-Inpainting and Two-Phase Denoising”, Variational, Geometric, and Level Set Methods in Computer Vision (VLSM), Springer, Vol. 3752, pp. 149–160, 2005.

[18] R. Chan, C. Hu, and M. Nikolova. “An iterative procedure for removing random-valued impulse noise,” IEEE Signal Process. Letters, pp. 921–924, 2004.

[19] M. Nikolova, “A variational approach to remove outliers and impulse noise,” Journal of Mathematical Imaging and Vision, 20 (2004), pp. 99–120.

[20] M. Lindenbaum, M. Fischer, and A.M. Bruckstein, “On gabor contribution to image enhancement”, Pattern Recognition, 27 (1994).

[21] T.F. Chan, J. Shen, “Mathematical models of local non-texture inpaintings”, SIAM Journal on Applied Mathematics, vol. 62, no. 3, pp. 1019-1043, 2001.