Domination in Operations on Intuitionistic Fuzzy Graphs

J. John Stephan
Assistant professor,
Department of Mathematics
Dhanalakshmi srinivasan Engineering college,
Perambalur

A. Muthaiyan
Assistant professor
P.G& Research department of Mathematics,
Government Arts College.
Ariyalur

N. Vinoth Kumar
Assistant professor,
Department of Mathematics,
M.A.M. School of Engineering,
Trichy

ABSTRACT
In this paper we discuss several operations on intuitionistic fuzzy graph such as union, join, composition, Cartesian product and study their domination parameters.

Keywords
Intuitionistic fuzzy graph , domination , domination number

1. INTRODUCTION
The first definition of fuzzy graphs was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram [6] and A. Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs . C. Natarajan and S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph [2]. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov [1]. The concept of domination in intuitionistic fuzzy graphs was investigated by R.parvathi and G.Thamizhendhi [8]. In this paper develop the concept of Domination in operations inuitionistic fuzzy graph.

2. DEFINITIONS
An intuitionistic fuzzy graph (IFG) is of the form G=(V,E) , where V={v_1,v_2,…,v_n} such that $\mu_1 : V \to [0,1]$, $\gamma_1 : V \to \{0,1\}$ denote the degree of membership and nonmember ship of the element $v_i \in V$ respectively and $0 \le \mu_1 + \gamma_1 \le 1$ for every $v_i \in V$,(i=1,2,…n) (ii) E ⊆ V×V where $\mu_2 : V \times V \to [0,1]$ and $\gamma_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i v_j) \le \mu_1(v_1) \wedge \mu_1(v_j),$$
$$\gamma_2(v_i v_j) \le \gamma_1(v_1) \vee \gamma_1(v_j), \text{ and}$$
$$0 \le \mu_2(v_i v_j) + \gamma_2(v_i v_j) \le 1.$$

An arc (v_i, v_j) of an IFG G is called an strong arc if $\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j)$, $\gamma_2(v_i, v_j) = \gamma_1(v_i) \vee \gamma_1(v_j)$.

Let G=(V,E) be an IFG on V. Let $u, v \in V$ we say that u dominates v in G if

$$\mu_2(v_i v_j) = \mu_1(v_1) \wedge \mu_1(v_j),$$
$$\gamma_2(v_i v_j) = \gamma_1(v_1) \vee \gamma_1(v_j)$$

A sub set S of V is called a dominating set in G if for every $v \in V - S$, there exist $u \in S$ such that u dominates v.

The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$ or γ.

A vertex u of an IFG is said to be an isolated vertex if $\mu_2(v_i v_j) < \mu_1(v_1) \wedge \mu_1(v_j)$, $\gamma_2(v_i v_j) < \gamma_1(v_1) \vee \gamma_1(v_j)$ for all $v \in V - \{u\}$.

Let $G = (V, E)$ be an IFG on V. A subset S of V is said to be an independent set if $\mu_2(v_i v_j) < \mu_1(v_1) \wedge \mu_1(v_j)$, $\gamma_2(v_i v_j) < \gamma_1(v_1) \vee \gamma_1(v_j)$ for all $u, v \in S$.S is said to be a maximal independent set if $S \cup \{v\}$ is not an independent set for any $v \in V - S$.

The maximum cardinality of an independent set in G is called the independence number of G is denoted by $\beta_0(G)$ .the maximum fuzzy cardinality of an independent dominating ser of G is called the independent dominating number of G and is denoted by $\gamma_i(G)$.

3. MAIN RESULT
3.1 Union of IFG
Let G_1 =(V_1,E_1) and G_2 =(V_2,E_2) be intuitionistic fuzzy graphs on V_1,V_2 respectively with $V_1 \cap V_2 = \phi$. The union of G_1 and G_2 denoted by $G_1 + G_2$, is the intuitionistic fuzzy graph G on $V_1 \cup V_2$ defined by $G = (G_1 \cup G_2) = ((\mu_1 \cup \mu_1'), (\gamma_1 \cup \gamma_1'), (\mu_2 \cup \mu_2'), (\gamma_2 \cup \gamma_2'))$ where

$$(\mu_1 \cup \mu_1')(u) = \begin{cases} \mu_1(u) \text{ if } u \in V_1 \\ \mu_1'(u) \text{ if } u \in V_2 \end{cases}$$

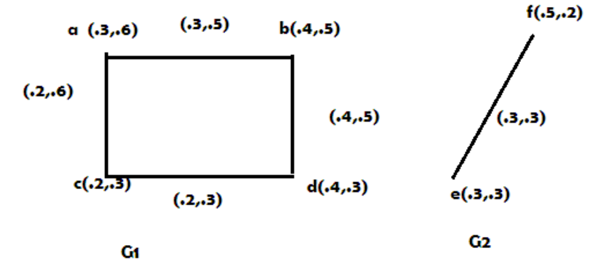
$$(\gamma_1 \cup \gamma_1^{'})(u) = \begin{cases} \gamma_1(u) \ if \ u \in V_1 \\ \gamma_1^{'}(u) \ if \ u \in V_2 \end{cases}$$

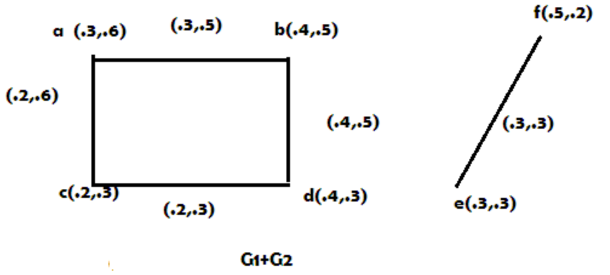
$$(\mu_2 \cup \mu_2^{'})(uv) = \begin{cases} \mu_2(uv) \ if \ \ uv \in E_1 \\ \mu_2^{'}(uv) \ if \ \ uv \in E_2 \\ 0 \ otherwise \end{cases}$$ and

$$(\gamma_2 \cup \gamma_2^{'})(uv) = \begin{cases} \gamma_2(uv) \ if \ \ uv \in E_1 \\ \gamma_2^{'}(uv) \ if \ \ uv \in E_2 \\ 0 \ otherwise \end{cases}$$

Remark: since dominating set D of $G_1 \cup G_2$ is of the form $D = D_1 \cup D_2$, where D_1 is the dominating set of G_1 and D_2 is the dominating set of G_2, it follows that $\gamma(G_1 \cup G_2) = \gamma(G_1) + \gamma(G_2)$.

Example:





In Fig. the dominating set of G_1={a,d}and $\gamma(G_1) = (.7,.9)$

In Fig. the dominating set of G_2={e}and $\gamma\ (G_2) = (.3,.3)$.

The dominating set of G_1+G_2 is{a,d,e} and $\gamma\ (G_2 + G_2) = (.7,.9)$

3.2 Join of IFG

Let G_1 =(V_1,E_1) and G_2 =(V_2,E_2) be intuitionistic fuzzy graphs on V_1,V_2 respectively with $V_1 \cap V_2 = \phi$. The join of G_1 and G_2 is the intuitionistic fuzzy graph G on $V_1 \cup V_2$ defined by

$$G = (G_1 + G_2) = ((\mu_1 + \mu_1^{'}), (\gamma_1 + \gamma_1^{'}), (\mu_2 + \mu_2^{'}), (\gamma_2 + \gamma_2^{'}))$$

where

$$(\mu_1 + \mu_1^{'})(u) = \begin{cases} \mu_1(u) \ if \ u \in V_1 \\ \mu_1^{'}(u) \ if \ u \in V_2 \end{cases}$$

$$(\gamma_1 + \gamma_1^{'})(u) = \begin{cases} \gamma_1(u) \ if \ u \in V_1 \\ \gamma_1^{'}(u) \ if \ u \in V_2 \end{cases}$$

$$(\mu_2 + \mu_2^{'})(uv) = \begin{cases} \mu_2(uv) \ if \ \ uv \in E_1 \\ \mu_2^{'}(uv) \ if \ \ uv \in E_2 \\ \mu_1(u) \wedge \mu_1^{'}(v) \ if \ \ u \in V_1 \ \& v \in V_2 \end{cases}$$

and

$$(\gamma_2 + \gamma_2^{'})(uv) = \begin{cases} \gamma_2(uv) \ if \ \ uv \in E_1 \\ \gamma_2^{'}(uv) \ if \ \ uv \in E_2 \\ \gamma_1(u) \vee \gamma_1^{'}(v) \ if \ \ u \in V_1 \ \& v \in V \end{cases}$$

3.2.1 *Theorem*

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two IFG on V_1 andV_2 respectively with $V_1 \cap V_2 = \phi$ then

i. $\gamma(G_1 + G_2) = \min\{\gamma(G_1), \gamma(G_2), \{\mu_1(u) + \mu_1^{'}(v), \gamma_1(u) + \gamma_1^{'}(v)\}\}$ where $u_1 \in V_1, v \in V_2$

ii. $\gamma_i(G_1 + G_2) = \min\{\gamma_1(G_1), \gamma_1(G_2)\}$

Proof

(i). It follows from the definition of $G_1 + G_2$ any edges of the form uv, where $u_1 \in V_1, v \in V_2$ is an effective edge. Hence any vertex of V_1 dominates all the vertices of V_2. Now let D be any minimal dominating set of $G_1 + G_2$. Then D is of the following form

1) D=D_1 where D_1 is a minimal dominating set of G_1
2) D=D_2 where D_2 is a minimal dominating set of G_2
3) D={u,v} where $u \in V_1$ *and* $v \in V_2$,{u} is not a dominating set of G_1 and {u} is not a dominating set of G_2.

Hence

$$\gamma(G_1 + G_2) = \min\{\gamma(G_1), \gamma(G_2), \{\mu_1(u) + \mu_1^{'}(v), \gamma_1(u) + \gamma_1^{'}(v)\}\}$$

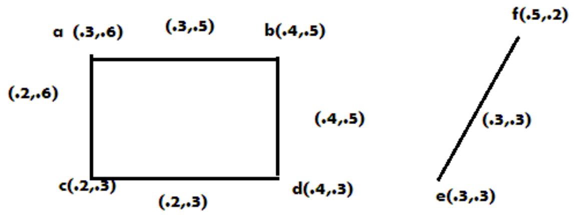
where $u_1 \in V_1, v \in V_2$.

(ii). Since every vertex of V_1 dominates every vertex of V_2 in G_1+G_2. Any independent set in $G_1 + G_2$ is either a subset of V_1or a subset of V_2. Hence any minimal dominating set D of $G_1 + G_2$ is one of the following forms

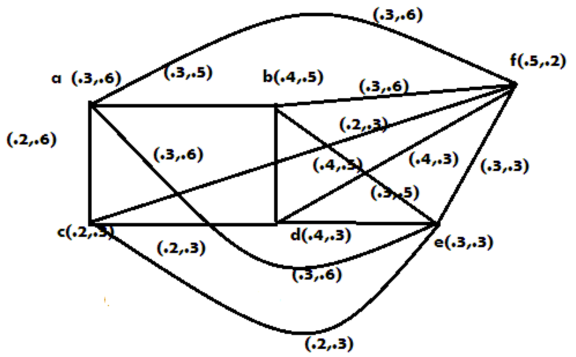
1) D=D_1, where D_1 is a minimal independent dominating set of G_1
2) D=D_2, where D_2 is a minimal independent dominating set of G_2.

Thus $\gamma_i(G_1 + G_2) = \min\{\gamma_1(G_1), \gamma_1(G_2)\}$

Example:



G1 G2



G_1+G_2

The dominating set is {e} and $\gamma(G_1+G_2)=(.3,.3)$

3.3 Composition of IFG

Let $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ be intuitionistic fuzzy graphs on V_1,V_2 respectively then the composition of G_1 and G_2 denoted by $G_1\circ G_2$, is the intuitionistic fuzzy graph G on $V_1\times V_2$ defined by $G=(G_1\circ G_2)=((\mu_1\circ\mu_1'),(\gamma_1\circ\gamma_1'),(\mu_2\circ\mu_2'),(\gamma_2\circ\gamma_2'))$ where

$$(\mu_1\circ\mu_1')(u_1,u_2)=\mu_1(u_1)\wedge\mu_1'(u_2)$$
$$(\gamma_1\circ\gamma_1')(u_1,u_2)=\gamma_1(u_1)\vee\gamma_1'(u_2)$$

$$(\mu_2\circ\mu_2')(u_1u_2)(v_1v_2)=\begin{cases}\mu_1(u_1)\wedge\mu_2'(u_2v_2)\ if\ u_1=v_1\ \&u_2\neq v_2\\ \mu_1'(u_2)\wedge\mu_1'(v_2)\wedge\mu_2(u_1v_1)\ otherwise\end{cases}$$

and

$$(\gamma_2\cup\gamma_2')(u_1u_2)(v_1v_2)=\begin{cases}\gamma_1(u_1)\vee\gamma_2'(u_2v_2)\ if\ u_1=v_1\ \&u_2\neq v_2\\ \gamma_1'(u_2)\vee\gamma_1'(v_2)\vee\gamma_2(u_1v_1)\ otherwise\end{cases}$$

3.3.1 Theorem

Let D_1 and D_2 be dominating sets of the intuitionistic fuzzy graph $G_1=(V_1,E_1)\ and\ G_2(V_2,E_2)$ respectively. Then $D_1\times D_2$ is a dominating set of $G_1\circ G_2$.

Proof:

Let $(u,v)\in D_1\times D_2$. Then $u\in D_1$ or $v\notin D_2$

Case (i) $u\notin D_1 and v\in D_2$

Let $u_1\in D_1$ be such that u_1 dominates v. Then

$$\mu_2(uu_1)=\mu_1(u)\wedge\mu_1(u_1)\ and$$
$$\gamma_2(uu_1)=\gamma_1(u)\wedge\gamma_1(u_1)$$

Now $(u_1,v)\in D_1\times D_2\ \ and$

$$(\mu_2\circ\mu_2')((u,v)(u_1,v))=\mu_2(uu_1)\wedge\mu_1'(v)$$
$$=\mu_1(u)\wedge\mu_1(u_1)\wedge\mu_1'(v)$$
$$=\mu_1(u)\wedge\mu_1'(v)\wedge\mu_1(u_1)\wedge\mu_1'(v)$$
$$=(\mu_1\circ\mu_1')(u,v)\wedge(\mu_1\circ\mu_1')(u_1,v_1)$$

And

$$(\gamma_2\circ\gamma_2')((u,v)(u_1,v))=\gamma_2(uu_1)\vee\gamma_1'(v)$$
$$=\gamma_1(u)\vee\gamma_1(u_1)\vee\mu_1'(v)$$
$$=\gamma_1(u)\vee\gamma_1'(v)\vee\gamma_1(u_1)\vee\gamma_1'(v)$$
$$=(\gamma_1\circ\gamma_1')(u,v)\vee(\gamma_1\circ\gamma_1')(u_1,v_1)$$

Hence (u_1,v) dominates (u,v) in $G_1\circ G_2$

Case (ii) $u\in D_1\ and\ v\notin D_2$

Let $v_1\in D_2$ be such that v_1 dominates v. Then

$$\mu_2'(v_1v)=\mu_1'(v_1)\wedge\mu_1'(v)\ and$$
$$\gamma_2'(v_1v)=\gamma_1'(v_1)\vee\gamma_1'(v)$$

Now $(u,v_1)\in D_1\times D_2\ \ and$

$$(\mu_2\circ\mu_2')((u,v)(u,v_1))=\mu_1(u)\wedge\mu_2'(uv_1)$$
$$=\mu_1(u)\wedge\mu_1'(v_1)\wedge\mu_1'(v)$$
$$=\mu_1(u)\wedge\mu_1'(v)\wedge\mu_1(u)\wedge\mu_1'(v_1)$$
$$=(\mu_1\circ\mu_1')(u,v)\wedge(\mu_1\circ\mu_1')(v,v_1)$$

And

$$(\gamma_2\circ\gamma_2')((u,v)(u,v_1))=\gamma_1(u)\vee\gamma_2'(uv_1)$$
$$=\gamma_1(u)\vee\gamma_1'(v_1)\vee\gamma_1'(v)$$
$$=\gamma_1(u)\vee\gamma_1'(v)\vee\gamma_1(u)\vee\gamma_1'(v_1)$$
$$=(\gamma_1\circ\gamma_1')(u,v)\vee(\gamma_1\circ\gamma_1')(v,v_1)$$

Hence (u,v_1) dominates (u,v) in $G_1\circ G_2$

Case (iii) $u\in D_1\ and\ v\in D_2$

Let $u_1\in D_1\ and\ v_1\in D_2$ be such that u_1 dominates u in G_1 and v_1 dominates v in G_2. Then

$$\mu_2(uu_1)=\mu_1(u)\wedge\mu_1(u_1)\ and$$
$$\gamma_2'(uu_1)=\gamma_1(u)\vee\gamma_1(u_1)$$

And

$$\mu_2(vv_1)=\mu_1(v)\wedge\mu_1(v_1)\ and$$
$$\gamma_2'(vv_1)=\gamma_1(v)\vee\gamma_1(v_1)$$

Now $(u_1,v_1)\in D_1\times D_2\ \ and$

$$(\mu_2 \circ \mu_2')\big((u,v)(u_1,v_1)\big) = \mu_2(uu_1) \wedge \mu_1'(v) \wedge \mu_1'(v_1)$$
$$= \mu_1(u) \wedge \mu_1(u_1) \wedge \mu_1'(v) \wedge \mu_1'(v_1)$$
$$= \mu_1(u) \wedge \mu_1'(v) \wedge \mu_1(u_1) \wedge \mu_1'(v_1)$$
$$= (\mu_1 \circ \mu_1')(u,v) \wedge (\mu_1 \circ \mu_1')(u,v_1)$$

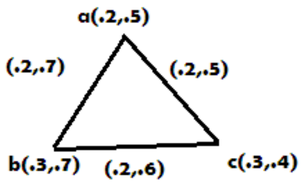
And

$$(\gamma_2 \circ \gamma_2')\big((u,v)(u_1,v_1)\big) = \gamma_2(uu_1) \vee \gamma_1'(v) \vee \gamma_1'(v_1)$$
$$= \gamma_1(u) \vee \gamma_1(u_1) \vee \gamma_1'(v) \vee \gamma_1'(v_1)$$
$$= \gamma_1(u) \vee \gamma_1'(v) \vee \gamma_1(u_1) \vee \gamma_1'(v_1)$$
$$= (\gamma_1 \circ \gamma_1')(u,v) \vee (\gamma_1 \circ \gamma_1')(u,v_1)$$

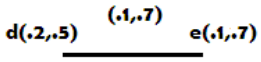
Hence (u_1,v_1) dominates (u, v) in $G_1 \circ G_2$. Thus $D_1 \times D_2$ is a dominating set of $G_1 \circ G_2$.

Example:

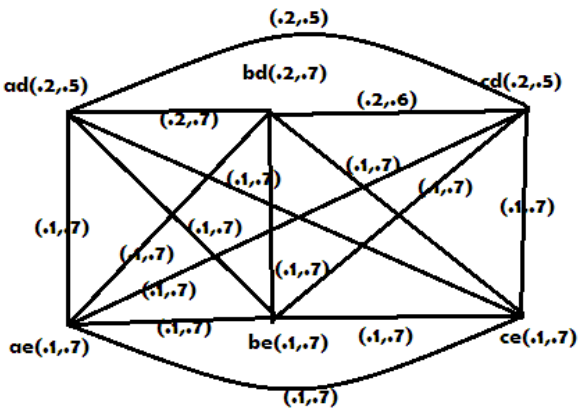
.



G1



G2



$G_1 \circ G_{2)}$

Dominating set of $G_1 \circ G_{2)}$ is {ae}

3.4 Cartesian product of IFG

Let $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ be intuitionistic fuzzy graphs on V_1,V_2 respectively then the Cartesian product of G_1 and G_2 denoted by $G_1 \times G_2$, is the intuitionistic fuzzy graph G on $V_1 \times V_2$ defined by $G = (G_1 \times G_2) = ((\mu_1 \times \mu_1'),(\gamma_1 \times \gamma_1'),(\mu_2 \times \mu_2'),(\gamma_2 \times \gamma_2'))$ where

$$(\mu_1 \times \mu_1')(u_1,u_2) = \mu_1(u_1) \wedge \mu_1'(u_2)$$
$$(\gamma_1 \times \gamma_1')(u_1,u_2) = \gamma_1(u_1) \vee \gamma_1'(u_2)$$

$$(\mu_2 \times \mu_2')(u_1u_2)(v_1v_2) = \begin{cases} \mu_1(u_1) \wedge \mu_2'(u_2v_2)\, if\ u_1 = v_1 \\ \mu_1'(u_2) \wedge \mu_1(u_1v_1)\, if\ u_2 = v_2 \\ otherwise \end{cases}$$

and

$$(\gamma_2 \times \gamma_2')(u_1u_2)(v_1v_2) = \begin{cases} \gamma_1(u_1) \vee \gamma_2'(u_2v_2)\, if\ u_1 = v_1 \\ \gamma_1'(u_2) \vee \gamma_1(u_1v_1)\, if\ u_2 = v_2 \\ otherwise \end{cases}$$

3.4.1 Theorem

Let D_1 and D_2 be minimum dominating set of the intuitionistic fuzzy graph $G_1 = (V_1, E_1)$ *and* $G_2 = (V_2, E_2)$ respectively. Then $\gamma_1(G_1 \otimes G_2) \le \{|D_1 \times V_2|, |V_1 \times D_2|\}$.

Proof :

We first prove that $D_1 \times V_2$ is a dominating set of $G_1 \otimes G_2$. Let $(u_1,u_2) \notin D_1 \times V_2$. Hence $u_1 \notin D_1$ since D_1 is a dominating set of G_1, there exist $v_1 \in D_1$ such that

$$\mu_2(u_1v_1) = \mu_1(u_1) \wedge \mu_1(v_1)\ and$$
$$\gamma_2(u_1v_1) = \gamma_1(u_1) \vee \gamma_1(v_1)$$

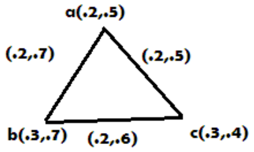
Now $(v_1u_2) \in (D_1 \times V_2)$ and

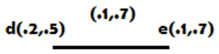
$$(\mu_2 \otimes \mu_2')((u_1u_2)(v_1u_2)) = \mu_1'(u_2) \wedge \mu_2(u_1v_1)$$
$$= \mu_1'(u_2) \wedge \mu_1(u_1) \wedge \mu_1(v_1)$$
$$= \mu_1(u_1) \wedge \mu_1'(u_2) \wedge \mu_1(v_1) \wedge \mu_1'(u_2)$$
$$= (\mu_1 \otimes \mu_1')(u_1u_2) \wedge (\mu_1 \otimes \mu_1')(v_1u_2)$$
$$(\gamma_2 \otimes \gamma_2')((u_1u_2)(v_1u_2)) = \gamma_1'(u_2) \vee \gamma_2(u_1v_1)$$
$$= \gamma_1'(u_2) \vee \gamma_1(u_1) \vee \gamma_1(v_1)$$
$$= \gamma_1(u_1) \vee \gamma_1'(u_2) \vee \gamma_1(v_1) \vee \gamma_1'(u_2)$$
$$= (\gamma_1 \otimes \gamma_1')(u_1u_2) \vee (\gamma_1 \otimes \gamma_1')(v_1u_2)$$

Thus (v_1,u_2) dominates (u_1,u_2) in $G_1 \otimes G_2$. So that $D_1 \times V_2$ is a dominating set of $G_1 \otimes G_2$. Similarly $V_1 \times D_2$ is a dominating set of $G_1 \otimes G_2$ and hence it follows that $\gamma_1(G_1 \otimes G_2) \le \{|D_1 \times V_2|, |V_1 \times D_2|\}$.

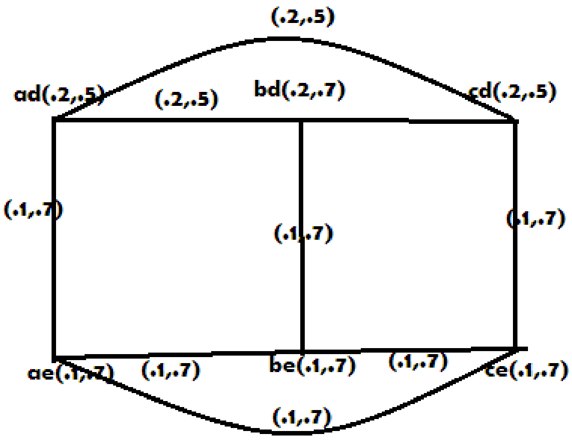
Example:

.





G_1 G_2



$G_1 \times G_2$

Dominating set of $G_1 \times G_2$ **is {ae, ad}**

4. CONCLUSION

In this paper we have prove some rests on operations on intuitionistic fuzzy graph . Further, the authors proposed to introduce new dominating parameters in intuitionistic fuzzy graph and apply these concepts to the intuitionistic fuzzy graph models in computer networks.

.

5. REFERENCES

[1] Atanasson , intuitionistic fuzzy set theory and applications, Physcia- verlag, New York, (199).

[2] Ayyaswamy.S, and Natarajan.C, Strong (weak) domination in fuzzy graphs, International Journal of Computational and Mathematical sciences, 2010.

[3] Balakrishnan and K.Ranganathan, A Text Book of Graph theory, Springer, 2000.

[4] Harary.F., Graph Theory, Addition Wesely, Third Printing, October 1972.

[5] Rosenfeld A. Fuzzy Graphs ,Fuzzy sets and their Applications (Acadamic Press, New York)

[6] Mordeson, J.N., and Nair, P.S., Fuzzy graphs and Fuzzy Hyper graphs, Physica-Verlag, Heidelberg, 1998, second edition, 2001.

[7] R.Parvathi and G.Thamizhendhi, Domination in Intuitionistic Fuzzy Graphs, Fourteenth Int.Conf. On IFSs, Sofia 15-16 may 2010.

[8] Somasundaram,A.,Somasundaram,S.,1998, Domination in Fuzzy Graphs-I, Pattern Recognition Letters, 19, pp. 787–791.

[9] Somasundaram, A., 2004, Domination in product Fuzzy Graph-II, Journal of Fuzzy Mathematics

[10] R.Parvathi and G.Thamizhendhi, Domination in Intuitionistic Fuzzy Graphs, Fourteenth Int.Conf. On IFSs, Sofia 15-16 may 2010.