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Control and Synchronization Chaotic Satellite using Active Control

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ABSTRACT
Satellite attitude dynamics, nonlinear systems with high dimension and are nonlinear and chaotic. In this paper, attitude control and synchronization two identical chaotic satellite with different initial conditions based on the control design is proposed. Using the Lyapunov theory stability controller has been demonstrated. Finally, according to the simulation results, the synchronization is complete, the control signal is low that changes are the ability to build and implement.

Keywords
Chaotic Synchronization, Satellite Dynamic, Active Control

1. INTRODUCTION
A chaotic system has complex dynamical behaviors, such as depending sensitively on tiny variations of initial conditions; having bounded trajectories in the phase space, etc. In recent years, there has been increasing interest in the study of synchronizing chaotic systems. Chaos synchronization has many potential applications in laser physics [1,2], secure communication [3,4], power electrical systems [5], aerospace [6,7] and gyro [8] and so on. Various control approaches were reported to realize the chaotic synchronization, such as Adaptive Control [9], Impulsive Control [10], Back Stepping [11], Fuzzy Control [12,13], Sliding Mode Control [14,15], Nonlinear Control [16], Active control [17,18]. After the pioneering work on chaos control by Ott et al [19] and synchronization of chaotic systems by Pecora and Carroll [1]. In this paper, attitude control and synchronization two identical chaotic satellites with different initial conditions based on the active control design is proposed. In synchronization theory, defined a Master system, which is the dominant system, and a bounded set of Slave systems.

The synchronization problem consists of creating either physical interconnections or control feedback loops, which forces the outputs of the slave systems to conform with those of the Master. As space technology progresses, the need for improved satellite systems by better understanding of satellite dynamics has continuously kept attention [20].Recently, non-linear dynamics, especially the chaotic attitude dynamics of a satellite have attracted the attention of many scientists [21,22].The control of the Slave satellite, on the other hand, is a synchronization problem. . A reference trajectory for the Slave satellite will therefore also depend on the states of the Master satellite. For many applications of formations of satellites the objective will be to point measuring instruments in the same direction. Let therefore the reference trajectory for the Slave satellite be the measured attitude of the Master satellite. In this paper, the synchronization of chaotic satellites systems is handled based on the active control approach.

2. ACTIVE CONTROL DESIGN
The active control method proposed in [23] is considered to synchronize two different chaotic systems. For this purpose, consider a master system:

$$\dot{X} = Ax + g(x) \tag{1}$$

Where $x \in R^n$ is the state vector, $A \in R^{n\times n}$ is a constant system matrix, and g(x) is a nonlinear sequence function. A slave system is defined as:

$$\dot{Y} = By + f(y) + \psi(t) \tag{2}$$

Where $y \in R^n$ is the state vector, $B \in R^{n\times n}$ is a constant system matrix, and $f(y)$ is a nonlinear sequence function, and $\psi(t) \in R^n$ is an active control function. A master-slave synchronization scheme is illustrated in Figure 1. The error state is defined as:

$$\begin{cases} e(t) = y(t) - x(t), \\ \lim_{t\to\infty} e = 0 \end{cases} \tag{3}$$

Therefore, the error dynamics are written as follows:

$$\dot{e} = \dot{Y} - \dot{X} = Ce + G(x,y) + \psi(t) \tag{4}$$

Where $C = \hat{B} - \hat{A}$ is the common part of the system matrices in the master and slave systems; the non-common parts and nonlinear functions are gathered in $G(x,y)$ as:

$$G(x,y) = f(y) - g(x) + (B - \overline{B})y - (A - \overline{A})x \tag{5}$$

and $\psi(t)$ is the controller matrix. Error vectors with an appropriate controller $\psi(t)$ satisfying $\forall(x,y) \in R^n$ and $\forall e \in R^n$ converge to zero. Hence, an appropriate controller should eliminate nonlinear terms and non-common parts, and contain another part to achieve stability, such as:

$$\psi(t) = -G(x,y) + U(t) \tag{6}$$

Where $U(t) = -Ke$ is a linear controller and $K \in R^n$ is a linear gain matrix. Substitution of equation (6) into (4) leads to:

$$\dot{e} = Ce + U(t) \tag{7}$$

With replacing $U(t) = -Ke$ in the equation (7), error dynamic is defined by

$$\dot{e} = Ce - Ke = (C - K)e \tag{8}$$

Synchronization of chaos by using active control can be realized when master and slave systems are completely different. If the eigenvalues $\lambda_{i(i=1,2,\ldots,n)}$ of the matrix

$C-K$ are negative $\lambda_i < 0$. Then the error state vectors exponentially converge to zero. That is, the master and slave systems exponentially synchronize.

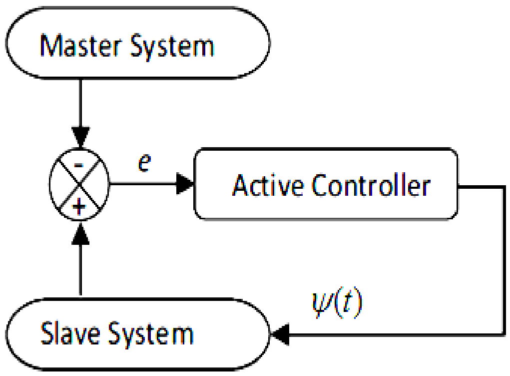

Figure 1: The master-slave synchronization scheme

3. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

3.1 Satellite System with Chaotic Dynamic

The orientation of the satellite at a given point can be locally described in terms of three angles φ, θ and ψ which are successive clockwise rotations about inertial axes I, J and K respectively. The kinematic equation of a satellite or spacecraft can be written as:

$$\begin{cases} \omega_x = \dot{\phi}.Cos\psi.Cos\theta + \dot{\theta}.Sin\psi \\ \omega_y = -\dot{\phi}.Sin\psi.Cos\theta + \dot{\theta}.Cos\psi \\ \omega_z = \dot{\psi} + \phi.Sin\theta \end{cases} \quad (9)$$

And on collecting terms and inverting, the following form is resulted, which is more appropriate for solving by numerical integration [24].

$$\begin{cases} \dot{\phi} = (\omega_x.Cos\psi - \omega_y Sin\psi)/Cos\theta. \\ \dot{\theta} = \omega_x.Sin\psi + \omega_y.Cos\psi \\ \dot{\psi} = \omega_z - (\omega_x.Cos\psi - \omega_y Sin\psi)/\tan\theta \end{cases} \quad (10)$$

The rotational motion for general rigid spacecraft acting under the influence of external torques is given by [24].The dynamical equation of a satellite, similar to a rigid body can be expressed as:

$$I\dot{\omega} = -\Omega I\omega + H + U \quad (11)$$

Where I is the moment of inertia tensor, ω is the angular velocity vector, U is the control torque, and h contains any external disturbance torques. The dynamical equations of a satellite are:

$$\begin{aligned} I_x\dot{\omega}_x &= \omega_y\omega_z(I_y - I_z) + H_x + U_x \\ I_y\dot{\omega}_y &= \omega_x\omega_z(I_z - I_x) + H_y + U_y \\ I_z\dot{\omega}_z &= \omega_y\omega_y(I_x - I_y) + H_z + U_z \end{aligned} \quad (12)$$

Where I_x, I_y and I_z are the principal moments of inertia, ω_x, ω_y and ω_z are the angular velocities of the satellite, U_x, U_y and U_z are the three control torques; H_x, H_y and H_z are perturbing torques. principal moments of inertia and perturbing torques such as:

$$\left\{ H = \begin{bmatrix} -1.2 & 0 & \frac{\sqrt{6}}{2} \\ 0 & 0.35 & 0 \\ -\sqrt{6} & 0 & -0.4 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \begin{cases} I_x = 3 \\ I_y = 2 \\ I_z = 1 \end{cases} \right. \quad (13)$$

This torques is chosen so as to force the satellite into chaotic motion [24]. By changing the elements value of system matrices, many various dynamical behavior could be observed. For example, let H=0 and U=0, the attitude motion of a satellite has a twisted periodic trajectory, which is shown in Fig.2 and Fig.3.

3.2 Synchronization Problem Formulation

Consider the following two identical satellites attitudes systems, where the Master system and Slave system are denoted with m and s, respectively. Master system:

$$\begin{aligned} \dot{\omega}_{xm} &= \delta_x\omega_{ym}\omega_{zm} - \frac{1.2}{I_x}\omega_{xm} + \frac{\sqrt{6}}{2I_x}\omega_{zm} \\ \dot{\omega}_{ym} &= \delta_y\omega_{xm}\omega_{zm} + \frac{0.35}{I_y}\omega_{ym} \\ \dot{\omega}_{zm} &= \delta_z\omega_{ym}\omega_{ym} - \frac{\sqrt{6}}{I_z}\omega_{xm} - \frac{0.4}{I_z}\omega_{zm} \end{aligned} \quad (14)$$

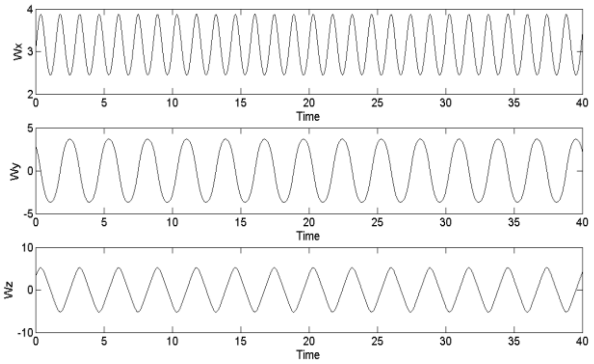

Fig.2. Chaotic Attitude of Satellite

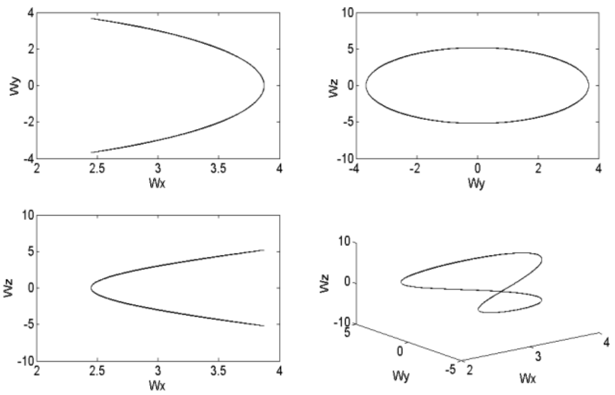

Fig.3. Phase Portraits of Chaotic Satellite

And Slave system:

$$
\begin{aligned}
\dot{\omega}_{xs} &= \delta_x \omega_{ys}\omega_{zs} - \frac{1.2}{I_x}\omega_{xs} + \frac{\sqrt{6}}{2I_x}\omega_{zs} \\
\dot{\omega}_{ys} &= \delta_y \omega_{xs}\omega_{zs} + \frac{0.35}{I_y}\omega_{ys} \\
\dot{\omega}_{zs} &= \delta_z \omega_{ys}\omega_{ys} - \frac{\sqrt{6}}{I_z}\omega_{xs} - \frac{0.4}{I_z}\omega_{zs}
\end{aligned} \tag{15}
$$

The aim is to design the controller $\psi \in R^n$ such that:

$$
\begin{cases}
e_x = \omega_{xs} - \omega_{xm} \\
e_y = \omega_{ys} - \omega_{ym} \\
e_x = \omega_{zs} - \omega_{zm}
\end{cases} \tag{16}
$$

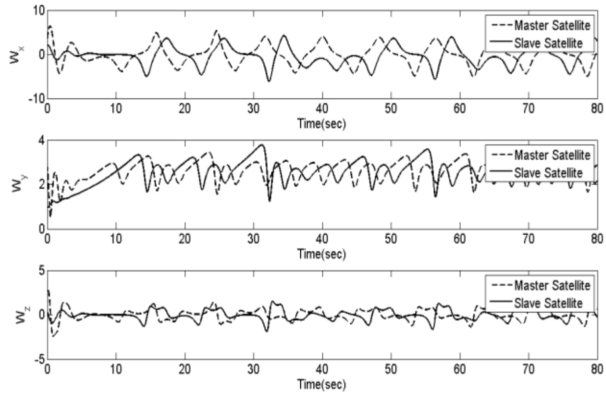

Fig.4.Chaotic Attitude of Satellites (Master & Slave)

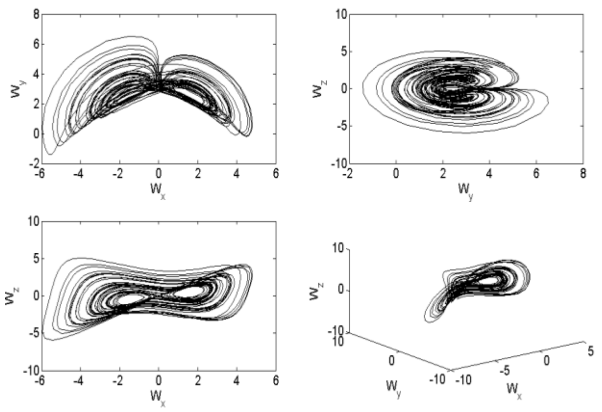

Fig.5.Phase Portraits of Chaotic Satellite

Then:

$$
\begin{cases}
\dot{e}_x = \delta_x \omega_{ys}\omega_{zs} - \frac{1.2}{I_x}\omega_{xs} + \frac{\sqrt{6}}{2I_x}\omega_{zs} \\
\quad -\delta_x \omega_{ym}\omega_{zm} - \frac{1.2}{I_x}\omega_{xm} + \frac{\sqrt{6}}{2I_x}\omega_{zm} + \psi_x(t) \\
\dot{e}_y = \delta_y \omega_{xs}\omega_{zs} + \frac{0.35}{I_y}\omega_{ys} \\
\quad -\delta_y \omega_{xm}\omega_{zm} - \frac{0.35}{I_y}\omega_{ym} + \psi_y(t) \\
\dot{e}_z = \delta_z \omega_{ys}\omega_{xs} - \frac{\sqrt{6}}{I_z}\omega_{xs} - \frac{0.4}{I_z}\omega_{zs} \\
\quad -\delta_z \omega_{ym}\omega_{xm} + \frac{\sqrt{6}}{I_z}\omega_{xm} + \frac{0.4}{I_z}\omega_{zm} + \psi_z(t)
\end{cases} \tag{17}
$$

Therefore vector control is:

$$
\begin{cases}
\psi_x = -\delta_x \omega_{ys}\omega_{zs} + \frac{1.2}{I_x}\omega_{xs} - \frac{\sqrt{6}}{2I_x}\omega_{zs} \\
\quad +\delta_x \omega_{ym}\omega_{zm} - \frac{1.2}{I_x}\omega_{xm} - \frac{\sqrt{6}}{2I_x}\omega_{zm} + U_x(t) \\
\psi_y = -\delta_y \omega_{xs}\omega_{zs} - \frac{0.35}{I_y}\omega_{ys} \\
\quad +\delta_y \omega_{xm}\omega_{zm} + \frac{0.35}{I_y}\omega_{ym} + U_y(t) \\
\psi_z = -\delta_z \omega_{ys}\omega_{xs} + \frac{\sqrt{6}}{I_z}\omega_{xs} + \frac{0.4}{I_{z2}}\omega_{zs} \\
\quad +\delta_z \omega_{ym}\omega_{xm} - \frac{\sqrt{6}}{I_z}\omega_{xm} - \frac{0.4}{I_z}\omega_{zm} + \psi_z(t)
\end{cases} \tag{18}
$$

With replacing equation (18) to (17), The controlled system and According to equation (8):

$$
\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{19}
$$

$$
\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{20}
$$

Then let the Lyapunov error function be $V(e) = \frac{1}{2}e^2$ where V(e) is a positive definite function. Assuming that the parameters of the master and slave systems are known and the states of both systems are measurable, may achieve the synchronization by selecting the controller U to make the first derivative of $V(e)$, i.e., $\dot{V}(e) < 0$. Then the states of slave system and master system are synchronized asymptotically globally.

$$
V(e) = \frac{1}{2}(e_x^2 + e_y^2 + e_z^2) \tag{21}
$$

With Eq. (22), the time derivative of the Lyapunov function along the trajectories of system (21) is:

$$\begin{aligned}\dot{V}(t) &= e_x\dot{e}_x + e_y\dot{e}_y + e_z\dot{e}_z \\ &= e_x(-Ke_x) + e_y(-Ke_y) + e_z(-Ke_z) \qquad (22) \\ &= -2KV(t)\end{aligned}$$

This implies that $\lim_{t\to\infty} e = 0$ which guarantees the global and exponential asymptotical stability of the origin of system (17). That is to say, systems (14) and (15) achieve global and exponential asymptotical synchronization. This completes the proof.

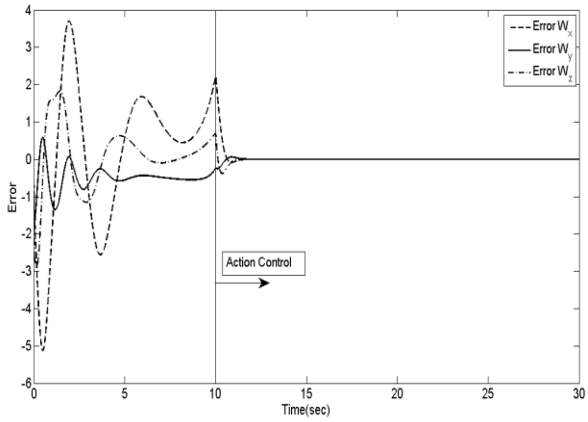

Fig.6. Synchronization errors of the satellites systems

4. SIMULATION RESULT

In this section, the fourth-order Runge–Kutta integration method is used to solve the system of differential equations. In the simulation process the initial states of the Master and the slave system are:

$$\begin{cases}[\omega_{xm}(0), \omega_{ym}(0), \omega_{zm}(0)]^T = [3 \quad 3 \quad 3]^T \\ \omega_{xs}(0), \omega_{ys}(0), \omega_{zs}(0)]^T = [2 \quad 1 \quad 0.5]^T\end{cases}.$$

The time responses of state variables of the satellite chaotic system are shown in Fig.4 and the phase Portraits of the satellite chaotic system are shown in Fig.5. Synchronization error is shown in Fig. 6. From the simulation results, it shows that the time responses of synchronization error under the proposed Active control converge quickly to zero; which means that perfect synchronization responses can be achieved. These results showed the efficiency of the control strategy. Signal control is shown Fig.7.

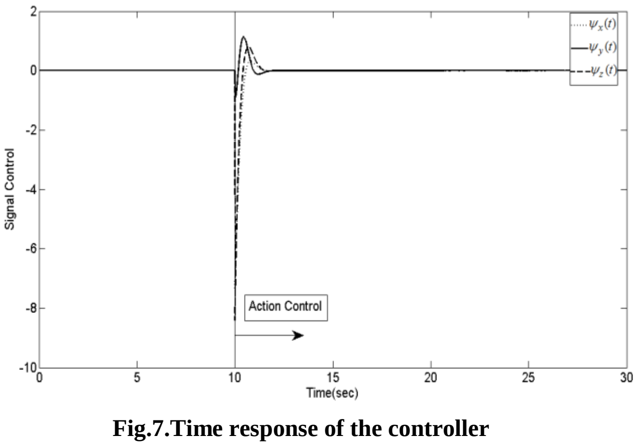

Fig.7.Time response of the controller

5. CONCLUSION

Subject to control and synchronize the satellites in space science is very important. Hence, the need for a controller that can control the satellite with chaotic dynamics.In this paper, an Active control has been proposed to synchronize two chaotic satellites systems identical, with initial condition different Master/Slave. The controller is designed to ensure perfect synchronization of the two systems. The numerical simulations have verified the effectiveness of the proposed method. Asymptotic stability of the closed-loop system is guaranteed by means of Lyapunov stability theory. The controller because of not large fluctuations can be used in practice.

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