Investigate the Performance of QAM Communication System by Transforming Linear Phase Filter Design using Parks-Mcclellan Algorithm into Minimum Phase Filter

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ABSTRACT
In this paper, the design of a low-pass minimum phase equiripple finite impulse response (FIR) digital filter from the given low-pass linear phase equiripple. The initial linear phase transfer function is obtained by standard Parks-Mcclellan algorithm. The shorter group delay should be minimal for efficient performance of digital pulse shaping filter like it improves the performance parameters like increased Capacity, reduced BER, better S/N ratio, and Reduced ISI (noise). The small group delay minimum phase filter coefficients were subjected to QAM (32, 64, and 128) system. The minimum phase filter performed well in terms of bit error rate (BER) parameter.

General Terms
Parks- Mcclellan Algorithm

Keywords
Linear phase FIR filter, Minimum phase filter, QAM, BER.

1. INTRODUCTION
Sometimes there is a requirement to design a FIR filter that not only does well but it is optimal. Optimization is the ability to state a maximum error on each band of interest. This error is expressed as the entire difference between the ideal or desired frequency response and the actual or resulting frequency response. The resulting filters are recognized as Equi-ripple FIR Filters. In order to minimize the error we need to define an error function E (θ) and a weight function G (θ) which defines the relative significance of the error at any given frequency θ. Then, the error function can be defined as follows [10]

$$E(\theta) = G(\theta)\,[Hd(\theta) - H(\theta)] \quad (1)$$

Where Hd(θ) is the desired amplitude response, and H(θ) is the actual amplitude response.

The weight function G (θ), defined as follows [12]

$$G(\theta)=\begin{Bmatrix} 1, \theta \in (\text{passband}) \\ 0, \theta \in (\text{stopband}) \end{Bmatrix} \quad (2)$$

And the resulting amplitude response, H(θ) is defined by:

$$H(\theta) = F(\theta)\,K(\theta) \quad (3)$$

And

$$K(\theta) = \sum_{k=0}^{m} b[k]\cos(k\theta) \quad (4)$$

Where

F (θ) =1, and M= $\frac{N}{2}$ for Type I FIR filter, N= Even order of filter.

Now, the main task here is to obtain the coefficients b[k] that minimize the maximum absolute weighted error |E (θ)|, that is, to obtain

$$\boldsymbol{\varepsilon} = \max|E(\theta)| \quad (5)$$

Where θ is in the operating frequency range of the filter.

2. THE ALTERNATION THEOREM
This theorem states that there exist at least 2 +K frequencies θi, {0 ≤ i ≤ K+1} where the maximum error, $\boldsymbol{\varepsilon}$, occurs. That is,

$$|E(\theta i)| = \quad 0 \le i \le K+1 \quad (6)$$

And

$$E(\theta_{i+1}) = -E(\theta), 0 \le i \le K \quad (7)$$

The last equation shows that the sign changes K+1 times, resulting in ripple on the band of interest.

3. PARKS-MCCLELLAN METHOD
The most common implementation of the Remez Exchange Algorithm is the descripted by Parks, McClellan and Rabiner [1] [4]. Its objective is to obtain the coefficients b[k] that minimize ε and it uses the properties of the Alternation Theorem mentioned above. Parks-McClellan method (also known as the Equiripple, Optimal, or Minimax method) with the Remez exchange algorithm is used to find an optimal equiripple set of coefficients to design an optimal linear phase filter. It minimizes the filter length for a particular set of design constraints. This method is used to design linear phase, symmetric or anti-symmetric filters of any standard type. Better filters result from minimization of maximum

error in both, the stop band and the pass band of the filter which leads to equiripple filters. In this algorithm to design FIR filters, some of its parameters such as the filter length (M), passband and stop band normalized frequencies (wp, ws), ripple in the pass band and stop band (δp, δs) are fixed and the remaining parameters are to be optimized. Parameters M, δp, and δs are fixed while the remaining parameters are optimized. The Parks–McClellan (PM) algorithm is the most widespread method for finest FIR filters design due to its flexibility and computational efficiency.

The first step is to find the order N of the desired filter. The following is an experiential formula for this [11]

$$N = \frac{-20\, log_{10}(\sqrt{\delta p \delta s}\,)-13}{2.32\,|\theta_{p-}\ \theta_s|} \qquad (8)$$

Where θ_p is the passband-edge digital frequency, θ_s is the stopband-edge digital frequency, δp is the passband deviation, δs is the stopband deviation.

$$\delta_p = (10^{Ap/20} - 1)\ (10^{Ap/20} + 1) \qquad (9)$$

$$\delta_s = 10^{-As/20} \qquad (10)$$

Where:

A_p And A_s are pass-band and stop-band attenuations respectively.

4. MINIMUM PHASE FILTER

FIR filters that have symmetric or anti-symmetric impulse responses have no phase distortion (their phase is linear). However, the difficulty with linear-phase filters is that the delay can be too great. The delay of a linear-phase filter is equal to $\frac{(N-1)}{2}$, where N is the length of the filter. To obtain a linear-phase FIR filter with a narrow transition-band and high stop-band attenuation requires making the filter long. Therefore, linear-phase FIR filters satisfying challenging specifications will have a large delay. This large delay could be a major drawback.

1. If the filter is used inside a feedback loop in a control system it could cause instability.

2. If the filter is used in a communication system it could cause delays that are longer than is tolerable.

For applications where it is vital to minimize the delay caused by a filter, a minimum-phase filter can be a good choice. Minimum- phase filters have all their zeros inside or on the unit circle. A minimum-phase filter can be obtained from a linear-phase filter by reflecting all of the zeros that are outside the unit circle to inside the unit circle. In other words, those zeros located at z = r ejθ are moved to z = $\frac{1}{r}e_{j\theta}$. This modification results in a minimum-phase filter that has the same frequency response magnitude |H ($e_{j\omega}$)| as the linear-phase filter. However, the minimum-phase filter obtained in this way will not be optimal in general. There will generally be a minimum-phase filter that is superior to the one gotten by simply reflecting the zeros in this way.

5. GROUP DELAY OF MINIMUM PHASE FILTER

The minimum phase system has the minimum group delay. Suppose, consider one zero α of the transfer function H(z). Lets place this zero inside unit circle (│α│<1) and see how the group delay is affected.

$$\alpha = │\alpha│e^{i\theta\alpha} \text{ where } \theta\alpha = Arg(\alpha) \qquad (11)$$

Since the zero α contributes the factor 1- αz^{-1} to the transfer function, the phase contributed by this term is the following.

$$\Phi\alpha(\omega) = \text{Arg}\ (1-\alpha e^{-i\omega}) \qquad (12)$$

=

Arg(1- │α│$e^{i\theta\alpha}e^{-i\omega}$)

= Arg(1- │α│$e^{-i(\omega-\theta\alpha)}$)

=Arg(1- │α│cos $(\omega-\theta\alpha)$+ i(│α│sin $(\omega-\theta\alpha)$

=Arg(1- │α│$^{-1}$ $-$ cos $(\omega-\theta\alpha)$+ i(sin $(\omega-\theta\alpha)$ (13)

Φ α(ω) contributes the following to the group delay [14]

$$-\frac{d\Phi\alpha(\omega)}{d\omega} = \frac{\sin^2(\omega-\theta\alpha)+\cos^2(\omega-\theta\alpha)-│\alpha│^{-1}\cos(\omega-\theta\alpha)}{\sin^2(\omega-\theta\alpha)+\cos^2(\omega-\theta\alpha)+│\alpha│^{-2}-2│\alpha│^{-1}\cos(\omega-\theta\alpha)}$$

$$-\frac{d\Phi\alpha(\omega)}{d\omega} = -\frac{│\alpha│-\cos(\omega-\theta\alpha)}{│\alpha│+│\alpha│^{-1}-2\cos(\omega-\theta\alpha)} \qquad (14)$$

The denominator and $\boldsymbol{\theta}$α are invariant to reflecting the zeros α outside the unit circle, bi. e., replacing α with $(\alpha^{-1})*$.However, by reflecting α outside unit circle, the increases the magnitude of |α| in the numerator, Thus having α inside the unit circle minimizes the group delay.

6. MEASUREMENT RESULTS

The measurement results carried out by using MATLAB. As the group delay of the filter play an important role in the performance of system. Smaller the group delay minimum the number of errors occurred [13].The minimum phase filter gives shorter group delay then simple linear phase filter.QAM (32,64,128) communication system subjected to minimum phase filter's coefficients

Table1. Performance analysis based on BER parameter (32QAM)

EbNo	BER(theoretical)	BER(Original)	BER(minimum Phase)
0	0.1895	0.1325	0.1301
1	0.1673	0.1193	0.1166
2	0.1461	0.1045	0.1016
3	0.1260	0.0887	0.0855

4	0.1066	0.0722	0.0690
5	0.0880	0.0559	0.0529
6	0.0702	0.0406	0.0379
7	0.0536	0.0274	0.0251
8	0.0388	0.0168	0.0151
9	0.0262	0.0092	0.0081
10	0.0162	0.0043	0.0037
11	0.0090	0.0017	0.0014
12	0.0044	0.0006	0.0004
13	0.0018	0.0001	0.0000
14	0.0006	0.0000	0.0000
15	0.0002	0.0000	0.0000
16	0.0000	0.0000	0.0000

Table2. Performance analysis based on BER parameter (64QAM)

EbNo	BER(theoretical)	BER(Original)	BER(minimum Phase)
0	0.1998	0.1018	0.0996
1	0.1779	0.0898	0.0873
2	0.1570	0.0767	0.0741
3	0.1372	0.0630	0.0604
4	0.1185	0.0494	0.0468
5	0.1008	0.0364	0.0341
6	0.0838	0.0249	0.0230
7	0.0676	0.0156	0.0141
8	0.0523	0.0088	0.0077
9	0.0385	0.0043	0.0037
10	0.0265	0.0018	0.0015
11	0.0169	0.0006	0.0005
12	0.0097	0.0002	0.0001
13	0.0049	0.0000	0.0000
14	0.0022	0.0000	0.0000
15	0.0008	0.0000	0.0000
16	0.0002	0.0000	0.0000

Table3. Performance analysis based on BER parameter128QAM

EbNo	BER(theoretical)	BER(Original)	BER(minimum Phase)
0	0.2477	0.0805	0.0784
1	0.2272	0.0695	0.0673
2	0.2065	0.0579	0.0557
3	0.1858	0.0461	0.0439
4	0.1656	0.0348	0.0327
5	0.1463	0.0244	0.0227
6	0.1281	0.0158	0.0144
7	0.1111	0.0092	0.0082
8	0.0950	0.0047	0.0041
9	0.0796	0.0021	0.0017
10	0.0650	0.0008	0.0006
11	0.0511	0.0002	0.0002
12	0.0383	0.0000	0.0000
13	0.0271	0.0000	0.0000
14	0.0177	0.0000	0.0000
15	0.0106	0.0000	0.0000
16	0.0057	0.0000	0.0000

17	0.0012	0.0000	0.0000
18	0.0002	0.0000	0.0000
19	0.0001	0.0000	0.0000

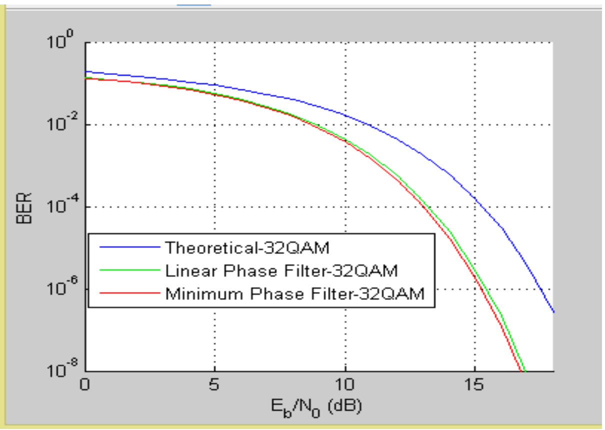

Figure 1. BER versus EbNo Performance Curves for 32QAM

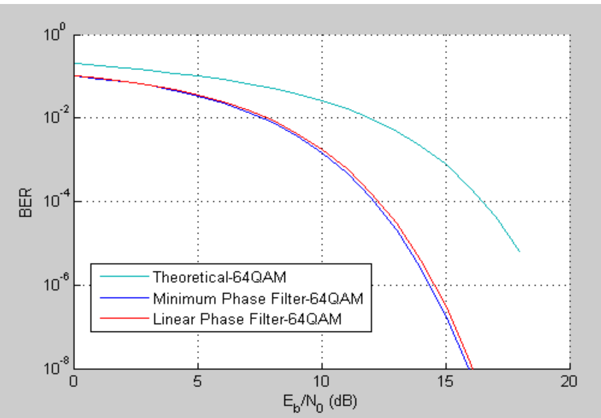

Figure 2. BER versus EbNo Performance Curves for 64QAM

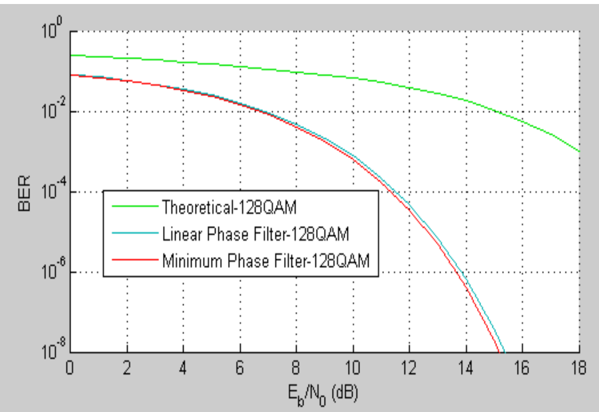

Figure3. BER versus EbNo Performance Curves for 128QAM

7. CONCLUSION

In this paper, A Minimum Phase FIR digital filter design approach from a given low-pass FIR transfer function, which has identical amplitude response, author deal with the equiripple amplitude response and observe that by using a minimum-phase filter obtained from linear phase parks-mcclellan algorithm, got a filter that has a better frequency response magnitude as it reduces the group delay. The group delay plays a crucial role in pulse shaping digital finite impulse response filter. The group delay actualy is the time interval between the signal fed into the system (filter) and and comes out of the system. The during processing the signal, some kind of system errors affects the signal (insertion loss) and thus reduced the output SNR and corresponding system performance. Insertion loss is the decrease in power delivered to the load when a filter is inserted between the source and the load. To reduce the insertion loss signal must come out as soon as posible (shorter group delay). The value of group delay should be minimal for efficient performance of digital pulse shaping filter like it improves the performance parameters like increased Capacity, reduced BER, better S/N ratio, and Reduced ISI (noise).The minimum phase filter has all zeros inside the unit circle, thus this arrangement corresponds to the minimum increase in total phase, which corresponds to minimum average total phase delay, which corresponds to maximum compactness in time, for any given(stable) set of poles and zeros with the exact same frequency magnitude response. The coefficients obtained from the minimum phase filter, when subjected to system with QAM, gives better result than original one, in terms BER metric.

8. FUTURE SCOPE

In this paper Parks McClellan (PM) Algorithm is used to design filter.A number of algorithms are avaliable in literature like Evolutionary Algorithm, Root Moments, complex cepstrum etc. can be used.

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10. REFERENCES

[1] J. H. McCllelan, T. W. Parks, and L. R. Rabiner, "A program for designing optimum FIR linear phase digital filters," IEEE Trans. Audio Electroacoust., vol. AU-21, pp. 506–526, Dec. 1973.Ding, W. and Marchionini, G. 1997 A Study on Video Browsing Strategies. Technical Report. University of Maryland at College Park.

[2] G. A. Mian and A. P. Nainer, "A fast procedure to design equiripple minimum-phase FIR filters," IEEE Trans. Circuits Syst., vol. CAS-29, pp. 327–331, 1982.Tavel, P. 2007 Modeling and Simulation Design. AK Peters Ltd.

[3] O. Herrmann, H. W. Schubler, Design of Nonrecursive Digital Filters with Minimum Phase, Electronic Letters, vol. 6, 1970, pp. 329-330

[4] Vijay K. Madisetti, Douglas B. Williams, The Digital Signal Processing Handbook, CRC Press and IEEE Press, 1998.

[5] L. R. Rabiner,J. H.McCllelan, and T. W.Parks, "FIR digital filter design techniques using weighted Chebychev approximation," Proc. IEEE, vol. 63, pp. 595–610, Apr. 1975.

[6] Y. Kamp and C. J. Wellekens, "Optimal design of minimum phase FIR filters," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-31, pp. 922–926, 1983.

[7] F. E. Burris, "Design of minimum-phase digital filters from linear phase prototypes," in Proc. 13th Annu. Lerton Conf. Circuits and Systems, Oct. 1–3, 1975, pp. 485–497.

[8] N. N. Chit and J. S. Mason, "Design of minimum phase FIR digital filters," Proc. Inst. Elect. Eng., pt. G, vol. 135, no. 6, Dec. 1988.

[9] T. Stathaki, A. G. Constantinides, and G. Stathakis, "Equiripple min- imum phase piecewise flat FIR filter design from linear phase systems using root moments," in Proc. IEEE ICASSP 98, Seattle, WA.

[10] Amritpal Singh, Dr. Naveen Dhillon, Sukhpreet Singh Bains, ''A Review on Fractional Delay FIR Digital Filters Design and Optimization Techniques,'' International Journal of Computer & Organization Trends –Volume 3 Issue 10 – Nov 2013

[11] Yuan-Pei Lin, P. P. Vaidyanathan, "A Kaiser Window Approach for the Design of Prototype Filters of Cosine Modulated Filterbanks," IEEE Signal processing letters,vol.5,no.6,june 1998

[12] A.ENIS CETIN,OMER N.GEREK,AND YASEMIN YARDIMCI, , "Equiripple FIR filter design by FFT algorithm," signal processing magazine 1997 IEEE

[13] A S Kang, Vishal Sharma, "Analysis of Simulation Parameters of Pulse Shaping FIR Filter for WCDMA" Vol 1, No 1 (June 2010) © IJoAT

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