

Applications of λ -Closed Sets in Intuitionistic Fuzzy Topological Space

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ABSTRACT

In this paper we study the relationship between λ -closed sets and some other intuitionistic fuzzy sets already exists. We also define intuitionistic fuzzy λ -irresolute map and study some of its properties.

KEYWORDS

Intuitionistic fuzzy topology, intuitionistic fuzzy λ -closed sets, intuitionistic fuzzy λ - open sets, and intuitionistic fuzzy λ -irresolute maps.

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1. INTRODUCTION

After the introduction of fuzzy sets by L.A Zadeh [17] in 1965, there we have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] in 1983. Using the notion of intuitionistic fuzzy sets, Coker [5] introduced the notion of intuitionistic fuzzy topology in 1997. This approach provides a wide field for investigation in the area of fuzzy topology and its application. The aim of this paper is to study the relations between intuitionistic fuzzy λ -closed sets and the other intuitionistic fuzzy sets already exists. Moreover we investigate intuitionistic fuzzy λ -irresolute map and study some of its properties

2. PRELIMINARIES

Definition 2.1: [1] Let X be a nonempty set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$, where the function $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\nu_A(x)$ of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 :[1]: Let A and B be intuitionistic fuzzy sets of the form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$, and form $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$

(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$

(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$.

The intuitionistic fuzzy sets $\tilde{0} = \{ \langle x, 0, 1 \rangle : x \in X \}$ and $\tilde{1} = \{ \langle x, 0, 1 \rangle : x \in X \}$ are respectively the empty set and whole set of X .

Definition 2.3 :[1]: Let $(\alpha, \beta) \in [0,1]$ with $\alpha + \beta \leq 1$, An intuitionistic fuzzy point (IFP), written as $p_{(\alpha, \beta)}$ is defined to be IFS of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p \\ (0, 1) & \text{otherwise} \end{cases}$$

Definition 2.4 : [5]: An intuitionistic fuzzy topology (IFT) on X is a family of IFSs which satisfying the following axioms.

(i) $\tilde{0}, \tilde{1} \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in I\} \subseteq \tau$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS) and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X .

The complement A of an IFOS in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in (X, τ) .

Definition 2.5 : [5]: Let (X, τ) be an intuitionistic fuzzy topology and

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$, be an intuitionistic fuzzy set in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{Int}(A) = \cup \{ G / G \text{ is an intuitionistic fuzzy open set in } X \text{ and } G \subseteq A \}$$

$$\text{Cl}(A) = \cap \{ K / K \text{ in an intuitionistic fuzzy closed set in } X \text{ and } A \subseteq K \}$$

Remark 2.6 :[5]: For any intuitionistic fuzzy set A in (X, τ) , we have

(i) $\text{cl}(A^c) = [\text{int}(A)]^c$,

(ii) $\text{int}(A^c) = [\text{cl}(A)]^c$,

(iii) A is an intuitionistic fuzzy closed set in $X \Leftrightarrow \text{Cl}(A) = A$

- (iv) A is an intuitionistic fuzzy open set in $X \Leftrightarrow \text{int}(A) = A$

Definition 2.7 :[6]: An intuitionistic fuzzy set $A = \{ \langle x, \mu_A(x), \nu_B(x) \rangle : x \in X \}$ in an intuitionistic fuzzy topological space (X, τ) is said to be

- (i) Intuitionistic fuzzy semi closed if $\text{int}(\text{cl}(A)) \subseteq A$.
 (ii) Intuitionistic fuzzy pre closed if $\text{cl}(\text{int}(A)) \subseteq A$.

Definition 2.8: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) called

(i) intuitionistic fuzzy generalized closed set [15] (intuitionistic fuzzy g -closed) if $\text{cl}(A) \subseteq U$ whenever

$A \subseteq U$ and U is intuitionistic fuzzy semi open

(ii) intuitionistic fuzzy g -open set [14], if the complement of an intuitionistic fuzzy g -closed set is called intuitionistic fuzzy g -open set.

(iii) intuitionistic fuzzy semi open (resp. intuitionistic fuzzy semi closed) [6] if there exists an intuitionistic fuzzy open (resp. intuitionistic fuzzy closed) such that $U \subseteq A \subseteq \text{Cl}(U)$ (resp. $\text{int}(U) \subseteq A \subseteq U$).

Remark 2.9 : [15]: Every intuitionistic fuzzy closed set (intuitionistic fuzzy open set) is intuitionistic fuzzy g -closed (intuitionistic fuzzy g -open set) but the converse may not be true

Definition 2.10 An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called

(i) an intuitionistic fuzzy w -closed [14] if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.

(X, τ)

(ii) an intuitionistic fuzzy rw -closed set [16] if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular semi open. (X, τ)

(iii) an intuitionistic fuzzy rg -closed set [16] if $\text{cl}(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular open. (X, τ)

(iv)) an intuitionistic fuzzy generalized α -closed set [8] (IFG α CS if $\text{acl}(A) \subseteq O$ whenever $A \subseteq O$ and O is IF α OS in (X, τ))

(iv)) an intuitionistic fuzzy α -generalized closed set [12] (IF α GCS if $\text{acl}(A) \subseteq O$ whenever $A \subseteq O$ and O is IFOS in (X, τ))

Definition 2.11 :[12] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) called an

(i) intuitionistic fuzzy α -open set (IF α OS in short) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$

(ii)) intuitionistic fuzzy α -closed set (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

Definition 2.12 :[8] An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) called intuitionistic

fuzzy α generalised closed set (IF α GCS in short) if $\text{acl}(A) \subseteq U$ and U is an IFOS in (X, τ) .

Definition 2.13 : [5] Let X and Y are nonempty sets and $f: X \rightarrow Y$ is a function.

(a) If $B = \{ \langle y, \mu_B(y), \nu_B(y) \rangle : y \in Y \}$ is an intuitionistic fuzzy set in Y , then the pre image of B

under f denoted by $f^{-1}(B)$, is defined by

$$f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x)) \rangle : x \in X \}$$

(b) If $A = \{ \langle x, \mu_A(x), \nu_B(x) \rangle / x \in X \}$ is an intuitionistic fuzzy set in X , then the image of A under f denoted by $f(A)$ is the intuitionistic fuzzy set in Y defined by

$$f(A) = \{ \langle y, f(\mu_A(y)), f(\nu_A(y)) \rangle : y \in Y \} \text{ where } f(\nu_A) = 1 - f(1 - \nu_A).$$

Definition 2.14: [10] An intuitionistic fuzzy set A of an intuitionistic topological space (X, τ) is called an

(i) intuitionistic fuzzy λ -closed set (IF λ -CS) if $A \supseteq \text{cl}(U)$ whenever $A \supseteq U$ and U is intuitionistic fuzzy open set in X

(ii) intuitionistic fuzzy λ -open set (IF λ -OS) if the complement A^c of an intuitionistic fuzzy λ -closed set

Definition 2.15 :[13] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

(i) intuitionistic fuzzy open mapping (IF open mapping) if $f(A)$ is an IFOS in Y for every IFOS A in X .

(ii) intuitionistic fuzzy closed mapping (IF closed mapping) if $f(A)$ is an IFCS in Y for every IFCS A in X .

Definition 2.16 :[11] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic fuzzy λ -continuous if the inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy λ -closed in X

Definition 2.17 : [11]: A topological space (X, τ) is called intuitionistic fuzzy λ - $T_{1/2}$ space

(IF λ - $T_{1/2}$ space in short) if every intuitionistic fuzzy λ -closed set is intuitionistic

fuzzy closed in X .

3. APPLICATIONS OF INTUITIONISTIC FUZZY λ -CLOSED SET.

In this section we study the relations between Intuitionistic fuzzy λ -closed sets and some other Intuitionistic fuzzy sets already exists.

Definition 3.1. Let A be an IFS in an IFTS (X, τ) . Then the intuitionistic fuzzy λ -interior and intuitionistic fuzzy λ -closure of A are defined as follows.

$$\lambda\text{-int}(A) = \cup \{ G \mid G \text{ is an IF}\lambda\text{-OS in } X \text{ and } G \subseteq A \},$$

$$\lambda\text{-cl}(A) = \cap \{ K \mid K \text{ is an IF}\lambda\text{-CS in } X \text{ and } A \subseteq K \}.$$

Theorem 3.2: Every IF preclosed set is IF λ -closed set .

Proof : Let A be a IF preclosed set. Let G is an IF open set such that $A \supseteq G$. Then $\text{cl}(\text{int}(A)) \supseteq \text{cl}(\text{int}(G)) \supseteq \text{cl}(G)$. Therefore $\text{cl}(\text{int}(A)) \supseteq \text{cl}(G)$. Since A is IF preclosed set we have $A \supseteq \text{cl}(\text{int}(A)) \supseteq \text{cl}(G)$. Thus $A \supseteq \text{cl}(G)$. Therefore A is IF λ -closed set.

Remark 3.3 : The converse above theorem need not be true as seen from the following example.

Example 3.4 : Let $X = \{ a,b \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X. where $U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.3, 0.6 \rangle \}$. Then the intuitionistic fuzzy set

$A = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.8, 0.2 \rangle \}$ is IF λ -closed set but not pre closed set.

Remark 3.5 : IF λ – closed sets and IF w-closed sets are independent to each other example

Example 3.6: Let $X = \{ a,b \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X. where $U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.2 \rangle \}$. Then the intuitionistic fuzzy set $A = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.4 \rangle \}$ is not intuitionistic fuzzy IF λ -closed set but not IF w-closed set

Example 3.7: Let $X = \{ a,b \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X. where $U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle \}$. Then the intuitionistic fuzzy set

$A = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.5 \rangle \}$ is IF w-closed set not IF λ -closed set.

Remark 3.8 : IF λ – closed sets and IF rw-closed sets are independent to each other example

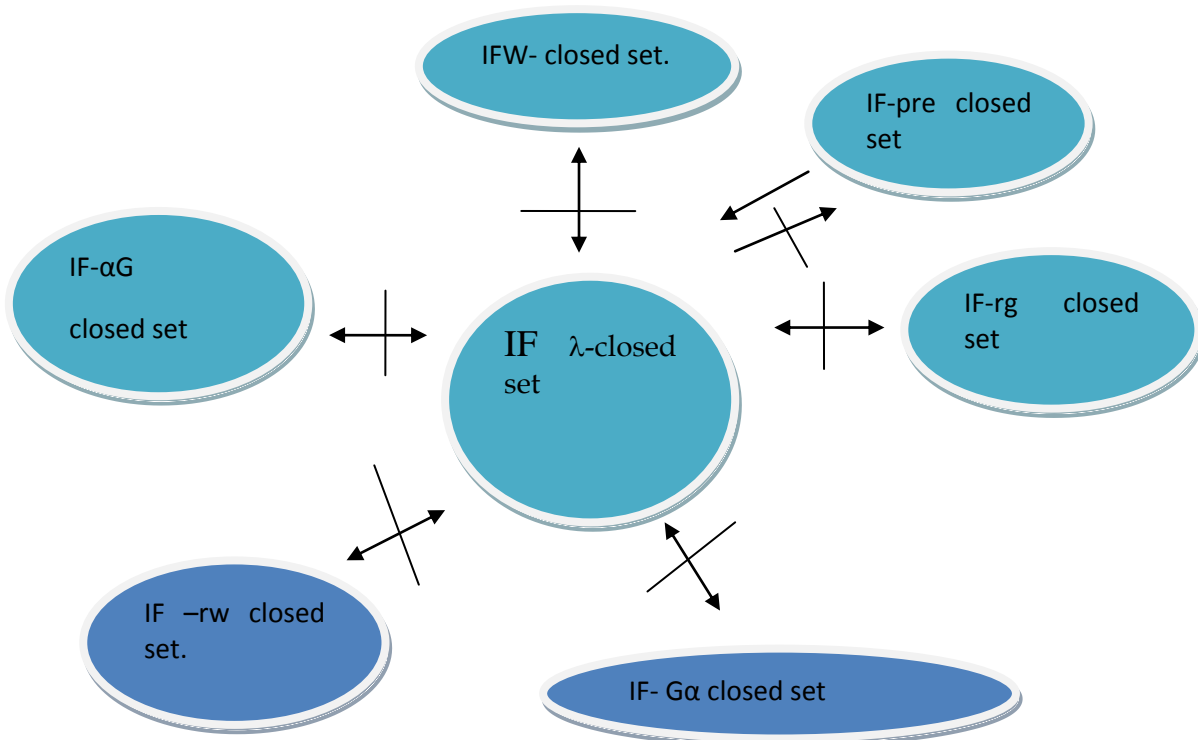
Example 3.9 Let $X = \{ a,b \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X. where $U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.2 \rangle \}$. Then the intuitionistic fuzzy set $A = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.4 \rangle \}$ is not intuitionistic fuzzy IF λ -closed set but not IF rw-closed set

Example 3.10 : Let $X = \{ a,b \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X. where $U = \{ \langle a, 0.7, 0.2 \rangle, \langle b, 0.6, 0.3 \rangle \}$. Then the intuitionistic fuzzy set

$A = \{ \langle a, 0.7, 0.2 \rangle, \langle b, 0.6, 0.3 \rangle \}$ is IF rw-closed set but not IF λ -closed set.

Remark 3.11: IF λ – closed set and IF rg-closed set are independent to each other example.

Example 3.12: Let $X = \{ a,b \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X. where $U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.2 \rangle \}$. Then the intuitionistic fuzzy set $A = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.4 \rangle \}$ is not intuitionistic fuzzy IF λ -closed set but not IF rg-closed set



Example 3.13: Let $X = \{ a,b,c,d \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U, V, W \}$

be an intuitionistic fuzzy topology on X . where $U = \{ \langle a, 0.7, 0.2 \rangle, \langle b, 0.6, 0.3 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \}$

$V = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \}$,

$W = \{ \langle a, 0.7, 0.2 \rangle, \langle b, 0.6, 0.3 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \}$

Then the intuitionistic fuzzy set $A = \{ \langle a, 0, 1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0.7, 0.2 \rangle, \langle d, 0, 1 \rangle \}$ is IF τ -closed set but not IF λ -closed set

Remark 3.14: α G closed sets and IF closed sets are independent to each other for example

Example 3.15: Let $X = \{ a,b \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X . where $U = \{ \langle a, 0.2, 0.8 \rangle, \langle b, 0.3, 0.7 \rangle \}$. Then the intuitionistic fuzzy set $A = \{ \langle a, 0.6, 0.4 \rangle, \langle b, 0.7, 0.3 \rangle \}$ is IF α G closed set but not IF λ -closed set.

Example 3.16: Let $X = \{ a, b \}$ and let $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X . where $U = \{ \langle a, 0.8, 0.2 \rangle, \langle b, 0.8, 0.1 \rangle \}$. Then the intuitionistic fuzzy set $A = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.7, 0.3 \rangle \}$ is IF λ -closed set but is not IF α G closed set .

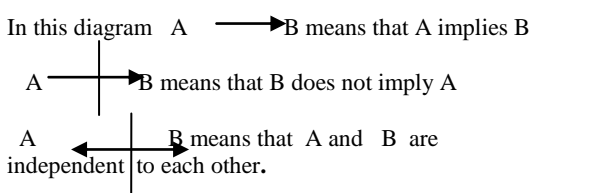
Remark 3.17 α G closed sets and IF closed sets are independent to each other for example

Example 3.18: Let $X = \{ a,b \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X . where $U = \{ \langle a, 0.2, 0.6 \rangle, \langle b, 0.2, 0.7 \rangle \}$. Then the intuitionistic fuzzy set $A = \{ \langle a, 0.4, 0.6 \rangle, \langle b, 0.2, 0.7 \rangle \}$ is IF α G closed set but not IF λ -closed set.

Example 3.19: Let $X = \{ a,b \}$ and $\tau = \{ \tilde{0}, \tilde{1}, U \}$ be an intuitionistic fuzzy topology on X . where $U = \{ \langle a, 0.2, 0.4 \rangle, \langle b, 0.3, 0.5 \rangle \}$. Then the intuitionistic fuzzy set $A = \{ \langle a, 0.5, 0.1 \rangle, \langle b, 0.6, 0 \rangle \}$ is IF λ -closed set but not IF α G λ -closed set.

Remark 3.20 : From above examples and remarks we get following diagram of implications.

In this diagram $A \implies B$ means that A implies B



4. λ - irresolute mappings in intuitionistic fuzzy topological spaces.

In this section we introduce intuitionistic fuzzy λ - irresolute mapping and study some of its properties.

Definition 4.1: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called intuitionistic fuzzy λ - irresolute (IF- λ - irresolute) if

$f^{-1}(V)$ is IF λ - closed set in (X, τ) for every IF λ -closed set V of (Y, σ)

Theorem 4.2: If a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called IF λ - irresolute mapping then it is IF λ -continuous mapping but not conversely.

Proof : Let V be any closed set in Y . since every closed set is IF λ - closed set, V is IF λ - closed set in Y . By assumption $f^{-1}(V)$ is IF λ -closed set in X . Thus f is IF λ -continuous.

The converse of the above theorem need not be true as seen in the following example:

Example 4.3: Let $X = Y = \{a, b\}$, and intuitionistic fuzzy sets U and V are defined as follows.

$U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.2, 0.7 \rangle \}$,

$V = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.6, 0.4 \rangle \}$ be the intuitionistic fuzzy sets respectively and Let $\tau = \{ \tilde{0}, \tilde{1}, U \}$ and $\sigma = \{ \tilde{0}, \tilde{1}, V \}$ be intuitionistic fuzzy topologies on X and Y respectively.

And let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity mapping. Clearly f is IF λ - continuous mapping. $f^{-1}(\{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle \}) = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle \}$ in not IF λ -closed set in X . Therefore f is not IF λ - irresolute mapping.

Theorem 4.4 : Let $X, Y,$ and Z be any topological space. For any IF λ -irresolute map $f: (X, \tau) \rightarrow (Y, \sigma)$ and IF λ -continuous map $g: (Y, \sigma) \rightarrow (Z, \upsilon)$. The composition

$g \circ f: (X, \tau) \rightarrow (Z, \upsilon)$ is IF λ -continuous.

Proof : Let V be any closed set in Z . since g is IF λ - continuous map, $g^{-1}(V)$ is IF λ -closed in Y . Since IF λ - f is irresolute map $f^{-1}(g^{-1}(V))$ is IF λ -closed in X . But

$f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ which is λ -closed in X . Thus $g \circ f$ is IF λ -continuous map.

Theorem 4.5 : If $f: (X, \tau) \rightarrow (Y, \sigma)$ is IF open mapping and IF continuous then f is IF λ - irresolute map.

Proof : Let A be a λ -closed set in Y . Let $f^{-1}(A) \supseteq O$ where O is open in X . since f is IF open,

$f(O)$ is open in Y . Thus we have $A \supseteq f(O)$. This implies $A \supseteq Cl(f(O))$. But $cl(f(O)) \supseteq f(cl(O))$ because f is IF continuous. Therefore $A \supseteq f(cl(O))$. Hence $f^{-1}(A)$ is IF λ -closed set in X . Therefore f IF λ - is irresolute.

Theorem:4. 6 : If X, Y and Z are topological spaces. and $f: (X, \tau) \rightarrow (Y, \sigma)$ and

$g: (Y, \sigma) \rightarrow (Z, \upsilon)$ are IF λ -irresolute then composition $g \circ f: (X, \tau) \rightarrow (Z, \upsilon)$ is IF λ -irresolute map.

Proof : Let V be a IF λ -closed set in Z . Since g is IF λ - irresolute map , $g^{-1}(V)$ is IF λ -closed set in (Y, σ) . Also we are given that f is IF λ -irresolute and therefore $f^{-1}g^{-1}(V)$ is

IF λ -closed set in (X, τ) . But $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$. Hence $(g \circ f)$ is IF λ -irresolute map from X to Z .

Theorem 4.7: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF λ - irresolute, then f is an IF irresolute

mapping if X is an IF λ - $T_{1/2}$ space.

Proof: Let A be an IFCS in Y . Then A is an IF λ -CS in Y . Therefore $f^{-1}(A)$ is an

IF λ -CS in X , by hypothesis. Since X is an IF λ - $T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X . Hence.

f is an IF irresolute mappings.

Theorem 4.8: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y .

Then the following conditions are equivalent if X and Y are IF λ - $T_{1/2}$ spaces.

- (i) f is an IF λ - irresolute mapping
- (ii) $f^{-1}(B)$ is an IF λ -OS in X for each IF λ -OS in Y
- (iii) $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each IFS B of Y .

Proof: (i) \Rightarrow (ii): It can be proved by using complement and definition 4.1.

(ii) \Rightarrow (iii): Let B be any IFS in Y and $B \subseteq cl(B)$. Then $f^{-1}(B) \subseteq f^{-1}(cl(B))$. Since $cl(B)$

is an IFCS in Y , $cl(B)$ is an IF λ -CS in Y . Therefore $f^{-1}(cl(B))$ is an IF λ -CS in X ,

by hypothesis. Since X is IF λ - $T_{1/2}$ space. $f^{-1}(cl(B))$ is an IFCS in X . Hence

$$cl(f^{-1}(B)) \subseteq cl(f^{-1}(cl(B))) = f^{-1}(cl(B)). \text{ That is } cl(f^{-1}(B)) \subseteq f^{-1}(cl(B)).$$

(iii) \Rightarrow (i): Let B be an IF λ -CS in Y . Since Y is an IF λ - $T_{1/2}$ space, B is an IFCS in Y and

$$cl(B) = B. \text{ Hence } f^{-1}(B) = f^{-1}(cl(B)) \subseteq cl(f^{-1}(B)). \text{ But clearly } f^{-1}(B) \supseteq cl(f^{-1}(B)).$$

Therefore $cl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFCS and hence it is an IF λ -CS in X . Thus f is an IF λ -irresolute mapping.

Theorem 4.9: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a an IF λ - irresolute mapping from an IFTS X into an IFTS Y . Then $f^{-1}(B) \subseteq \lambda$ -int($f^{-1}(cl(int(cl(B))))$) for every IF λ -OS B in Y , if X and Y are IF λ - $T_{1/2}$ spaces.

Proof: Let B be an IF λ -OS in Y . Then by hypothesis $f^{-1}(B)$ is an IF λ -OS in X . Since X is an IF λ - $T_{1/2}$ space, $f^{-1}(B)$ is an IFOS in X . Therefore λ -int($f^{-1}(B)$) = $f^{-1}(B)$. since Y is an IF λ - $T_{1/2}$ space, B is an IF λ -OS in Y and $B \subseteq cl(int(cl(B)))$. Now $f^{-1}(B) = \lambda$ -int($f^{-1}(B)$) implies,

$$f^{-1}(B) \subseteq \lambda$$
-int($f^{-1}(cl(int(cl(B))))$).

Theorem 4.10: If a mapping $f: X \rightarrow Y$ is intuitionistic fuzzy λ -irresolute mapping, then

$$f(\lambda$$
-cl(B)) $\subseteq \lambda$ -cl($f(B)$) for every IFS B of X .

proof : Let B be an IFS of X . Since $cl(f(B))$ is an IF λ -CS in Y , by our assumption

$$f^{-1}(cl(f(B))) \text{ is an IF}\lambda\text{-CS in } X. \text{ Furthermore } B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(cl(f(B))) \text{ and hence}$$

$$\lambda$$
-cl(B) $\subseteq f^{-1}[cl(f(B))]$ and consequently $f[\lambda$ -cl(B)] $\subseteq f[f^{-1}[cl(f(B))]] \subseteq cl(f(B))$.

Theorem 4.11 : If any union of IF λ -CS is an IF λ -CS, then a mapping $f: X \rightarrow Y$ from an IFTS

X into an IFTS Y is intuitionistic fuzzy λ - irresolute if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IF λ -CS B in Y such that $f(p_{(\alpha,\beta)}) \in B$, there exists an IF λ -CS A in X such that $p_{(\alpha,\beta)} \in A$ and

$$f(A) \subseteq B.$$

Proof : Let f be any intuitionistic fuzzy-irresolute mapping, $p_{(\alpha,\beta)}$ an IFP in X and B be any IF λ -CS in Y , such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) = \lambda$ -cl($f^{-1}(B)$). We take

$A = \lambda$ -cl($f^{-1}(B)$). Then A is an IF λ -CS in X , containing IFP $p_{(\alpha,\beta)}$ and

$$f(A) = f[\lambda$$
-cl($f^{-1}(B))]$ $\subseteq f[f^{-1}(B)] \subseteq B.$

Conversely assume that B be any IF λ -CS in Y and IFP $p_{(\alpha,\beta)}$ (x) in X , such that

$p_{(\alpha,\beta)} \in f^{-1}(B)$. By assumption there exists IF λ -CS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. Therefore $p_{(\alpha,\beta)}(x) \in A \subseteq f^{-1}(B)$ and $p_{(\alpha,\beta)} \in A = \lambda$ -cl(A) $\subseteq \lambda$ -cl($f^{-1}(B)$). Since $p_{(\alpha,\beta)}$ is an arbitrary IFP and $f^{-1}(B)$ is union of all IFP contained in $f^{-1}(B)$, $f^{-1}(B)$ is an IF λ -CS in X , so f is an intuitionistic fuzzy λ -irresolute mapping.

Corollary 4.12. A mapping $f: X \rightarrow Y$ from an IFTS X into an IFTS Y is intuitionistic fuzzy

λ -irresolute if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IF λ -CS B in Y such that $f(p_{(\alpha,\beta)})$

$\in B$, there exists an IF λ -CS A in X such that $p_{(\alpha,\beta)} \in A$ and $A \subseteq f^{-1}(B)$.

Proof. Follows from Theorem 4.11

5. CONCLUSION

In this paper we have studied the relations between intuitionistic fuzzy λ -closed sets and the other intuitionistic fuzzy sets already exists. Also we studied the intuitionistic fuzzy λ -irresolute map and some of its properties.

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