Transient Analysis of a Two Node Tandem Communication Network with Two Stage Compound Poisson Binomial Bulk Arrivals and DBA

Nakka.Thirupathi Rao
Dept. of CS&SE
Andhra University
Visakhapatnam, India

K.Srinivasa Rao
Dept. of Statitics
Andhra University
Visakhapatnam, India

Kuda.Nageswara Rao
Dept. of CS&SE
Andhra University
Visakhapatnam, India

P.Srinivasa Rao
Dept. of CS&SE
Andhra University
Visakhapatnam, India

ABSTRACT
For efficient design and development of communication networks the stochastic modelling serves the basic frame work. This paper addresses the novel idea of utilizing compound Poisson binomial process for developing and analyzing a two node tandem communication network with two stage arrivals and Dynamic bandwidth allocation (DBA).Here it is assumed that two nodes are connected in tandem and messages arrive to the first and second buffers are connected to a random number of packets and stored in buffers for forward transmission. Arrivals are characterized by compound Poisson binomial processes in both buffers which match close with the realistic situation. The transmission processes in both the transmitters are assumed to follow dynamic bandwidth allocation which is characterized by load dependent on time. Using difference differential equations and joint probability generating function the transient behaviour of the system is studied. The performance of the network is evaluated by deriving explicit expressions for the performance measures such as mean content of the buffers, mean delays, throughput of the nodes and utilization of transmitters. Numerical illustrations are presented to study the effect of changes in input parameters on system performance measures. With suitable cost considerations, the optimal operating policies of the communication network are derived and analyzed. It is observed that the compound Poisson binomial bulk arrivals distribution parameters have significant influence on system performance measures. Analyzing the two stage direct arrivals improve the network performance and reduce congestion in buffers and mean delays. This model is useful to analyze the communication networks at LAN, WAN and MAN.

Keywords
Tandem Communication Networks, Compound Poisson Binomial bulk arrivals, Dynamic Bandwidth Allocation (DBA), Performance Measures.

1. INTRODUCTION
Communication network modelling is a prerequisite for design and analysis of many communication systems. It is difficult to conduct laboratory experiments under variable load conditions, the communication network models are developed with various assumptions on arrival processes, transmission processes, allocation, routing and flow control mechanisms. For better utilization of resources and to improve quality of service packet switching is used over circuit or message switching. Another important consideration is the statistical multiplexing of communication networks which can reduce the delay in transmission (Srinivasa Rao et al (2000), 2007, 2010, 2011, 2012), Gaujal and Hyon (2002)). Much work has been reported in literature regarding communication networks with congestion control strategies. Bit dropping is one of the usual method adopted for congestion control. In this method, the idea is discarding certain portion of profit such as least significant bits in order to reduce the load and the transmission line (Sriram K (1993), Kin K Leung (2002). But the bit dropping creates fluctuations in voice quality due to a dynamically varying bit rate during a cell transmission. To maintain quality of service and to reduce the congestion in buffers another transmission strategy dynamic bandwidth allocation strategy is utilized as an alternative and efficient control strategy. In dynamic bandwidth allocation, the unutilized available bandwidth in the transmission lines is utilized by changing the transmission rate depending on the content of the buffer connected to it. It is demonstrated the dynamic bandwidth allocation strategy can improve the throughput of nodes and reduce congestion (Varma et al (2007), Padmavathy et al (2009), Srinivasa Rao et al (2010), Trinadha Rao et al (2012), Ramasundari et al (2013)).

In all the papers referred above, they assumed that the arrivals are single and follow Poisson process. But, in packetized switching the message that arrives to the source are converted into a random number of packets and arrive to the buffers in bulk. Therefore, each arriving module is to be characterized with a Compound Poisson process (Nageswara Rao et al (2010, 2011) and Srinivasa Rao et al (2011)) have developed and analyzed some communication network models with dynamic bandwidth allocation having bulk arrivals. Recently Suhasini et al (2013a, 2013b) have developed and analyzed some communication network models having dynamic bandwidth allocation with non-homogeneous compound Poisson process. But, in these papers also the authors considered that the arrivals to the network are to be first buffer only. But, in some communication systems like satellite and wireless communications, there is a two stage arrival i.e., the arrivals of packets are to the first buffer and also to the second buffer directly. For example, in tele communications there are some local calls and some STD calls where the STD calls may directly arrive to the second buffer. To analyze this sort of systems, a two node tandem communication network with dynamic bandwidth allocation having two stage direct compound binomial Poisson arrivals is developed and analyzed.

Using the difference differential equations the joint probability density function of queue size distribution is derived in section 2. In section 3, the system performance measures such as average number of packets in each buffers, throughput of the nodes and mean delay in transmission are derived explicitly. In section 4, the performance of the communication network is evaluated through numerical methods. With suitable cost considerations, the optimal operating policies of the communication network are derived in section 5. The sensitivity analysis of the model is presented in section 6. Section 7 deals with the conclusions and the scope for further.

2. QUEUING MODEL

Consider a two-transmitter tandem communication network in which the messages arrive to the network are converted into a random number of packets. The arrival process of the messages is random and a number of packets (X) that a message can be converted follows a binomial distribution with parameters m and p i.e., the arrival modules follows a compound Poisson binomial process with composite arrival rate α_1. E(X), α_2. E(X) where $\alpha_1$and α_2 are the mean arrival rate of modules.
The probability mass function of the number of packets that a message can be converted is

$$C_k = \frac{{}^{m}C_k p^k (1-p)^{m-k}}{1-(1-p)^m} \qquad K=1, \quad 2,\ldots.m.$$

0<p<1,

Consider a Communication network in which two nodes are in tandem and the messages arrive to the first node are converted into number of packets and stored in first buffer connected to the first node. It is further assumed that the arrivals of packets in two stages follow compound Poisson processes and the number of transmissions in both nodes follows Poisson processes. The arrival of packets to the first buffer is in bulk with random batch size having the probability mass function {$\mathbf{C_k}$}. In both the nodes the transmission is carried with dynamic bandwidth allocation. i.e. the transmission rate at each node is adjusted instantaneously depending upon the content of the buffer connected to it. This can be modeled by considering that the transmission rates are linearly dependent on the content of the buffers.

Here, it is assumed that the arrival of packets follows compound Poisson process with parameters $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$ and the number of transmissions at nodes 1 and 2 also follow compound Poisson processes with parameters β_1 and β_2. The queue discipline is First-In-First-Out (FIFO). The schematic diagram representing the communication network model is shown in figure 1.

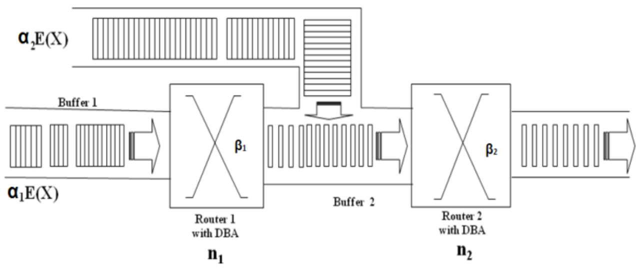


Figure 1: Communication network with two stage bulk arrivals and dynamic bandwidth allocation

Let $P_{n1,\,n2}$ (t) be the probability that there are n_1 packets in the first buffer and n_2 packets in the second buffer at time t. Then, the difference-differential equations governing the network are

$$\frac{\partial P_{n_1,n_2}(t)}{\partial t} = -\left(\alpha_1 + n_1\beta_1 + n_2\beta_2 + \alpha_2\right)P_{n_1,n_2}(t) + \left(n_1+1\right)\beta_1 P_{n_1+1,n_2-1}(t) + (n_2+1)\beta_2 P_{n_1,n_2+1}(t) + \alpha_1\left[\sum_{i=1}^{n_1} P_{n_1-i,n_2}(t)\frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\right] + \alpha_2\left[\sum_{j=1}^{n_2} P_{n_1,n_2-j}(t)\frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}\right]$$

(1)

$$\frac{\partial P_{n_1,0}(t)}{\partial t} = -\left(\alpha_1 + n_1\beta_1 + \alpha_2\right)P_{n_1,0}(t) + +\beta_2 P_{n_1,1}(t) + \alpha_1\left[\sum_{i=1}^{n_1} P_{n_1-i,0}(t)\frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\right]$$

(2)

$$\frac{\partial P_{0,n_2}(t)}{\partial t} = -\left(\alpha_1 + n_2\beta_2 + \alpha_2\right)P_{0,n_2}(t) + \beta_1 P_{1,n_2-1}(t) + (n_2+1)\beta_2 P_{0,n_2+1}(t) + \alpha_2\left[\sum_{j=1}^{n_2} P_{0,n_2-j}(t)\frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}\right]$$

(3)

$$\frac{\partial P_{1,0}(t)}{\partial t} = -\left[\alpha_1 + \beta_1 + \alpha_2\right]P_{1,0}(t) + \beta_2 P_{1,1}(t) + \alpha_1\left[P_{0,0}(t)\frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\right]$$

(4)

$$\frac{\partial P_{0,1}(t)}{\partial t} = -\left(\alpha_1 + \beta_2 + \alpha_2\right)P_{0,1}(t) + \beta_1 P_{1,0}(t) + 2\beta_2 P_{0,2}(t) + \alpha_2\left[P_{0,0}(t)\frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}\right]$$

(5)

$$\frac{\partial P_{0,0}(t)}{\partial t} = -(\alpha_1 + \alpha_2)P_{0,0}(t) + \beta_2 P_{0,1}(t)$$

(6)

with initial conditions

$$P_{00}(0) = 1; P_{i,j}(0) = 0 \quad \text{for i, j >0}$$

Let P(Z_1, Z_2, t) be the joint probability generating function of $P_{n1,\,n2}$ (t) and C_i(Z) is the probability generating function of { C_k }. Then

$$P(Z_1, Z_2, t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} Z_1^{n_1} Z_2^{n_2} P_{n_1,n_2}(t) \qquad \text{and}$$

$$C_i(Z) = \sum_{k=1}^{\infty} C_i Z^k \quad , \qquad i=1,\ 2$$

(7)

Multiplying the equations (1) to (6) with corresponding $Z_1^{n_1}, Z_2^{n_2}$ and summing over all n_1=0, 1, 2, 3, …, and n_2=0, 1, 2, 3, …, one can get

$$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\frac{\partial}{\partial t}P_{n_1,n_2}(t)Z_1^{n_1}Z_2^{n_2} = -\left[\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}Z_1^{n_1}Z_2^{n_2}\left(n_1\beta_1+n_2\beta_2+\alpha_2\right)P_{n_1,n_2}(t)\right]$$
$$+\sum_{n_1=0}^{\infty}\sum_{n_2=1}^{\infty}(n_1+1)\beta_1 Z_1^{n_1}Z_2^{n_2}P_{n_1+1,n_2-1}(t)$$
$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}(n_2+1)\beta_2 Z_1^{n_1}Z_2^{n_2}P_{n_1,n_2+1}(t)$$
$$+\alpha_1\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}Z_1^{n_1}Z_2^{n_2}.\left[\sum_{i=1}^{n_1}P_{n_1-i,n_2}(t).\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\right]$$
$$+\alpha_2\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}Z_1^{n_1}Z_2^{n_2}.\left[\sum_{j=1}^{n_2}P_{n_1,n_{2-j}}(t).\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}\right] \qquad (8)$$

After simplification, we have

$$\frac{\partial P(Z_1,Z_2,t)}{\partial t} = \left[\alpha_1(C_1(Z_1)-1)+\alpha_2(C_2(Z_2)-1)\right]P(Z_1,Z_2,t)$$
$$+\left[\beta_1(Z_2-Z_1)\right]\frac{\partial P(Z_1,Z_2,t)}{\partial Z_1}+\left[\beta_2(1-Z_2)\right]\frac{\partial P(Z_1,Z_2,t)}{\partial Z_2}$$

Rearranging the terms

$$\left[\alpha_1(C_1(Z_1)-1)+\alpha_2(C_2(Z_2)-1)\right]P(Z_1,Z_2,t)$$
$$=\frac{\partial P(Z_1,Z_2,t)}{\partial t}-\left[\beta_1(Z_2-Z_1)\right]\frac{\partial P(Z_1,Z_2,t)}{\partial Z_1}-\left[\beta_2(1-Z_2)\right]\frac{\partial P(Z_1,Z_2,t)}{\partial Z_2} \qquad (9)$$

Using the Lagrangian’s method, the auxiliary equations of the equation (9) are

$$\frac{dt}{1}=\frac{-dZ_1}{\beta_1(Z_2-Z_1)}=\frac{-dZ_2}{\beta_2(1-Z_2)}=\frac{dP(Z_1,Z_2,t)}{\left[\alpha_1(C_1(Z_1)-1)+\alpha_2(C_2(Z_2)-1)\right]P(Z_1,Z_2,t)} \qquad (10)$$

Solving the equations (10), one can get

$$\mathbf{u}=(Z_2-1)e^{-\beta_2 t} \qquad (11)$$

$$\mathbf{v}=\left[(Z_1-1)+\frac{\beta_1}{\beta_2-\beta_1}(Z_2-1)\right]e^{-\beta_1 t} \qquad (12)$$

and

$$w=P(Z_1,Z_2;t).\exp\left\{\left[-\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^J u^J v^{(r-J)}\left(\frac{e^{[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\right]\right.$$
$$\left.-\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)u^S\left(\frac{e^{\beta_2 k_2 t}}{\beta_2 k_2}\right)\right]\right\} \qquad (13)$$

where, **u** and **v** are as given in (11) and (12) respectively, **u**, **v** and **w** are the arbitrary integral constants.

Therefore,

$$P(Z_1,Z_2;t)=w.\exp\left\{\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^J u^J v^{(r-J)}\left(\frac{e^{[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\right]\right.$$
$$\left.+\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)u^S\left(\frac{e^{\beta_2 k_2 t}}{\beta_2 k_2}\right)\right]\right\}$$

Substituting the value of ‘**w**’ and using the initial conditions, we get

$$P(Z_1,Z_2;t)=\left[\exp\left\{\left\{-\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^J u^J v^{(r-J)}\left(\frac{1}{J\beta_2+(r-J)\beta_1}\right)\right]\right.\right.\right.$$
$$\left.\left.\left.+\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)u^S\left(\frac{1}{\beta_2 k_2}\right)\right]\right\}\right\}\right].$$
$$\left[\exp\left\{\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^J u^J v^{(r-J)}\left(\frac{e^{[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\right]\right.\right.$$
$$\left.\left.+\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)u^S\left(\frac{e^{\beta_2 k_2 t}}{\beta_2 k_2}\right)\right]\right\}\right]$$

This implies

$$P(Z_1,Z_2;t)=\exp\left\{\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^J u^J v^{(r-J)}\left(\frac{e^{[J\beta_2+(r-J)\beta_1]t}-1}{J\beta_2+(r-J)\beta_1}\right)\right]\right.$$
$$\left.+\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)u^S\left(\frac{e^{\beta_2 k_2 t}-1}{\beta_2 k_2}\right)\right]\right\}$$

Substituting the values of **u** and **v** in the above equation and simplifying, one can get the joint probability generating function of the number of packets in first node and second nodes as

$$P(Z_1,Z_2;t)=\exp\left\{\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{2r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^J (Z_2-1)^J\left((Z_1-1)+\frac{\beta_1(Z_2-1)}{\beta_2-\beta_1}\right)^{(r-J)}\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\right]\right.$$
$$\left.+\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(Z_2-1)^S\left(\frac{1-e^{-\beta_2 k_2 t}}{\beta_2 k_2}\right)\right]\right\} \qquad (14)$$

3. PERFORMANCE MEASURES OF THE NETWORK

In this section, we derive and analyze the performance measures of the communication network under transient conditions. From equation (14), the joint probability generating function of the number of packets in both the buffers is

$$P(Z_1,Z_2;t)=\exp\left\{\begin{bmatrix}\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{2r-J}\dfrac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\\ \left(\dfrac{\beta_1}{\beta_2-\beta_1}\right)^J(Z_2-1)^J\left((Z_1-1)+\dfrac{\beta_1(Z_2-1)}{\beta_2-\beta_1}\right)^{(r-J)}\left(\dfrac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\end{bmatrix} +\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(Z_2-1)^S\left(\frac{1-e^{-\beta_2k_2t}}{\beta_2k_2}\right)\right]\right\} \tag{15}$$

Taking $Z_2=1$, we get the probability generating function of the first buffer size distribution as

$$P(Z_1,t)=\exp\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}(-1)^{2r}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}{}^{k_1}C_r(Z_1-1)^r\frac{\left(1-e^{-r\beta_1t}\right)}{r\beta_1}\right] \tag{16}$$

Expanding the equation $P(Z_1,t)$ and collecting the constant terms, we get the probability that the first buffer is empty as

$$p_{0.}(t)=\exp\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}{}^{k_1}C_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1t}\right)}{r\beta_1}\right] \tag{17}$$

The utilization of the first node is

$$U_1=1-p_{0.}(t)$$

$$=1-\exp\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}{}^{k_1}C_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1t}\right)}{r\beta_1}\right] \tag{18}$$

The mean number of packets in the first buffer is

$$L_1=\frac{\alpha_1}{\beta_1}\left[\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}.k_1\left(1-e^{-\beta_1t}\right)\right] \tag{19}$$

Throughput of the first node is

$$Thp_1=\beta_1.U_1$$

$$=\beta_1\left[1-\exp\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}{}^{k_1}C_r(-1)^{3r}\frac{\left(1-e^{-r\beta_1t}\right)}{r\beta_1}\right]\right] \tag{20}$$

The average delay in the first buffer is

$$W(N_1)=\frac{L_1}{Thp_1}=\frac{\dfrac{\alpha_1}{\beta_1}\left[\sum_{k_1=1}^{m_1}\dfrac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}.k_1\left(1-e^{-\beta_1t}\right)\right]}{=\beta_1\left[1-\exp\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\dfrac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}{}^{k_1}C_r(-1)^{3r}\dfrac{\left(1-e^{-r\beta_1t}\right)}{r\beta_1}\right]\right]} \tag{21}$$

The variance of the number of packets in the first buffer is

$$Var(N_1)=E\left[N_1^2-N_1\right]+E\left[N_1\right]-\left(E\left[N_1\right]\right)^2$$

$$=\frac{\alpha_1}{2\beta_1}\left[\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}k_1(k_1-1)\left(1-e^{-2\beta_1t}\right)\right]+\frac{\alpha_1}{\beta_1}\left[\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}k_1\left(1-e^{-\beta_1t}\right)\right]$$

(22)

The coefficient of variation of the number of packets in the first buffer is

$$cv(N_1)=\frac{\sqrt{Var(N_1)}}{L_1} \tag{23}$$

Similarly, taking $Z_1=1$ in (15), we get the probability generating function of the second buffer size distribution as

$$P(Z_2,t)=\exp\left\{\begin{bmatrix}\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{2r-J}\dfrac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\\ \left(\dfrac{\beta_1}{\beta_2-\beta_1}\right)^r(Z_2-1)^r\left(\dfrac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\end{bmatrix} +\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(Z_2-1)^S\left(\frac{1-e^{-\beta_2k_2t}}{\beta_2k_2}\right)\right]\right\} \tag{24}$$

Expanding the equation $P(Z_2,t)$ and collecting the constant terms, we get the probability
that the second buffer is empty as

$$p_{.0}(t)=\exp\left\{\begin{bmatrix}\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\dfrac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\\ \left(\dfrac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\dfrac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\end{bmatrix} +\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\beta_2k_2t}}{\beta_2k_2}\right)\right]\right\} \tag{25}$$

The utilization of the second node is

$$U_2=1-p_{.0}(t)$$

$$U_2=1-\exp\left\{\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\right] +\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\beta_2k_2t}}{\beta_2k_2}\right)\right]\right\} \tag{26}$$

The mean number of packets in the second buffer is

$$L_2 = \frac{\alpha_1}{\beta_2}\left(\sum_{k_1=1}^{m_1} \frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}} k_1\right)\left[\left(1-e^{-\beta_2 t}\right)+\frac{\beta_2}{\beta_2-\beta_1}\left(e^{-\mu_2 t}-e^{-\mu_1 t}\right)\right]$$
$$+\frac{\alpha_2}{\beta_2}\left(\sum_{k_2=1}^{m_2} \frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}} k_2\right)\left(1-e^{-\beta_2 t}\right) \quad (27)$$

Throughput of the second node is

$$Thp_2 = \beta_2 . U_2$$

$$Thp_2 = \beta_2 . \left\{1-\exp\left\{\left[\alpha_1 \sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J} \frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}} ({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\right]\right.\right.$$
$$\left.\left.+\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^s\left(\frac{1-e^{-\mu_2 k_2 t}}{\mu_2 k_2}\right)\right]\right\}\right\} \quad (28)$$

The average delay in the second buffer is

$$W(N_2) = \frac{L_2}{Thp_2} = \frac{\frac{\alpha_1}{\beta_2}\left(\sum_{k_1=1}^{m_1} \frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}} k_1\right)\left[\left(1-e^{-\beta_2 t}\right)+\frac{\beta_2}{\beta_2-\beta_1}\left(e^{-\beta_2 t}-e^{-\beta_1 t}\right)\right]+\frac{\alpha_2}{\beta_2}\left(\sum_{k_2=1}^{\infty} \frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}} k_2\right)\left(1-e^{-\beta_2 t}\right)}{\beta_2 . \left\{1-\exp\left\{\left[\alpha_1 \sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J} \frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}} ({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\right] +\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^s\left(\frac{1-e^{-\beta_2 k_2 t}}{\beta_2 k_2}\right)\right]\right\}\right\}} \quad (29)$$

The variance of number of packets in the second buffer is

$$Var(N_2) = E\left[N_2^2 - N_2\right] + E[N_2] - \left(E[N_2]\right)^2$$

$$= \alpha_1\left\{\left(\sum_{k_1=1}^{m_1} \frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}} .k_1(k_1-1)\right)\left(\frac{\beta_1}{\beta_1-\beta_2}\right)^2\left[\left(\frac{1-e^{-2\beta_1 t}}{2\beta_1}\right)-2\left(\frac{1-e^{-(\beta_1+\beta_2)t}}{\beta_1+\beta_2}\right)+\left(\frac{1-e^{-2\beta_2 t}}{2\beta_2}\right)\right]\right\}$$
$$+\left\{\alpha_2\sum_{k_2=1}^{m_2} \frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}} k_2(k_2-1)\left(\frac{1-e^{-2\beta_2 t}}{2\beta_2}\right)\right\}+\left\{\alpha_1\left(\sum_{k_1=1}^{m_1} \frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}} .k_1\right)\left[\left(\frac{1-e^{-\beta_1 t}}{\beta_2}\right)-\left(\frac{e^{-\beta_2 t}-e^{-\beta_1 t}}{\beta_1-\beta_2}\right)\right]\right\}$$
$$+\left\{\frac{\alpha_2}{\beta_2}\left(\sum_{k_2=1}^{m_2} \frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}} .k_2\right)\left(1-e^{-\beta_2 t}\right)\right\} \quad (30)$$

The coefficient of variation of the number of packets in the second buffer is

$$cv(N_2) = \frac{\sqrt{Var(N_2)}}{L_2} \quad (31)$$

Expanding the equation (15) and collecting the constant terms, we get the probability that the network is empty as

$$p_{00}(t) = \exp\left\{\left[\alpha_1 \sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J} \frac{{}^{m_1}C_{k_1} p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}} ({}^{k_1}C_r)({}^{r}C_J)(\beta_1)^J \frac{(\beta_2)^{r-J}}{(\beta_2-\beta_1)^r}\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\right]\right.$$
$$\left.+\alpha_2\left[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2} p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^s\left(\frac{1-e^{-\beta_2 s t}}{\beta_2 s}\right)\right]\right\} \quad (32)$$

The mean number of packets in the network is

$$L_N = L_1 + L_2 \quad (33)$$

4. PERFORMANCE EVALUATION OF THE NETWORK

In this section, the performance of the proposed network is discussed through numerical illustration. Different values of the parameters are considered for bandwidth allocation and arrival of packets. Considered a communication network with two-stage bulk arrivals and dynamic bandwidth allocation β_1 and β_2 are the arrival rates of messages at buffer 1 and buffer 2 respectively. The number of packets that can be converted from a message varies from 1 to 4 depending on the length of the message. The number of arrivals of packets to the buffers is in batches of random size. The batch size of arrivals are assumed to follow binomial distribution with parameters (m_1, p_1) and (m_2, p_2) for buffer 1 and buffer 2respectively. The transmission rate of first node β_1 which varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec. the packets leave the second node with a transmission rate of β_2 which varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec. In both the nodes, dynamic bandwidth allocation is considered i.e. the transmission rate of each packet depends on the number of packets in the buffer connected to it at that instant.

Since the performance characteristics of the communication network are highly sensitive with respect to time, the transient behaviour of the model is studied through computing the performance measures with the following set of values for the model parameters:

t= 0.2, 0.4, 0.6, 0.8 seconds
m_1= 1, 2, 3, 4
p_1= 0.1, 0.2, 0.3, 0.4
m_2= 3, 4, 5, 6
p_2= 0.4, 0.5, 0.6, 0.7
α_1= 2, 3, 4, 5
(with multiplication of 10^4 packets/sec)
α_2= 0.5, 0.6, 0.7, 0.8
(with multiplication of 10^4 packets/sec)
β_1= 1, 2, 3, 4
(with multiplication of 10^4 packets/sec) and
β_2= 6, 7, 8, 9
(with multiplication of 10^4 packets/sec)

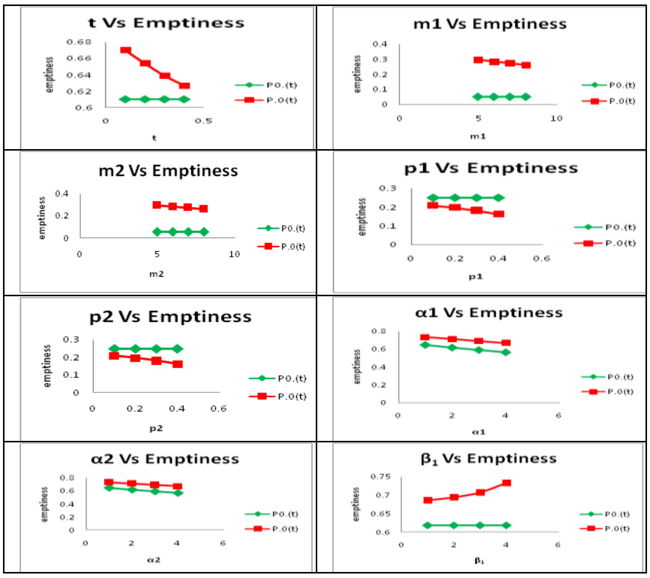
From equations (21) to (31), the transmission rate of first transmitter and the transmission rate of second transmitter are computed for different values of t, m_1, m_2, p_1, p_2, α_1, α_2, β_1 and β_2 given in Table 1.The relationship between the parameters and the probability of emptiness are shown in figure 2.

Table 1: Values of Network and Buffers Emptiness Probabilities of the Communication Network with Dynamic Bandwidth Allocation and Binomial Bulk Arrivals

t*	m_1	m_2	P_1	P_2	α_1 #	α_2 #	β_1 $	β_2 $	$P_{00}(t)$	$P_{0.}(t)$	$P_{.0}(t)$
0.2	2	3	0.4	0.8	1	0.8	2	6	0.765	0.610	0.670
0.4	2	3	0.4	0.8	1	0.8	2	6	0.740	0.610	0.654
0.6	2	3	0.4	0.8	1	0.8	2	6	0.713	0.610	0.639
0.8	2	3	0.4	0.8	1	0.8	2	6	0.693	0.610	0.627
1	**1**	3	0.4	0.8	1	0.8	2	6	0.602	0.372	0.519
1	**2**	3	0.4	0.8	1	0.8	2	6	0.469	0.227	0.438
1	**3**	3	0.4	0.8	1	0.8	2	6	0.398	0.138	0.369
1	**4**	3	0.4	0.8	1	0.8	2	6	0.360	0.084	0.311
1	2	**3**	0.4	0.8	1	0.8	2	6	0.221	0.051	0.295
1	2	**4**	0.4	0.8	1	0.8	2	6	0.216	0.051	0.284

1	2	**5**	0.4	0.8	1	0.8	2	6	0.200	0.051	0.273
1	2	**6**	0.4	0.8	1	0.8	2	6	0.189	0.051	0.262
1	2	3	**0.1**	0.8	1	0.8	2	6	0.364	0.150	0.339
1	2	3	**0.2**	0.8	1	0.8	2	6	0.349	0.105	0.252
1	2	3	**0.3**	0.8	1	0.8	2	6	0.329	0.101	0.223
1	2	3	**0.4**	0.8	1	0.8	2	6	0.301	0.098	0.211
1	2	3	0.4	**0.4**	1	0.8	2	6	0.443	0.249	0.207
1	2	3	0.4	**0.5**	1	0.8	2	6	0.407	0.249	0.196
1	2	3	0.4	**0.6**	1	0.8	2	6	0.381	0.249	0.181
1	2	3	0.4	**0.7**	1	0.8	2	6	0.362	0.249	0.162
1	2	3	0.4	0.8	**2**	0.8	2	6	0.957	0.756	0.559
1	2	3	0.4	0.8	**3**	0.8	2	6	0.863	0.732	0.542
1	2	3	0.4	0.8	**4**	0.8	2	6	0.787	0.664	0.537
1	2	3	0.4	0.8	**5**	0.8	2	6	0.734	0.620	0.529
1	2	3	0.4	0.8	1	**0.5**	2	6	0.746	0.649	0.733
1	2	3	0.4	0.8	1	**0.6**	2	6	0.721	0.619	0.713
1	2	3	0.4	0.8	1	**0.7**	2	6	0.698	0.592	0.691
1	2	3	0.4	0.8	1	**0.8**	2	6	0.677	0.566	0.670
1	2	3	0.4	0.8	1	0.8	**1**	6	0.759	0.618	0.686
1	2	3	0.4	0.8	1	0.8	**2**	6	0.761	0.618	0.694
1	2	3	0.4	0.8	1	0.8	**3**	6	0.772	0.618	0.707
1	2	3	0.4	0.8	1	0.8	**4**	6	0.790	0.618	0.733
1	2	3	0.4	0.8	1	0.8	2	**6**	0.878	0.643	0.636
1	2	3	0.4	0.8	1	0.8	2	**7**	0.883	0.656	0.641
1	2	3	0.4	0.8	1	0.8	2	**8**	0.896	0.668	0.657
1	2	3	0.4	0.8	1	0.8	2	**9**	0.909	0.689	0.672

*=Seconds, #=Multiples of 10, 000 messages/seconds, $=Multiples of 10,000 packets/second



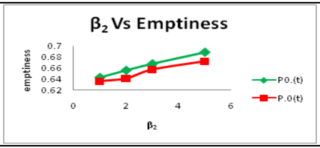


Figure 2: The relationship between probability of Emptiness and various parameters

It is observed that the probability of emptiness of the communication network and the two buffers are highly sensitive with respect to changes in time. As time (t) varies from 0.2 to 0.8 seconds, the probability of emptiness in the network reduces from 0.765 to 0.693 when other parameters are fixed at (2, 3, 0.4, 0.8,1, 0.8, 2, 6) for (m_1, m_2, p_1, p_2, α_1, α_2, β_1, β_2). Similarly, the probabilities of emptiness of the first buffer remain unchanged as 0.610 and the probabilities of emptiness of the first buffer reduce from 0.670 to 0.627 for first node and second node respectively.

When the batch size distribution parameter (m_1) varies from 1 to 4, the probability of emptiness of the network decreases from 0.602 to 0.360 when other parameters are fixed at (1, 3, 0.4, 0.8,1, 0.8, 2, 6) for (t, m_2, p_1, p_2, α_1, α_2, β_1, β_2). The same phenomenon is observed with respect to the first and second nodes. The probabilities of emptiness of the first and second buffers decrease from 0.372 to 0.084 and 0.519 to 0.311 respectively. When the batch size distribution parameter (m_2) varies from 3 to 6, the probability of emptiness of the network decreases from 0.221 to 0.189 when other parameters are fixed at (1, 2, 0.4, 0.8,1, 0.8, 2, 6) for (t, m_1, p_1, p_2, α_1, α_2, β_1, β_2). The probabilities of emptiness of the first buffer remains unchanged as 0.051 and the second buffers decrease from 0.295 to 0.265 respectively.

When the batch size distribution parameter (p_1) varies from 0.1 to 0.4, the probability of emptiness of the network decreases from 0.364 to 0.301 when other parameters are fixed at (1, 2, 3, 0.8, 1, 0.8, 2, 6) for (t, m_1, m_2, p_2, α_1, α_2, β_1, β_2). The same phenomenon is observed with respect to the first and second nodes. The probabilities of emptiness of the first and second buffers decrease from 0.150 to 0.098 and 0.339 to 0.211 respectively. When the batch size distribution parameter (p_2) varies from 0.4 to 0.7, the probability of emptiness of the network decreases from 0.443 to 0.362 when other parameters are fixed at (1, 2, 3, 0.4, 1, 0.8, 2, 6) for (t, m_1, m_2, p_1, α_1, α_2, β_1, β_2). The probabilities of emptiness of the first buffer remains unchanged as 0.249 and the second buffers decrease from 0.207 to 0.196 respectively.

The influence of arrival of messages on system emptiness is also studied. As the arrival rate (α_1) varies from $2x10^4$ messages/sec to $5x10^4$ message/sec, the probability of emptiness of the network decreases from 0.957 to 0.734 when other parameters are fixed at (1, 2, 3, 0.4, 0.8, 0.8, 2, 6) for (t, m_1, m_2, p_1, p_2, α_2, β_1, β_2). The same phenomenon is observed with respect to the first and second nodes. As the direct arrival rate at buffer 2 (α_2) varies from $0.5x10^4$ messages/sec to $0.8x10^4$ message/sec, the probability of emptiness of the network decreases from 0.746 to 0.677 when other parameters are fixed at (1, 2, 3, 0.4, 0.8, 1, 2, 6) for (t, m_1, m_2, p_1, p_2, α_1, β_1, β_2). The probabilities of emptiness of the first and second buffers decrease from 0.649 to 0.566 and 0.733 to 0.630 respectively.

When the transmission rate of first transmitter (β_1) varies from $1x10^4$ packet/sec to $4x10^4$ packet/sec, the probability of emptiness of the network and the second buffer increases from 0.759 to 0.790 and 0.686 to 0.733 respectively and the probability of emptiness of the first buffer remain unchanged as 0.618 when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 0.8, 6) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_2).

When the transmission rate of second transmitter (β_2) varies from $6x10^4$ packet/sec to $9x10^4$ packet/sec, the probability of emptiness of the network and the first and second buffers increases from 0.878 to 0.909, 0.643 to 0.689 and 0.636 to 0.672 respectively when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 0.8, 2) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_1).

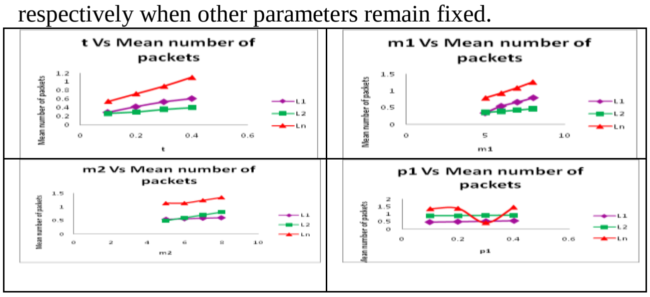
From the equations (15 to 26 and 27), the mean number of packets and the utilization of the network are computed for different values of t, m_1, m_2, p_1, p_2, α_1, α_2, β_1 and β_2 are given in Table 2. The relationship between mean number of packets in the buffers, utilization of the nodes with the parameters t, m_1, m_2, p_1, p_2, α_1, α_2, β_1 and β_2 is shown in figure 3. It is observed that after 0.2 seconds, the first buffer is having on an average of 2900 packets, after 0.4 seconds it rapidly raised to an average of 6110 packets for fixed values of other parameters (2, 3, 0.4, 0.8, 1, 0.8, 2, 6) for (m_1, m_2, p_1, p_2, α_1, α_2, β_1, β_2). It is also observed that as time (t) varies from 0.2 to 0.8 seconds, average content of the second buffer and the network increase from 2640 packets to 3990 packets and from 5400 to 11010 packets respectively.

Table 2: Values of mean number of packets and utilization of the communication network with dynamic bandwidth allocation and bulk arrivals

t*	m_1	m_2	P_1	P_2	α_1#	α_2#	$\beta_1$$	$\beta_2$$	L_1	U_1	L_2	U_2	L_n
0.2	2	3	0.4	0.8	1	0.8	2	6	0.290	0.244	0.264	0.141	0.540
0.4	2	3	0.4	0.8	1	0.8	2	6	0.421	0.268	0.297	0.218	0.718
0.6	2	3	0.4	0.8	1	0.8	2	6	0.535	0.336	0.359	0.263	0.894
0.8	2	3	0.4	0.8	1	0.8	2	6	0.611	0.380	0.399	0.291	1.101
1	**1**	3	0.4	0.8	1	0.8	2	6	0.332	0.351	0.355	0.267	0.787
1	**2**	3	0.4	0.8	1	0.8	2	6	0.540	0.381	0.388	0.287	0.928
1	**3**	3	0.4	0.8	1	0.8	2	6	0.662	0.408	0.425	0.309	1.087
1	**4**	3	0.4	0.8	1	0.8	2	6	0.795	0.434	0.466	0.330	1.261
1	2	**3**	0.4	0.8	1	0.8	2	6	0.541	0.381	0.488	0.319	1.128
1	2	**4**	0.4	0.8	1	0.8	2	6	0.560	0.392	0.593	0.336	1.133
1	2	**5**	0.4	0.8	1	0.8	2	6	0.582	0.401	0.698	0.353	1.239
1	2	**6**	0.4	0.8	1	0.8	2	6	0.598	0.425	0.805	0.367	1.345
1	2	3	**0.1**	0.8	1	0.8	2	6	0.455	0.357	0.885	0.364	1.340
1	2	3	**0.2**	0.8	1	0.8	2	6	0.480	0.364	0.893	0.369	1.373
1	2	3	**0.3**	0.8	1	0.8	2	6	0.509	0.372	0.901	0.373	0.410
1	2	3	**0.4**	0.8	1	0.8	2	6	0.540	0.381	0.911	0.378	1.452
1	2	3	0.4	**0.4**	1	0.8	2	6	0.396	0.396	0.561	0.330	1.137
1	2	3	0.4	**0.5**	1	0.8	2	6	0.402	0.396	0.647	0.346	1.223
1	2	3	0.4	**0.6**	1	0.8	2	6	0.428	0.396	0.737	0.361	1.313
1	2	3	0.4	**0.7**	1	0.8	2	6	0.439	0.396	0.829	0.373	1.406
1	2	3	0.4	0.8	**2**	0.8	2	6	1.153	0.396	1.100	0.481	2.252
1	2	3	0.4	0.8	**3**	0.8	2	6	1.729	0.628	1.277	0.562	3.006
1	2	3	0.4	0.8	**4**	0.8	2	6	2.306	0.773	1.454	0.631	3.760
1	2	3	0.4	0.8	**5**	0.8	2	6	2.882	0.862	1.632	0.689	4.519
1	2	3	0.4	0.8	1	**0.5**	2	6	3.459	0.949	1.530	0.705	4.498
1	2	3	0.4	0.8	1	**0.6**	2	6	3.459	0.949	1.623	0.716	5.082
1	2	3	0.4	0.8	1	**0.7**	2	6	3.459	0.949	1.716	0.727	5.175
1	2	3	0.4	0.8	1	**0.8**	2	6	3.459	0.949	1.809	0.738	5.268
1	2	3	0.4	0.8	1	0.8	**1**	6	5.057	0.985	1.583	0.661	6.640
1	2	3	0.4	0.8	1	0.8	**2**	6	3.459	0.949	1.902	0.748	5.361
1	2	3	0.4	0.8	1	0.8	**3**	6	2.534	0.889	2.042	0.777	4.576
1	2	3	0.4	0.8	1	0.8	**4**	6	1.963	0.820	2.105	0.789	4.068
1	2	3	0.4	0.8	1	0.8	2	**6**	1.589	0.751	2.134	0.793	3.723
1	2	3	0.4	0.8	1	0.8	2	**7**	1.589	0.751	1.138	0.744	3.427
1	2	3	0.4	0.8	1	0.8	2	**8**	1.589	0.751	1.612	0.699	3.202
1	2	3	0.4	0.8	1	0.8	2	**9**	1.589	0.751	1.435	0.658	3.025

*=Seconds, #=Multiples of 10, 000 messages/seconds, $=Multiples of 10,000 packets/second

As the batch size distribution parameter (m_1) varies from 1 to 4, the first buffer values increases from 3320 packets to 7950 packets, the second buffer and the network average content increases from 3550 packets to 4660 packets and 7870 packets to 12610 packets. As the batch size distribution parameter (m_2) varies from 3 to 6, the first buffer value increases from 5410 packets to 5940 packets and the second buffer increases from 4880 packets to 8050 packets, network average content increase from 11280 packets to 13450 packets respectively when other parameters remain fixed.



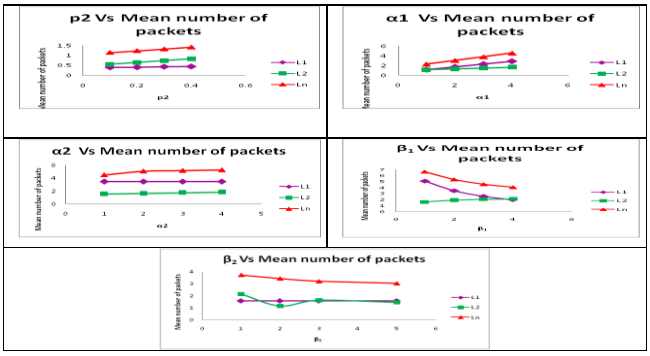


Fig 3: The relation between Mean number of packets and utilization for various parameters

As the batch size distribution parameter (p_1) varies from 0.1 to 0.4, the first buffer values increases from 4550 packets to 5400 packets, the second buffer and the network average content increases from 8850 packets to 9110 packets and 13400 packets to 14520 packets. As the batch size distribution parameter (p_2) varies from 0.4 to 0.7, the first buffer value increases from 3960 packets to 4390 packets and the second buffer value remain unchanged as 3960 packets, network average content increase from 11370 packets to 14060 packets respectively when other parameters remain fixed.

As the arrival rate of messages at node 1 (α_1) varies from $2x10^4$ messages/sec to $5x10^4$ messages/sec the first buffer, second buffer and the network average content increases from 11530 packets to 28820 packets, 11000 packets to 16320 packets and 22520 packets to 45190 packets respectively when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 0.8, 2, 6) for (t, m_1, m_2, p_1, p_2, α_2, β_1, β_2). As the arrival rate of messages at node 2 (α_2) varies from $0.5x10^4$ messages/sec to $0.8x10^4$ messages/sec, the first buffer value remains unchanged as 34590 packets and the second buffer content, network average content increase from 15300 packets to 18090 packets and from 44980 packets to 52680 packets respectively when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 2, 6) for (t, m_1, m_2, p_1, p_2, α_1, β_1, β_2).

As the transmission rate of first transmitter (β_1) varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec, the first buffer and the network average content decreases from 50570 packets to 19630 packets and from 66400 packets to 40680 packets respectively when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 0.8, 6) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_2). As the transmission rate of second transmitter (β_2) varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec, the second buffer and the network average content decreases from 21340 packets to 14350 packets and from 37230 packets to 30250 packets respectively when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 0.8, 2) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_1). It is revealed that the utilization characteristics are similar to mean number of packet characteristics. Here also, as the time (t) and the arrival rate of messages at buffer 1 (α_1) and buffer 2 (α_2) increases the utilization of both the nodes increases for fixed values of the other parameters. As the batch size distribution parameters (m_1) and (p_1) increases, the utilization of both the nodes increases when the other parameters are fixed. As the batch size distribution parameters (m_2) and (p_2) increases, the utilization of both the nodes increases when the other parameters are fixed. It is also noticed that as the transmission rate of first transmitter (β_1) and second transmitter (β_2) increases, the utilization of the second node increases while the utilization of the first node decreases when other parameters remain fixed.

From the equations (14 and 15), the throughput and average delay of the network are computed for different values of t, m_1, m_2, p_1, p_2, α_1, α_2, β_1, β_2 and are given in Table 3. The relationship between throughput, average delay and parameters is shown in figure 4 and figure 5. It is observed that as the time (t) increases from 0.2 seconds to 0.8 seconds, the throughput of the first and second nodes increase from 4750 packets to 7590 packets and from 12260 packets to 16460 packets respectively and there after stabilized when other parameters remain fixed at (2, 3, 0.4, 0.8, 1, 0.8, 2, 6) for (m_1, m_2, p_1, p_2, α_1, α_2, β_1, β_2). As the batch size distribution parameter (m_1) varies from 1 to 4, the throughput of the first and second node increases from 7020 packets to 8680 packets and 16000 packets to 19820 packets respectively when other parameters remain fixed. As the batch size distribution parameter (m_2) varies from 3 to 6, the throughput of the first node remain unchanged as 7610 packets and the throughput of the second node increases from 18850 packets to 22000 packets respectively when other parameters remain fixed. As the batch size distribution parameter (p_1) varies from 0.1 to 0.4, the first buffer values increases from 4550 packets to 5400 packets, the second buffer and the network average content increases from 8850 packets to 9110 packets and 13400 packets to 14520 packets. As the batch size distribution parameter (p_2) varies from 0.4 to 0.7, the first buffer value increases from 3960 packets to 4390 packets and the second buffer value remain unchanged as 3960 packets, network average content increases from 11370 packets to 14060 packets respectively when other parameters remain fixed.

As the arrival rate of messages at node 1 (α_1) varies from $2x10^4$ messages/sec to $5x10^4$ messages/sec the first buffer, second buffer and the network average content increase from 11530 packets to 28820 packets, 11000 packets to 16320 packets and 22520 packets to 45190 packets respectively when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 0.8, 2, 6) for (t, m_1, m_2, p_1, p_2, α_2, β_1, β_2). As the arrival rate of messages at node 2 (α_2) varies from $0.5x10^4$ messages/sec to $0.8x10^4$ messages/sec, the first buffer value remains unchanged as 34590 packets and the second buffer content, network average content increase from 15300 packets to 18090 packets and from 44980 packets to 52680 packets respectively when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 2, 6) for (t, m_1, m_2, p_1, p_2, α_1, β_1, β_2). As the transmission rate of first transmitter (β_1) varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec, the first buffer and the network average content decrease from 50570 packets to 19630 packets and from 66400 packets to 40680 packets respectively when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 0.8, 6) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_2). As the transmission rate of second transmitter (β_2) varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec, the second buffer and the network average content decrease from 21340 packets to 14350 packets and from 37230 packets to 30250 packets respectively when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 0.8, 2) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_1). It is revealed that the utilization characteristics are similar to mean number of packet characteristics. Here also, as the time (t) and the arrival rate of messages at buffer 1 (α_1) and buffer 2 (α_2) increases the utilization of both the nodes increases for fixed values of the other parameters. As the batch size distribution parameters (m_1) and (p_1) increase, the utilization of both the nodes increases when the other parameters are fixed. As the batch size distribution parameters (m_2) and (p_2) increases, the utilization of both the nodes increases when the other parameters are fixed.

It is also noticed that as the transmission rate of first transmitter (β_1) and second transmitter (β_2) increases, the utilization of the second node increase while the utilization of the first node decreases when other parameters remain fixed.

From the equations (14 and 15), the throughput and average delay of the network are computed for different values of t, m_1, m_2, p_1, p_2, α_1, α_2, β_1, β_2 and are given in Table 3. The relationship between throughput, average delay and parameters is shown in figure 4 and figure 5. It is observed that as the time (t) increases from 0.2 seconds to 0.8 seconds, the throughput of the first and second nodes increase from 4750 packets to 7590 packets and from 12260 packets to 16460 packets respectively and there after stabilized when other parameters remain fixed at (2, 3, 0.4, 0.8, 1, 0.8, 2, 6) for (m_1, m_2, p_1, p_2, α_1, α_2, β_1, β_2). As the batch size

distribution parameter (m_1) varies from 1 to 4, the throughput of the first and second node increases from 7020 packets to 8680 packets and 16000 packets to 19820 packets respectively when other parameters remain fixed. As the batch size distribution parameter (m_2) varies from 3 to 6, the throughput of the first node remain unchanged as 7610 packets and the throughput of the second node increases from 18850 packets to 22000 packets respectively when other parameters remain fixed.

Table 3: Values of Throughput and mean delay of the communication network with dynamic bandwidth allocation and bulk arrivals

t*	m_1	m_2	P_1	P_2	α_1 #	α_2 #	β_1 $	β_2 $	Thp_1	Thp_2	$W(N_1)$	$W(N_2)$
0.2	2	3	0.4	0.8	1	0.8	2	6	0.475	1.226	0.697	0.217
0.4	2	3	0.4	0.8	1	0.8	2	6	0.536	1.310	0.787	0.221
0.6	2	3	0.4	0.8	1	0.8	2	6	0.671	1.580	0.797	0.227
0.8	2	3	0.4	0.8	1	0.8	2	6	0.759	1.646	0.804	0.229
1	**1**	3	0.4	0.8	1	0.8	2	6	0.702	1.600	0.616	0.222
1	**2**	3	0.4	0.8	1	0.8	2	6	0.761	1.723	0.710	0.225
1	**3**	3	0.4	0.8	1	0.8	2	6	0.817	1.852	0.810	0.230
1	**4**	3	0.4	0.8	1	0.8	2	6	0.868	1.982	0.915	0.235
1	2	**3**	0.4	0.8	1	0.8	2	6	0.761	1.885	0.710	0.259
1	2	**4**	0.4	0.8	1	0.8	2	6	0.761	2.013	0.710	0.294
1	2	**5**	0.4	0.8	1	0.8	2	6	0.761	2.116	0.710	0.330
1	2	**6**	0.4	0.8	1	0.8	2	6	0.761	2.200	0.710	0.366
1	2	3	**0.1**	0.8	1	0.8	2	6	0.715	2.186	0.637	0.401
1	2	3	**0.2**	0.8	1	0.8	2	6	0.729	2.211	0.659	0.404
1	2	3	**0.3**	0.8	1	0.8	2	6	0.744	2.239	0.684	0.408
1	2	3	**0.4**	0.8	1	0.8	2	6	0.761	2.271	0.710	0.411
1	2	3	0.4	**0.4**	1	0.8	2	6	0.780	1.198	0.739	0.283
1	2	3	0.4	**0.5**	1	0.8	2	6	0.780	2.078	0.739	0.311
1	2	3	0.4	**0.6**	1	0.8	2	6	0.780	2.165	0.739	0.350
1	2	3	0.4	**0.7**	1	0.8	2	6	0.780	2.240	0.739	0.370
1	2	3	0.4	0.8	**2**	0.8	2	6	1.256	2.288	0.918	1.381
1	2	3	0.4	0.8	**3**	0.8	2	6	1.547	3.375	1.118	0.378
1	2	3	0.4	0.8	**4**	0.8	2	6	1.723	3.787	1.338	0.384
1	2	3	0.4	0.8	**5**	0.8	2	6	1.831	4.134	1.574	0.395
1	2	3	0.4	0.8	1	**0.5**	2	6	1.897	4.231	1.823	0.362
1	2	3	0.4	0.8	1	**0.6**	2	6	1.897	4.299	1.823	0.378
1	2	3	0.4	0.8	1	**0.7**	2	6	1.897	4.364	1.823	0.393
1	2	3	0.4	0.8	1	**0.8**	2	6	1.897	4.427	1.823	0.409
1	2	3	0.4	0.8	1	0.8	**1**	6	0.985	3.963	5.135	0.400
1	2	3	0.4	0.8	1	0.8	**2**	6	1.897	4.488	1.823	0.424
1	2	3	0.4	0.8	1	0.8	**3**	6	2.668	4.665	0.950	0.438
1	2	3	0.4	0.8	1	0.8	**4**	6	3.279	4.731	0.599	0.445
1	2	3	0.4	0.8	1	0.8	2	**6**	3.754	4.756	0.423	0.449
1	2	3	0.4	0.8	1	0.8	2	**7**	3.754	5.209	0.423	0.353
1	2	3	0.4	0.8	1	0.8	2	**8**	3.754	5.593	0.423	0.288
1	2	3	0.4	0.8	1	0.8	2	**9**	3.754	5.922	0.423	0.242

*=Seconds, #=Multiples of 10, 000 messages/seconds, $=Multiples of 10,000 packets/second

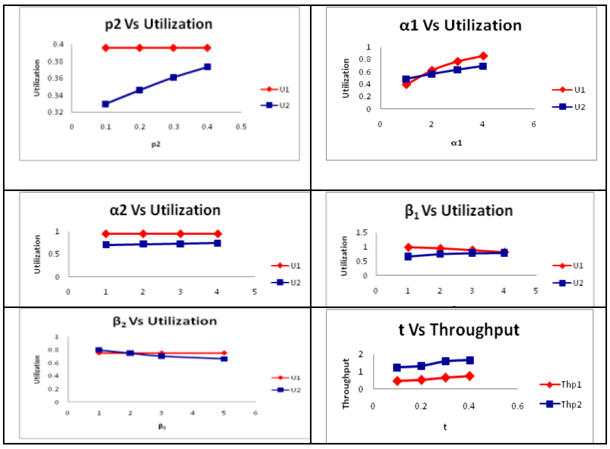


Fig 4: The relationship between Utilization, Throughput and various parameters

As the batch size distribution parameter (p_1) varies from 0.1 to 0.4, the throughput of the first and second nodes increase from 7150 packets to 7610 packets and 21860 packets to 22710 packets respectively when other parameters remain fixed. As the batch size distribution parameter (p_2) varies from 0.4 to 0.7, the throughput of the first node remain unchanged as 7800 packets and the throughput of the second node increase from 11980 packets to 22400 packets respectively when other parameters remain fixed. As the arrival rate of messages at buffer 1 (α_1) varies from $2x10^4$ messages/sec to $5x10^4$ messages/sec, it is observed that the throughput of the first and second nodes increase 12560 packets to 18310 packets and from 22880 packets to 41340 packets respectively, when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 0.8, 2, 6) for (t, m_1, m_2, p_1, p_2, α_2, β_1, β_2). As the arrival rate of messages at buffer 2 (α_2) varies from $0.5x10^4$ messages/sec to $0.8x10^4$ messages/sec, it is observed that the throughput of the first node remain unchanged as18970 packets and the throughput of the second node increases from 42310 packets to 44270 packets respectively, when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 2, 6) for (t, m_1, m_2, p_1, p_2, α_1, β_1, β_2).
As the transmission rate of first transmitter (β_1) varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec, the throughput of the first and second nodes increase from 9850 packets 32790 packets and from 39630 packets to 47310 packets respectively, when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 0.8, 6) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_2). As the transmission rate of second node (β_2) varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec, the throughput of second node increases from 47560 packets to 59220 packets, when other parameters remain fixed at (1, 2, 3, 0.4, 0.8, 1, 0.8, 2) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_1). From table 3, it is also observed that as time (t) varies from 0.2 to 0.8 seconds, the mean delay of the first and second buffers increase from 29.70 µs to 80.40 µs and from 21.70 µs to 22.90 µs respectively, when other parameters remain fixed (2, 3, 0.4, 0.8, 1, 0.8, 2, 6) for (m_1, m_2, p_1, p_2, α_1, α_2, β_1, β_2). As the batch size distribution parameter (m_1) varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec, the mean delay of the first and second buffers increase from 61.60 µs to 91.50 µs and 22.20 µs to 23.50 µs respectively when other parameters remain fixed at (1, 3, 0.4, 0.8, 1, 0.8, 2, 6) for (t, m_2, p_1, p_2, α_1, α_2, β_1, β_2). As the batch size distribution parameter (m_2) varies from $3x10^4$ packets/sec to $6x10^4$ packets/sec, the mean delay of the first buffer remain n unchanged as 71 µs and the mean delay of the second buffer increase from 25.90 µs to 36.60 respectively when other parameters remain fixed at (1, 2, 0.4, 0.8, 1, 0.8, 2, 6) for (t, m_1, p_1, p_2, α_1, α_2, β_1, β_2).
As the batch size distribution parameter (p_1) varies from $0.1x10^4$ packets/sec to $0.4x10^4$ packets/sec, the mean delay of the first and second buffers increase from 63.700 µs to 71.900 µs and 40.10 µs to 41.10 µs respectively when other parameters remain fixed at (1, 2, 3, 0.8, 1, 0.8, 2, 6) for (t, m_1, m_2, p_2, α_1, α_2, β_1, β_2). As the batch size distribution parameter (p_2) varies from $0.4x10^4$ packets/sec to $0.7x10^4$ packets/sec, the mean delay of the first buffer remain unchanged as 73.90 µs and the mean delay of the second buffer increases from 28.30 µs to 37 µs respectively when other parameters remain fixed at (1, 2, 3, 1, 0.8, 2, 6) for (t, m_1, m_2, p_1, α_1, α_2, β_1, β_2). When the arrival rate (α_1) varies from $2x10^4$ messages/sec to $5x10^4$ messages/sec, the mean delay of the first and second buffers increase from 91.80 µs to 15.74 µs and 13.81 µs to 39.50 µs respectively, when other parameters remain fixed at (1, 2, 3, 1, 0.4, 0.8, 2, 6) for (t, m_1, m_2, p_1, p_2, α_2, β_1, β_2). When the arrival rate (α_2) varies from $0.5x10^4$ messages/sec to $0.8x10^4$ messages/sec, the mean delay of the first buffer remain unchanged as 18.23 µs and the mean delay of the second buffer increase from 36.20µs to 40.90 µs respectively, when other parameters remain fixed at (1, 2, 3, 1, 0.4, 1, 2, 6) for (t, m_1, m_2, p_1, p_2, α_1, β_1, β_2). As the transmission rate of first transmitter (β_1) varies from $1x10^4$ packets/sec to $4x10^4$ packets/sec, the mean delay of the first buffer decreases from 513.5 µs to 59.90 µs and the mean delay of the second buffer increases from 40 µs to 44.50 µs, when other parameters remain fixed at (1, 2, 3, 1, 0.4, 1, 0.8, 6) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_2). As the transmission rate of second transmitter (β_2) varies from $6x10^4$ packets/sec to $9x10^4$ packets/sec, the mean delay of the first buffer remain unchanged as 42.30 µs and the mean delay of the second buffer decreases from 44.90 µs to 24.20 µs, when other parameters remain fixed at (1, 2, 3, 1, 0.4, 1, 0.8, 2) for (t, m_1, m_2, p_1, p_2, α_1, α_2, β_1).
If the variance increases then the burstness of the buffers will be high. Hence, the parameters are to be adjusted such that the variance of the buffer content in each buffer must be small. The coefficient of variation of the number of packets in each buffer helps us to understand the consistency of the traffic flow through buffers. If coefficient of variation is large then the flow is inconsistent. It also helps us to compare the smooth flow of packets in two or more nodes. It is observed that, as the time (t) and the batch size distribution parameters (m_1, m_2) increases, the variance of first and second buffers increase and the coefficient of variation of the number of packet in the first and second buffers decrease. As the batch size distribution parameters (p_1 and p_2) increase, the variance of first and second buffers are increasing and the coefficient of variation of the number of packets in the first buffer is increasing and for the second buffer is decreasing. As the arrival rate of packets at node 1 and node 2 (α_1 and α_2) increases, the variance of first and second buffers are increasing and the coefficient of variation of the number of packets in the first buffer is increasing and for the second buffer is decreasing. From this analysis it is observed that the dynamic bandwidth allocation strategy has a significant influence on all performance measures of the network. It is further observed that the performance measures are highly sensitive towards smaller values of time. Hence, it is optimal to consider bulk arrivals with dynamic bandwidth allocation and evaluate the performance under transient condition. It is also observed that the congestion in buffers and delays in transmission can be reduced to a minimum level by adopting dynamic bandwidth allocation. This phenomenon has a vital bearing on quality of transmission (service).

5. OPTIMAL POLICIES OF THE MODEL

In this section, we derive the optimal operating policies of the communication networks under study. Here, it is assumed that the service provider of the communication network is interested in maximization of the profit function at a given time t. Let the service provider gets an amount of R_i units per every unit of time of the system busy at i^{th} transmitter (i=1, 2). In other words, he gets revenue of R_i units per every unit of throughput of the i^{th} transmitter. Therefore, the total revenue of the communication network at time t is,

R (t) =R_1. (Throughput of first transmitter) + R_2. (Throughput of second transmitter)

(34)

$$R(t)=R_1.\beta_1\Bigg[1-\exp\Bigg[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\,{}^{k_1}C_r\,(-1)^{3r}\,\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\Bigg]\Bigg]$$

$$+R_2.\beta_2.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\Bigg]$$

$$+\alpha_2\Bigg[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\mu_2k_2t}}{\mu_2k_2}\right)\Bigg]\Bigg]\Bigg\}\Bigg\}$$

(35)

Let A is the set up cost for operating the communication network. C_1 is the penalty cost due to waiting of a customer in the first transmitter. C_2 is the penalty cost due to waiting of a customer in the second transmitter. Therefore, the total cost for operating the communication network at time t is,

C (t) = A- C_1*(Average waiting time of a customer in first transmitter) -C_2*(Average waiting time of a customer in second transmitter)

(36)

$$C(t)=A-C_1.\frac{\dfrac{\alpha_1}{\beta_1}\left[\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}.k_1\left(1-e^{-\beta_1 t}\right)\right]}{\beta_1\left[1-\exp\left[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\,{}^{k_1}C_r\,(-1)^{3r}\,\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]\right]}$$

$$-C_2.\frac{\dfrac{\alpha_1}{\beta_2}\left(\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}k_1\right)\left[\left(1-e^{-\beta_2 t}\right)+\frac{\beta_2}{\beta_2-\beta_1}\left(e^{-\beta_2 t}-e^{-\beta_1 t}\right)\right]+\dfrac{\alpha_2}{\beta_2}\left(\displaystyle\sum_{k_2=1}^{\infty}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}k_2\right)\left(1-e^{-\beta_2 t}\right)}{\beta_2.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\Bigg]+\alpha_2\Bigg[\displaystyle\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\beta_2k_2t}}{\beta_2k_2}\right)\Bigg]\Bigg]\Bigg\}\Bigg\}}$$

(37)

Substituting the values of R (t) and C (t) from equation (35) and (37) respectively we get total cost function as,

$$P(t)=R_1.\beta_1\Bigg[1-\exp\Bigg[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\,{}^{k_1}C_r\,(-1)^{3r}\,\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\Bigg]\Bigg]$$

$$+R_2.\beta_2.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\Bigg]$$

$$+\alpha_2\Bigg[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\mu_2k_2t}}{\mu_2k_2}\right)\Bigg]\Bigg]\Bigg\}\Bigg\}$$

$$-C_1.\frac{\dfrac{\alpha_1}{\beta_1}\left[\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}.k_1\left(1-e^{-\beta_1 t}\right)\right]}{=\beta_1\left[1-\exp\left[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\,{}^{k_1}C_r\,(-1)^{3r}\,\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]\right]}$$

$$-C_2.\frac{\dfrac{\alpha_1}{\beta_2}\left(\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}k_1\right)\left[\left(1-e^{-\beta_2 t}\right)+\frac{\beta_2}{\beta_2-\beta_1}\left(e^{-\beta_2 t}-e^{-\beta_1 t}\right)\right]+\dfrac{\alpha_2}{\beta_2}\left(\displaystyle\sum_{k_2=1}^{\infty}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}k_2\right)\left(1-e^{-\beta_2 t}\right)}{\beta_2.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\Bigg]+\alpha_2\Bigg[\displaystyle\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\beta_2k_2t}}{\beta_2k_2}\right)\Bigg]\Bigg]\Bigg\}\Bigg\}}$$

(38)

To obtain the optimal values of β_1 and β_2, maximizing P (t), with

respect to β_1 and β_2 and verify the hessian matrix implies

$$\frac{\partial P(t)}{\partial\beta_1}=0$$

$$\frac{\partial P(t)}{\partial\beta_1}=R_1.\beta_1\Bigg[1-\exp\Bigg[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\,{}^{k_1}C_r\,(-1)^{3r}\,\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\Bigg]\Bigg]$$

$$+R_2.\beta_2.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\Bigg]$$

$$+\alpha_2\Bigg[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\mu_2k_2t}}{\mu_2k_2}\right)\Bigg]\Bigg]\Bigg\}\Bigg\}$$

$$-C_1.\frac{\dfrac{\alpha_1}{\beta_1}\left[\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}.k_1\left(1-e^{-\beta_1 t}\right)\right]}{=\beta_1\left[1-\exp\left[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\,{}^{k_1}C_r\,(-1)^{3r}\,\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\right]\right]}$$

$$-C_2.\frac{\dfrac{\alpha_1}{\beta_2}\left(\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}k_1\right)\left[\left(1-e^{-\beta_2 t}\right)+\frac{\beta_2}{\beta_2-\beta_1}\left(e^{-\beta_2 t}-e^{-\beta_1 t}\right)\right]+\dfrac{\alpha_2}{\beta_2}\left(\displaystyle\sum_{k_2=1}^{\infty}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}k_2\right)\left(1-e^{-\beta_2 t}\right)}{\beta_2.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\Bigg]+\alpha_2\Bigg[\displaystyle\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\beta_2k_2t}}{\beta_2k_2}\right)\Bigg]\Bigg]\Bigg\}\Bigg\}}=0$$

(39)

$$\frac{\partial P(t)}{\partial\beta_2}=0$$

Implies

$$\frac{\partial P(t)}{\partial\beta_2}=R_1.\beta_1\Bigg[1-\exp\Bigg[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}\,{}^{k_1}C_r\,(-1)^{3r}\,\frac{\left(1-e^{-r\beta_1 t}\right)}{r\beta_1}\Bigg]\Bigg]$$

$$+R_2.\beta_2.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\frac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\Bigg]$$

$$+\alpha_2\Bigg[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\mu_2k_2t}}{\mu_2k_2}\right)\Bigg]\Bigg\}\Bigg\}$$

$$-C_1.\frac{\dfrac{\alpha_1}{\beta_1}\left[\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}.k_1\left(1-e^{-\beta_1t}\right)\right]}{=\beta_1\left[1-\exp\left[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}{}^{k_1}C_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\beta_1t}\right)}{r\beta_1}\right]\right]}$$

$$-C_2.\frac{\dfrac{\alpha_1}{\beta_2}\left(\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}k_1\right)\left[\left(1-e^{-\beta_2t}\right)+\dfrac{\beta_2}{\beta_2-\beta_1}\left(e^{-\beta_2t}-e^{-\beta_1t}\right)\right]+\dfrac{\alpha_2}{\beta_2}\left(\displaystyle\sum_{k_2=1}^{\infty}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}k_2\right)\left(1-e^{-\beta_2t}\right)}{\beta_2.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\dfrac{\beta_1}{\beta_2-\beta_1}\right)^r\left(\dfrac{1-e^{-[J\beta_2+(r-J)\beta_1]t}}{J\beta_2+(r-J)\beta_1}\right)\Bigg]+\alpha_2\Bigg[\displaystyle\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\dfrac{1-e^{-\beta_2k_2t}}{\beta_2k_2}\right)\Bigg]\Bigg\}\Bigg\}}=0$$

(40)

The determinant of the Hessian matrix is,

$$|D|=\begin{vmatrix}\dfrac{\partial^2P(t)}{\partial\beta_1^{\,2}} & \dfrac{\partial^2P(t)}{\partial\beta_1\partial\beta_2}\\ \dfrac{\partial^2P(t)}{\partial\beta_1\partial\beta_2} & \dfrac{\partial^2P(t)}{\partial\beta_2^{\,2}}\end{vmatrix}<0$$

(41)

Solving the equations (39) and (40) with respect to $\beta_1$and β_2 and verifying the condition (41), for the given parameters of α_1, α_2, m_1, m_2, p_1, p_2 and t we get the optimal values of transmission rates at transmitter 1 and transmitter 2 as β_1^* and β_2^*respectively.

Substituting the values of β_1^* and β_2^* in equation (40), we get the optimal value of the profit at given time t as

$$P^*(t)=R_1.\beta_1^*\left[1-\exp\left[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}{}^{k_1}C_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\beta_1^*t}\right)}{r\beta_1^*}\right]\right]$$

$$+R_2.\beta_2^*.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\frac{\beta_1^*}{\beta_2^*-\beta_1^*}\right)^r\left(\frac{1-e^{-[J\beta_2^*+(r-J)\beta_1^*]t}}{J\beta_2^*+(r-J)\beta_1^*}\right)\Bigg]$$

$$+\alpha_2\Bigg[\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\frac{1-e^{-\beta_2^*k_2t}}{\beta_2^*k_2}\right)\Bigg]\Bigg\}\Bigg\}$$

$$-C_1.\frac{\dfrac{\alpha_1}{\beta_1^*}\left[\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}.k_1\left(1-e^{-\beta_1^*t}\right)\right]}{\beta_1^*\left[1-\exp\left[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}{}^{k_1}C_r\left(-1\right)^{3r}\frac{\left(1-e^{-r\beta_1^*t}\right)}{r\beta_1^*}\right]\right]}$$

$$-C_2.\frac{\dfrac{\alpha_1}{\beta_2^*}\left(\displaystyle\sum_{k_1=1}^{m_1}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}k_1\right)\left[\left(1-e^{-\beta_2^*t}\right)+\dfrac{\beta_2^*}{\beta_2^*-\beta_1^*}\left(e^{-\beta_2^*t}-e^{-\beta_1^*t}\right)\right]+\dfrac{\alpha_2}{\beta_2^*}\left(\displaystyle\sum_{k_2=1}^{\infty}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}k_2\right)\left(1-e^{-\beta_2^*t}\right)}{\beta_2^*.\Bigg\{1-\exp\Bigg\{\Bigg[\alpha_1\displaystyle\sum_{k_1=1}^{m_1}\sum_{r=1}^{k_1}\sum_{J=0}^{r}(-1)^{3r-J}\frac{{}^{m_1}C_{k_1}p_1^{k_1}(1-p_1)^{m_1-k_1}}{1-(1-p_1)^{m_1}}({}^{k_1}C_r)({}^{r}C_J)\left(\dfrac{\beta_1^*}{\beta_2^*-\beta_1^*}\right)^r\left(\dfrac{1-e^{-[J\beta_2^*+(r-J)\beta_1^*]t}}{J\beta_2^*+(r-J)\beta_1^*}\right)\Bigg]+\alpha_2\Bigg[\displaystyle\sum_{k_2=1}^{m_2}\sum_{s=1}^{k_2}\frac{{}^{m_2}C_{k_2}p_2^{k_2}(1-p_2)^{m_2-k_2}}{1-(1-p_2)^{m_2}}({}^{k_2}C_s)(-1)^S\left(\dfrac{1-e^{-\beta_2^*k_2t}}{\beta_2^*k_2}\right)\Bigg]\Bigg\}\Bigg\}}$$

(42)

6. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

In this section, we demonstrate the solution procedure through a numerical illustration. Consider the service provider to a communication network is operating a two-transmitter tandem communication network. Let the estimated cost of revenue per unit throughput of transmitter 1 is R_1 varies from 0.3 to 0.6 and the revenue per unit of throughput of transmitter 2 R_2 varies from 0.3 to 0.6. It is estimated the composite arrival rate of messages per unit time at node 1 is α_1 varies from 0.5 to 0.8 and at node 2 is α_2 varies from 0.3 to 0.6. The batch size distribution parameters of the number of packets that are generated from a message are considered to varies from 1 to 4 for both m_1 and m_2 and the other distribution parameters varies from 0.2 to 0.5 and 0.3 to 0.6 for both p_1 and p_2. Let the penalty cost per a packet waiting time at transmitter 1 for transmission per unit time is C_1 varies from 0.06 to 0.09. The penalty cost per a packet waiting time at transmitter 2 for transmission per unit time is C_2 varies from 0.07 to 0.10. With these costs and parameters, the optimal values of transmission rates β_1 and β_2 are obtained using Newton Ramp sons Method and Mathcad. The optimal value of transmission rate of first transmitter and transmission rate of second transmitter, optimal profit parameters are computed and shown in Table 5.

It is observed that the transmission rate of first transmitter (β_1) and the transmission rate of second transmitter (β_2) are highly sensitive with respect to changes in time. As time (t) varies from 0.4 to 0.7 seconds, the transmission rate of first transmitter (β_1) reduces from 3.498 to 3.381, the transmission rate of second transmitter (β_2) reduce from 3.380 to 3.450. The total optimal cost function decreases from 3.015 to 3.051 when other parameters are fixed at (2, 3, 0.1, 0.2, 0.5, 0.4, 0.5, 0.5, 0.1, 0.2) for (m_1, m_2, p_1, p_2, α_1, α_2, R_1, R_2, C_1, C_2). When the batch size distribution parameter number of packets a message can be converted (m_1) varies from $1\text{x}10^4$ packets/sec to $4\text{x}10^4$ packets/sec, the transmission rate of first transmitter (β_1) increases from 2.387 to 3.125 and the transmission rate of second transmitter (β_2) increases from 3.392 to 3.511 and the total optimal cost function increases from 2.466 to 2.790 when other parameters are fixed at (0.5, 3, 0.1, 0.2, 0.5, 0.4, 0.5, 0.5, 0.1, 0.2) for (t, m_2, p_1, p_2, α_1, α_2, R_1, R_2, C_1, C_2). When the batch size distribution parameter (m_2) varies from $0.3\text{x}10^4$ packets/sec to $0.6\text{x}10^4$ packets/sec, the transmission rate of first transmitter (β_1) increases from 3.013 to 3.697, the transmission rate of second transmitter (β_2) remain unchanged as 3.442 when other parameters are fixed at (0.5, 3, 0.1, 0.2, 0.5, 0.4, 0.5, 0.5, 0.1, 0.2) for (t, m_1, p_1, p_2, α_1, α_2, R_1, R_2, C_1, C_2).

As the arrival rate of node 1 (α_1) varies from $0.5\text{x}10^4$ messages/sec to $0.8\text{x}10^4$ messages/sec, the transmission rate of first transmitter (β_1) increases from 3.421 to 3.612, the transmission rate of second transmitter (β_2) increases from 3.900 to 4.008 and the total optimal cost function increases from 3.228 to 3.697 when other parameters are fixed at (0.5, 3, 0.1, 0.2, 0.5, 0.4, 0.5, 0.5, 0.1, 0.2) for (t, m_1, m_2, p_1, p_2, α_2, R_1, R_2, C_1, C_2). As the arrival rate of node 2 (α_2) varies from $0.3\text{x}10^4$ messages/sec to $0.6\text{x}10^4$ messages/sec, the transmission rate of first transmitter (β_1) increases from 3.721 to 4.010, the transmission rate of second transmitter (β_2) increases from 3.938 to 4.010 and the total optimal cost function increases from 3.391 to 3.611 when other parameters are fixed at (0.5, 3, 0.1, 0.2, 0.5, 0.4, 0.5, 0.5, 0.1, 0.2) for (t, m_1, m_2, p_1, p_2, α_1, R_1, R_2, C_1, C_2). When the revenue parameter (R_1) varies from 0.3 to 0.6, the transmission rate of first transmitter (β_1*) increases from 3.852 to 3.956, the transmission rate of second transmitter (β_2*) increases from 3.922 to 3.987 and the total optimal cost function increases from 3.445 to 3.519 when other parameters are fixed at (0.5, 3, 0.1, 0.2, 0.5, 0.4, 0.5, 0.5, 0.1, 0.2) for (t, m_1, m_2, p_1, p_2, α_1, α_2, R_2, C_1, C_2).

Table 5: Numerical representation of Optimal Values of β_1*and β_2*

t	m_1	m_2	p_1	p_2	α_1	α_2	R_1	R_2	C_1	C_2	β_1*	β_2*	R	D_1
0.4	2	3	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.498	3.380	3.015	-0.56
0.5	2	3	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.461	3.449	3.038	-0.56
0.6	2	3	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.426	3.473	3.045	-0.56
0.7	2	3	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.381	3.450	3.051	-0.56
0.5	**1**	3	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	2.387	3.392	2.466	-0.92
0.5	**2**	3	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	2.669	3.405	2.639	-0.92
0.5	**3**	3	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	2.973	3.475	2.829	-0.92
0.5	**4**	3	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.125	3.511	2.920	-0.92
0.5	2	**1**	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.293	3.180	2.863	-0.65
0.5	2	**2**	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.451	3.335	2.999	-0.66
0.5	2	**3**	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.631	3.450	3.125	-0.65
0.5	2	**4**	0.1	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.876	3.568	3.271	-0.65
0.5	2	3	**0.2**	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.874	3.220	3.159	-0.18
0.5	2	3	**0.3**	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.942	3.224	3.190	-0.18
0.5	2	3	**0.4**	0.2	0.5	0.4	0.5	0.5	0.1	0.2	3.998	3.229	3.216	-0.18
0.5	2	3	**0.5**	0.2	0.5	0.4	0.5	0.5	0.1	0.2	4.084	3.325	3.293	-0.18
0.5	2	3	0.1	**0.3**	0.5	0.4	0.5	0.5	0.1	0.2	3.013	3.442	2.827	-0.46
0.5	2	3	0.1	**0.4**	0.5	0.4	0.5	0.5	0.1	0.2	3.214	3.442	2.936	-0.49
0.5	2	3	0.1	**0.5**	0.5	0.4	0.5	0.5	0.1	0.2	3.458	3.442	3.060	-0.49
0.5	2	3	0.1	**0.6**	0.5	0.4	0.5	0.5	0.1	0.2	3.697	3.442	3.174	-0.49
0.5	2	3	0.1	0.2	**0.5**	0.4	0.5	0.5	0.1	0.2	3.421	3.900	3.228	-0.82
0.5	2	3	0.1	0.2	**0.6**	0.4	0.5	0.5	0.1	0.2	3.487	3.922	3.270	-0.82
0.5	2	3	0.1	0.2	**0.7**	0.4	0.5	0.5	0.1	0.2	3.564	3.965	3.326	-0.83
0.5	2	3	0.1	0.2	**0.8**	0.4	0.5	0.5	0.1	0.2	3.612	4.008	3.367	-0.82
0.5	2	3	0.1	0.2	0.5	**0.3**	0.5	0.5	0.1	0.2	3.721	3.938	3.391	-0.27
0.5	2	3	0.1	0.2	0.5	**0.4**	0.5	0.5	0.1	0.2	3.875	4.011	3.492	-0.27
0.5	2	3	0.1	0.2	0.5	**0.5**	0.5	0.5	0.1	0.2	3.961	4.083	3.561	-0.27
0.5	2	3	0.1	0.2	0.5	**0.6**	0.5	0.5	0.1	0.2	4.010	4.153	3.611	-0.27
0.5	2	3	0.1	0.2	0.5	0.4	**0.3**	0.5	0.1	0.2	3.852	3.922	3.445	-0.31
0.5	2	3	0.1	0.2	0.5	0.4	**0.4**	0.5	0.1	0.2	3.875	3.944	3.465	-0.31
0.5	2	3	0.1	0.2	0.5	0.4	**0.5**	0.5	0.1	0.2	3.906	3.965	3.487	-0.31

When the revenue parameter (R_2) varies from 0.3 to 0.6, the transmission rate of first transmitter (β_1*) increases from 3.851 to 3.976, the transmission rate of second transmitter (β_2*) increases from 3.907 to 4.050 and the total optimal cost function increases from 3.438 to 3.554 when other parameters are fixed at (0.5, 3, 0.1, 0.2, 0.5, 0.4, 0.5, 0.5, 0.1, 0.2) for (t, m_1, m_2, p_1, p_2, α_1, α_2, R_1, C_1, C_2).

7. SENSITIVITY ANALYSIS:

The sensitivity analysis of the Transmission Rate parameters β_1* and β_2*, and the total cost function p*(t) are studied with respect to the parameters t, m_1, m_2, p_1, p_2, α_1, α_2, R_1, R_2, C_1 and C_2.

Table 6: **Sensitivity Analysis**

Parameter	Performance Measure	% change in parameters						
		-15%	-10%	-5%	0	+5%	+10%	+15%
t=0.7	β_1*	4.013	4.052	4.087	4.105	4.126	4.152	4.178
	β_2*	3.950	3.970	3.991	4.011	4.031	4.051	4.071
	R	3.581	3.605	3.628	3.643	3.660	3.683	3.697
P_1=0.3	β_1*	3.724	3.789	3.818	3.851	3.885	3.910	3.942
	β_2*	3.897	3.920	3.942	3.964	3.986	4.007	4.029
	R	3.442	3.479	3.501	3.524	3.547	3.566	3.589
P_2=0.2	β_1*	3.896	3.924	3.952	3.976	3.998	4.018	4.036
	β_2*	3.888	3.923	3.957	3.991	4.024	4.057	4.090
	R	3.508	3.535	3.561	3.585	3.608	3.630	3.651
α_1 =0.5	β_1*	3.862	3.889	3.922	3.942	3.958	3.982	4.003
	β_2*	3.856	3.880	3.905	3.929	3.953	3.977	4.001
	R	3.481	3.502	3.526	3.545	3.561	3.581	3.600
α_2=0.4	β_1*	3.796	3.823	3.849	3.878	3.892	3.918	3.947
	β_2*	3.763	3.794	3.825	3.855	3.885	3.915	3.945
	R	3.413	3.438	3.462	3.487	3.505	3.529	3.554
R_1=0.7	β_1*	3.781	3.804	3.831	3.862	3.886	3.911	3.937
	β_2*	3.803	3.827	3.851	3.875	3.899	3.922	3.946
	R	3.425	3.445	3.466	3.489	3.509	3.529	3.550
C_1=0.4	β_1*	3.851	3.898	3.925	3.968	3.987	4.019	4.041
	β_2*	3.854	3.879	3.903	3.927	3.951	3.974	3.998
	R	3.476	3.505	3.526	3.554	3.572	3.592	3.613
		-75%	-50%	-25%	0	+25%	+50%	+75%
m_1=1	β_1*	3.487	3.503	3.521	3.548	3.561	3.578	3.591
	β_2*	3.415	3.427	3.440	3.452	3.464	3.477	3.489
	R	3.146	3.159	3.174	3.193	3.204	3.219	3.230
m_2=3	β_1*	3.202	3.239	3.261	3.286	3.307	3.331	3.364
	β_2*	3.253	3.299	3.344	3.389	3.433	3.476	3.518
	R	2.927	2.966	2.997	3.030	3.060	3.091	3.127

Sensitivity analysis of the model is carried out with respect to the parameters t, m_1, m_2, p_1, p_2, α_1, α_2, R_1, R_2, C_1 and C_2 on the transmission rate of first transmitter (β_1) and the transmission rate of second transmitter (β_2).

The following data has been considered for the sensitivity analysis.

t = 0.7 sec, m_1=1, m_2=3, p1=0.3, p_2=0.2, $\alpha_1 = 0.5\text{x}10^4$ packets/sec, $\alpha_2 = 0.4\text{x}10^4$ packets/sec, R_1 = 0.7, R2 = 0.6, C_1 =0.4 and C_2 =0.2.

The performance measure of the model is computed by variation of -15%, -10%, -5%, 0%, +5%, +10% and +15% on the input parameters t, p, α, θ, R_1, R_2, C_1 and C_2 and -75%,

-50%, -25%, 0%, +25%, +50% and +75% on the batch size distribution parameters m_1 and m_2 to retain them as integers. The computed values of the performance measures are given in Table 6. The performance measures are highly affected by varying the time (t) and the batch size distribution parameters of arrivals, a time (t) increases to 15% the average number of packets transmitting through the two buffers increases along with the two transmitters. Similarly, as the arrival rate of messages (α_1 and α_2) increases by 15%, the average number of packets transmitted through two transmitter's increases. Over all analysis of the parameters reflects that, dynamic bandwidth allocation strategy for congestion control tremendously reduces the mean delay in communication and improve voice quality by reducing burstness in buffers.

The sensitivity analysis of the Transmission rate parameters β_1* and β_2*, Arrival rates α_1 and α_2 and the total cost function p*(t) are studied with respect to the parameters t, m_1, m_2, p_1, p_2, R_1, R_2, C_1 and C_2. Sensitivity analysis of the model is performed with respect to the parameters t, m_1, m_2, p1, p_2, R_1, R_2, C_1, and C_2 on the arrival rates (α_1 and α_2), transmission rate of first transmitter (β_1) and the transmission rate of second transmitter (β_2). The performance measures are highly affected with the variation in time (t) and the batch size distribution parameters of arrivals. As time (t) increases by 15% the average number of packets transmitting through the two buffers increases along with the two transmitters and the arrival rate of the packets increases. As the batch size distribution parameter p increases to 15%, the average number of packets transmitting through the two buffers increases along with the two transmitters and the arrival rate of the packets increases. Over all analysis of the parameters reflects that dynamic bandwidth allocation strategy for congestion control tremendously reduces the mean delay in communication and improve voice quality by reducing burstness in buffers.

8. CONCLUSION

This paper deals with a novel and new communication network model with bulk arrivals having two stage direct arrivals. Here it is assumed that the messages arrive directly to the first buffer and second buffer which are connected in tandem. Further it is assumed that the messages are converted into a random number of packets and stored in buffers for forward transmission. The arrival processes in both the buffers are characterized with compound Poisson binomial processes. The explicit expressions for the characteristics of the communication network model such as content of the buffers, throughput of nodes, mean delays in transmission, utilization of transmitters are derived. The sensitivity analysis of the model revealed that the direct arrivals for second node have significant influence in reducing the burstness in buffers and mean delays in transmission. With suitable cost considerations, the optimal operating policies of the communication network are also derived. Through numerical illustrations the sensitivity of the changes in input parameters and costs on the optimal operating policies is also studied. This communication network is much useful foe performance evaluation, control and monitoring of communication networks at data/voice transmissions, satellite communications, LAN, WAN scheduling and internet providers. This communication network model can also be extended to non-Markovian transformation processes which require further investigations.

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