Attitude Control of Chaotic Satellite with Unknown Input and uncertainties based on Sliding Control

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ABSTRACT
The attitude control of missiles, spacecraft and satellites is essential; in order to remain them fixed in space to perform their missions accurately. The attitude equation of a satellite is a six- dimensional nonlinear system which includes some types of nonlinear behavior such as periodic trajectory, chaotic dynamics. In this paper, a sliding mode control design method for stabilization of the attitude chaotic satellites with unknown inputs and uncertainties. Using Lyapunov theory, the stability control system is proven. Simulation results show that the proposed controller can be chaotic satellite attitude in the presence of unknown inputs and uncertainties will converge to the unstable equilibrium points.

Keywords
Chaotic Attitude, Dynamic Error, Sliding Control

1. INTRODUCTION
Chaos the concept and mathematical precision, seemingly random and complex phenomenon which is inherent deterministic nature. Chaotic dynamics of some features that are important, it is very sensitive to initial conditions (that is, very little difference in the initial conditions that will change future behavior of the rate difference is proportional to Lyapunov exponent) so at first thought that the dynamics are uncontrollable chaos. One of the concepts of chaos control stabilization of chaotic dynamics in unstable equilibrium points. First time in [1] proved that there is a problem of chaos control. They show that a very small control signal can be made to provide conditions for the control chaos dynamics. This is a characteristic of chaos, which is not possible at all nonlinear dynamics model. Then, many methods have been introduced, such as fuzzy control [2], adaptive feedback [3] sliding mode [4], impulsive control [5], backstepping control [6]. So in the past two decades, the problem of controlling chaos dynamics has attracted much interest from researchers. Control chaos, in many applications, including secure communication [7], gyroscopes [8], removal of heart rhythms [9], and many others in [10, 11] has been introduced. In this paper, chaotic satellite attitude control problem with unknown inputs and uncertainties are discussed. The research in [12] was proven to be chaotic attitude motion satellite. Hence, a sliding mode controller design method is proposed for stabilizing the attitude motion chaotic of satellite. Recently, various researches and publications introduced the chaotic dynamics of the satellites. Methods that have been introduced thus far include predictive control [13], impulsive control [14], and neural networks [15]. The second part of the describes the chaotic state of satellite dynamics. Next controller design method is explained. Fourth, the problem attitude control of the satellite with unknown inputs and uncertainties expressed. Finally, part five illustrates the simulation results.

2. SATELLITE DYNAMIC AND PROBLEM FORMULATION
In this section, the satellite system and the chaotic dynamics are studied. In [16] the problem of satellite attitude control with redundant thrusters, and in [17] satellite attitude control with an uneven inertia distribution has been investigated. However, given that the satellite attitude motion under external disturbances becomes chaotic mode, the control satellite system will be very complex. So, be very careful satellite control system is designed. In [13] and [14] satellite attitude control of chaotic behavior has been investigated. The orientation of the satellite at a given point can be locally described in terms of three angles ϕ, θ and ψ which are successive clockwise rotations about inertial axes I, J and K respectively. The kinematic equation of a satellite or spacecraft can be written as:

$$\begin{cases} \omega_x = \dot{\phi}.Cos\psi.Cos\theta + \dot{\theta}.Sin\psi \\ \omega_y = -\dot{\phi}.Sin\psi.Cos\theta + \dot{\theta}.Cos\psi \\ \omega_z = \dot{\psi} + \phi.Sin\theta \end{cases} \quad (1)$$

And on collecting terms and inverting, the following form is resulted, which is more appropriate for solving by numerical integration.

$$\begin{cases} \dot{\phi} = (\omega_x.Cos\psi - \omega_y Sin\psi)/Cos\theta. \\ \dot{\theta} = \omega_x.Sin\psi + \omega_y.Cos\psi \\ \dot{\psi} = \omega_z - (\omega_x.Cos\psi - \omega_y Sin\psi)/\tan\theta \end{cases} \quad (2)$$

The rotational motion for general rigid spacecraft acting under the influence of external torques is given by [18].The dynamical equation of a satellite, similar to a rigid body can be expressed as:

$$I\dot{\omega} = -\Omega I\omega + H + U \quad (3)$$

Where I is the moment of inertia tensor, ω is the angular velocity vector, U is the control torque, and H contains any external disturbance torques. The dynamical equations of a satellite are:

$$\begin{aligned} I_x\dot{\omega}_x &= \omega_y\omega_z(I_y - I_z) + H_x + U_x \\ I_y\dot{\omega}_y &= \omega_x\omega_z(I_z - I_x) + H_y + U_y \\ I_z\dot{\omega}_z &= \omega_y\omega_y(I_x - I_y) + H_z + U_z \end{aligned} \quad (4)$$

Where I_x, I_y and I_z are the principal moments of inertia, ω_x, ω_y and ω_z are the angular velocities of the satellite, U_x, U_y and U_z are the three control torques; H_x, H_y and

H_z are perturbing torques. Principal moments of inertia and perturbing torques such as:

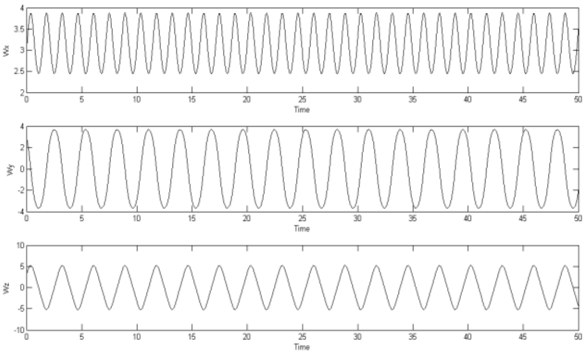

Fig.1 Period Attitude satellite

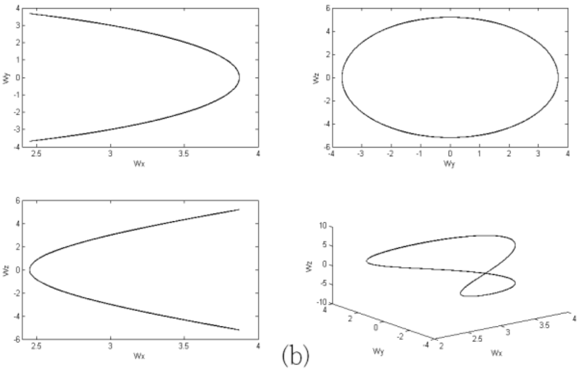

Fig.2 Phase Portraits of Period Satellite

$$M = \begin{bmatrix} -1.2 & 0 & \frac{\sqrt{6}}{2} \\ 0 & 0.35 & 0 \\ -\sqrt{6} & 0 & -0.4 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \begin{cases} I_x = 3Kg.m^2 \\ I_y = 2Kg.m^2 \\ I_z = 1Kg.m^2 \end{cases} \quad (5)$$

This torques is chosen so as to force the satellite into chaotic motion. By changing the elements value of system matrices, many various dynamical behaviors could be observed. For example, let H=0 and U=0, the attitude motion of a satellite has a twisted periodic trajectory, which is shown in Fig.1 and Fig.2. Nonlinear system (4), under the conditions express in (5) shows the chaotic attitude satellite.

$$\begin{aligned} \dot{\omega}_x &= \omega_y \omega_z \frac{(I_y - I_z)}{I_x} - \frac{1.2}{I_x}\omega_x + \frac{\sqrt{6}}{2I_x}\omega_z \\ \dot{\omega}_y &= \omega_x \omega_z \frac{(I_z - I_x)}{I_y} + \frac{0.35}{I_y}\omega_y \\ \dot{\omega}_z &= \omega_y \omega_x \frac{(I_x - I_y)}{I_z} - \frac{\sqrt{6}}{I_z}\omega_x - \frac{0.4}{I_z}\omega_z \end{aligned} \quad (6)$$

In equation (6) it is seen that the three coupled nonlinear relationship exists between the satellite dynamics. Thus, it can be seen in the attitude of the satellite is the most complex chaotic dynamics. See Fig.3.

As mentioned, a characteristic of chaos is sensitivity to initial conditions. That small change in initial conditions can to change the behavior of a dynamic future. For a better understanding see Fig.5. The control problem is to suppress the chaos and regulate the state trajectory of this system to a desire fixed point or around the equilibrium point is unstable (i.e. $\omega_{x,y,z} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$). Hence, the proposed controller will be described in the next section.

3. SLIDING CONTROLLER DESIGN

The object of the attitude control system is to regulate the angular velocities of a rigid-body in the space. Satellite chaotic dynamical system, consider the following:

$$\dot{\omega} = A\omega + f(\omega) + D + \Delta\omega + U(t) \quad (7)$$

Where $\omega \in R^n$ is the state vector, $A \in R^n$ matrix system parameters and $f : R^n \to R^n$ is a nonlinear function. $U \in R^n$ is the signal of control, D is the unknown input and $\Delta\omega$ is the uncertainty of system. In practice, there are uncertainties in the actual systems. For this reason, in this paper, the uncertainty of each variable is considered.

$$e_{x,y,z}(t) = \omega_{x,y,z} - \omega^*_{x,y,z} \quad (8)$$

There is $\omega^*_{x,y,z} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, the satellite system is the point of unstable equilibrium. Therefore, the error dynamics will be given by:

$$\dot{e} = Ae + \phi(x, y) + U(t) \quad (9)$$

Where is $\phi(\omega_{x,y,z}) = f(\omega_{x,y,z})$. The main goal of the design by controlling the vector, so that $\lim_{t\to\infty} \| e(t) \| = 0$ is obtained. Therefore, to solve the stabilization problem of consider such as control vector:

$$U = -\phi(x, y) + BG(t) \quad (10)$$

B is a constant column vector. Placement (10) in (9) simplifies as follow error dynamics.

$$\dot{e} = Ae + BG(t) \quad (11)$$

There (11), is an LTI system and $G(t)$ the input signal system. Now, the issue will focus on stabilizing an error around the point $e = 0$. An important consideration in the design of the controller is a sliding surface. This is surface:

$$S(e) = C_i e_i \qquad i = 1, 2, 3, \quad (12)$$

That C is a row vector with constant values. To achieve a zero error, tracking error dynamics of sliding surface will be completely, or have the following two conditions.

$$S(e) = 0 \quad (13)$$

$$\dot{S}(e) = 0 \quad (14)$$

Substituting by (12) in (14) can be obtained:

$$\dot{S}(e) = C\left[Ae + BG(t)\right] \quad (15)$$

By solving equation (15) for variable $G(t)$, we can achieve the desired control. Therefore equation (15) is equal to zero. Hence, it is clear that:

$$G = -(CB)^{-1} CAe \quad (16)$$

From equation (16) we can get the condition $CB \neq 0$ should be established. Placement (16) the dynamics equation (10), the closed-loop system be reformed as follow.

Vector C is chosen such that all eigenvalues of the matrix $\left[I - B(CB)C\right]A$ has a negative real part, so that the control system is asymptotically stable. To design the sliding mode controller we use the constant plus proportional rate reaching law [19]:

$$\dot{S} = -kS - w\frac{S}{|S| + \lambda} \tag{17}$$

In the equation (17), λ denotes a sufficiently small design constant, and the gains $w > 0$ and $\lambda > 0$ is determined such that the sliding condition is satisfied and sliding mode motion will occur. From equation (16) and (17), we can obtain $G(t)$:

$$G(t) = -(CB)^{-1}\left[C(kI + A)e + w\frac{S}{|S| + \lambda}\right] \tag{18}$$

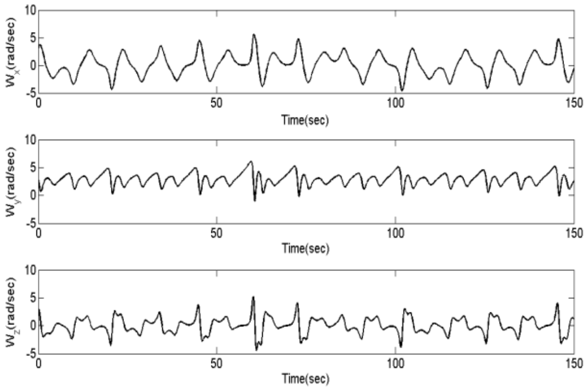

Fig.3 Chaotic Attitude of Satellite

Theorem1. The chaotic dynamics of the satellite system converges asymptotically to zero, if all initial conditions $x_0 \in \Re^n$ and the sliding control law such as is designed such as $u = -\phi(x, y) + BG(t)$. Where $G(t)$ is defined by the equation (19) and B is a column vector chosen such that (A, B) is completely controllable. Also, the gains w and λ are positive.

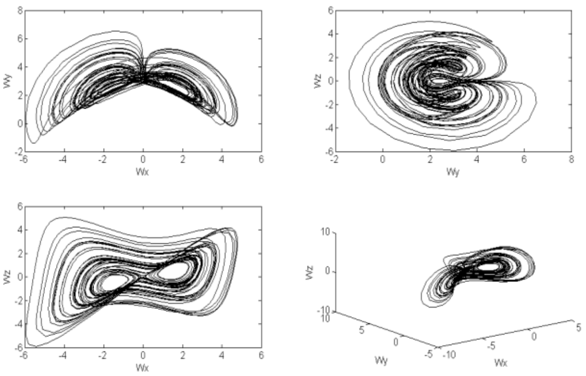

Fig.4 Phase Portraits of chaotic Satellite

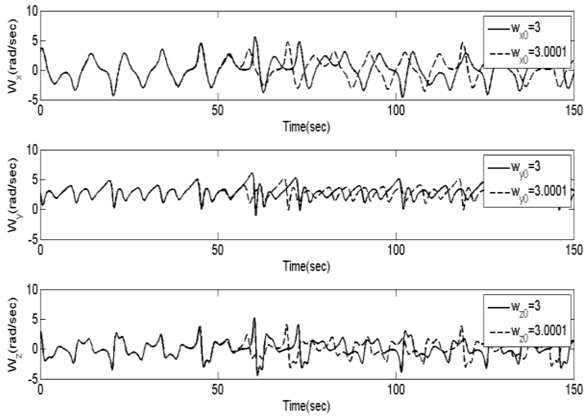

Fig4. Chaotic Attitude of Satellite with initial Different

Proof. By substituting the control laws (18) and (10) into the error dynamics (9), we get:

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + w\frac{S}{|S| + \lambda}\right] \tag{19}$$

In order to observe the stability of the error dynamics with the forgoing controller, as a possible Lyapunov function.

$$V(S) = \frac{1}{2}S^2 \tag{20}$$

Then the derivative of V becomes:

$$\dot{V} = S\dot{S}$$

$$\Rightarrow C(Ae - B(CB)^{-1}\left[C(kI + A)e + w\frac{S}{|S| + \lambda}\right])S \tag{21}$$

$$\Rightarrow -kS^2 - w\frac{S}{|S| + \lambda}S < 0$$

Which is a negative definite function on R^n. Hence, by Lyapunov stability theory, it is immediate that the error dynamics (19) is globally asymptotically stable for all initial conditions $e(0) \in R^n$.

4. CONTROL OF CHAOTIC ATTITUDE MOTION

In this paper, the results of previous section, the dynamic of chaotic satellite attitude control. According to the equation (8) and system (6), error dynamic such as:

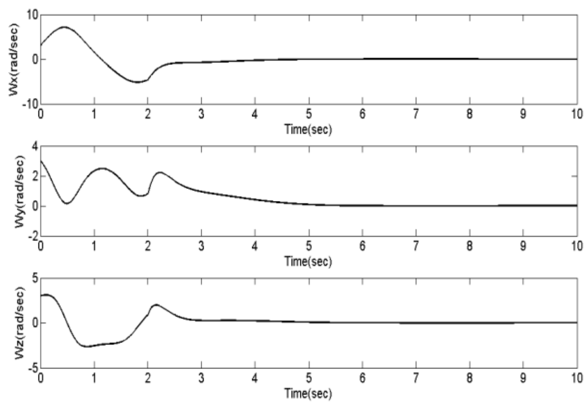

Fig6. Stabilizing the attitude motion chaotic satellite using proposed controller

$$\begin{cases} e_x = \omega_x - \omega_x^* \\ e_y = \omega_y - \omega_y^* \\ e_z = \omega_z - \omega_z^* \end{cases} \tag{22}$$

And:

$$\begin{aligned} \dot{e}_1 &= \omega_y \omega_z \frac{(I_y - I_z)}{I_x} - \frac{1.2}{I_x}\omega_x + \frac{\sqrt{6}}{2I_x}\omega_z + D_x \\ &\quad + \Delta\omega_x + U_x \\ \dot{e}_2 &= \omega_x \omega_z \frac{(I_z - I_x)}{I_y} + \frac{0.35}{I_y}\omega_y + D_y + \Delta\omega_y + U_y \\ \dot{e}_3 &= \omega_x \omega_y \frac{(I_x - I_y)}{I_z} - \frac{\sqrt{6}}{I_z}\omega_x - \frac{0.4}{I_z}\omega_z + D_z \\ &\quad + \Delta\omega_z + U_z \end{aligned} \tag{23}$$

In (23), $D, \Delta\omega$ is the unknown input and uncertainties respectively as:

$$D = \begin{bmatrix} 0.1Sin(\omega_x) \\ 0.2Sin(\omega_y) \\ 0.3Sin(\omega_z) \end{bmatrix}, \Delta\omega = \begin{bmatrix} 0.3\omega_z \\ 0 \\ 0.3\omega_x \end{bmatrix} \tag{24}$$

Hence, matric system and nonlinear part is:

$$A = \begin{bmatrix} \frac{-1.2}{I_x} & 0 & \frac{\sqrt{6}}{2I_x} \\ 0 & \frac{0.35}{I_y} & 0 \\ -\frac{\sqrt{6}}{I_z} & 0 & \frac{-0.4}{I_z} \end{bmatrix} \phi = \begin{bmatrix} \delta_x \omega_y \omega_z \\ \delta_y \omega_x \omega_z \\ \delta_z \omega_y \omega_x \end{bmatrix} \tag{25}$$

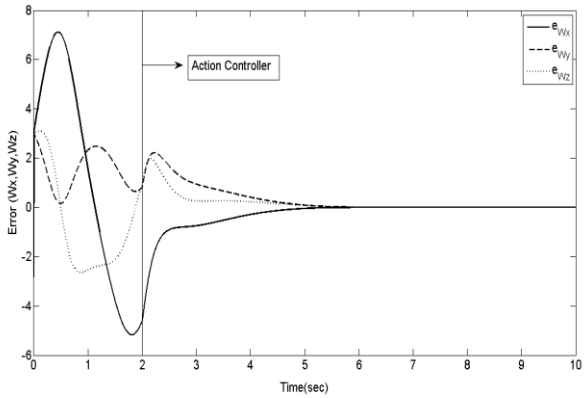

Fig.7 Error Controller

Which $\delta_x = \frac{I_y - I_z}{I_x}$, $\delta_y = \frac{I_z - I_x}{I_y}$, $\delta_z = \frac{I_x - I_y}{I_z}$. Now B is chosen such that there is a controllable condition.

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} \tag{26}$$

Finally, $G(t)$ can be determined by selecting gains $k = 7, w = 0.2$. So we have $\mathrm{G}(t)$ as follows:

$$\mathrm{G}(t) = -3.3e_1 - 3.5875e_2 + 0.2\frac{S}{|S| + \lambda} \tag{27}$$

By substituting equation (27) in (10) the chaotic attitude satellite control with unknown inputs and uncertainties are accessible.

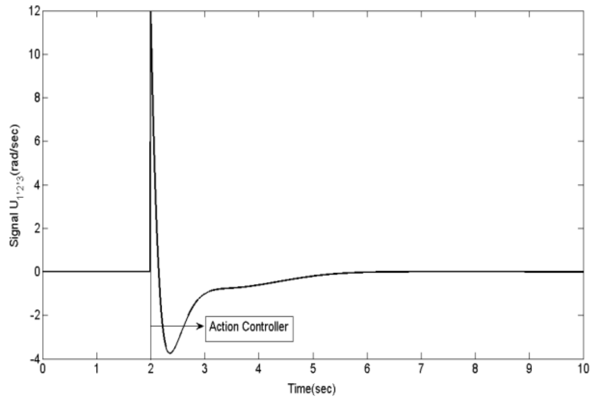

Fig.8 Sliding Control proposed

5. NUMERICAL SIMULATION RESULT

The simulation results use the fourth-order Runge-Kutte integration with step time $h = 10^{-6}$. The initial condition $\omega_0 = [3 \quad 3 \quad 3]^T$, and unknown inputs and uncertainties in (24) are considered, constant λ is selected as 0.1. Fig.3 chaotic motion of the system (9) and Fig.4 shows the phase space. The proposed controller is applied at the time $t = 2(\mathrm{sec})$. Fig.5 attitude control of the satellite with unknown inputs and uncertainties, sliding mode control has been demonstrated. Fig.6 and Fig.7 are show, the control signal proposed and error respectively. The simulation results can be seen that the proposed controller does not chattering, means that the controller can be implemented in the real world. (See Fig.7)

6. CONCLUSION

In this paper, was introduction a sliding mod control design method for stabilization of the satellite attitude chaotic state with unknown inputs and uncertainties. Using Lyapunov theory, the stability control system is proved. The numerical simulation results show that the proposed sliding mode controller the system could chaotic attitude motion of satellites under unknown inputs and uncertainties converge to the unstable equilibrium points. The controller because of not large fluctuations can be used in practice. Given the importance of the stability of the satellite position requires the controller to be suitable and accurate. Important point in this article, the unknown inputs and uncertainties, that the proposed controller able to reach the equilibrium point unstable.

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