

Multi-Warehouses Inventory Problem of Deteriorating Items with Fuzzy Lead-Time and Partial Lost Sales under Inflation and Time Value of Money

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ABSTRACT

In this paper, a realistic replenishment model with multiple warehouses (one is primary warehouse (PW) from where the items are sold and others are secondary warehouses (SWs) where the items are stored) is developed with fuzzy lead-time under the assumption that the capacities of the warehouses are finite. Inflation and time value of money are taken into account. The items of secondary warehouses are transported to the primary warehouse in continuous release pattern and associated transportation cost is proportional to the distance from PW to SWs. The holding cost of items in SWs has reverse effect with distance. Here, the demand of items is a deterministic function of selling price and the displayed inventory. Deterioration rates of the items are constant and different in different warehouses. The replenishment rate is infinite and the problem is constructed with shortages, which are the mixture of back orders and lost sales. The backlogged demand is assumed to be a function of currently backlogged amount. When an item is out of stock, the loyal and captive customers will wait until the outstanding orders arrive and are served. To compensate the inconvenience of backordering and to secure orders, the supplier may offer a price discount on the stock out item. There are three scenarios depending upon the time when the new order is placed for the next cycle. The problem is illustrated with the help of numerical examples.

Keywords

Inventory, Multi-warehouses, Deterioration, Fuzzy lead-time, Back order, Lost sale, Backorder discount, Inflation, Time value of money.

1. INTRODUCTION

In important markets like super market, corporation market, municipality market etc., the space for a showroom is limited. When an attractive price discount for bulk purchase is available or the cost of procuring goods is higher than the inventory related other costs or the demand of items is very high, management then decides to purchase a large amount of items at a time. These large numbers of items may not be accommodated in the existing showroom (viz., the Primary Warehouse, PW) due to limited capacity. To store the excess items, one or more additional warehouses (viz., the Secondary Warehouses, SWs) are hired. These secondary warehouses may be located near PW or a little / far away from it. The items are stored first in PW and only excess stocks are stored in SW. The actual service to the customer is done at PW only. The items of SWs are transferred to PW in a continuous/bulk release pattern to meet up the demand at PW until the stock levels in SWs are emptied and lastly the items of PW are released. There are several related papers presented in this

area such as Hartley [1], Sarma[2], Goswami and Chaudhuri[3], Pakkala Achary[4,5], Bhunia and Maiti[6] and others.

Many supermarket/corporation market managers have observed that for certain items, displayed stock level (DSL) has a positive impact on the demand rate. A large amount of stock generates high rate of consumption and consumption rate goes down when the stock level is low. Hence, it is concluded that there is a functional relationship between the demand of an item and its displayed stock level in a showroom / shop. This relationship was first considered by Levin et. al.[7] and then by Silver and Peterson[8], Backer and Urban[9], Mandal and Phauzdar[10], Datta and Pal[11,12] etc. According to real life situation, the demand rate is not only dependent on the stock level, it also depends on the selling price and it has a reverse effect.

A stock out occurs in the business of all types of items. During the stock out period, either all the demand is backordered, in which all customers wait until their demand is satisfied; or all the demand is lost. However, in many real inventory systems, demand can be captive partially. For customers whose needs are not urgent at that time can wait to be satisfied, while others who cannot wait will balk away to some other sources. The cost for a lost sale ranges from profit loss on the sale to some unspecified loss of good will. In order to compensate customers for the inconvenience of waiting, the idleness of equipment, or even lost production during the stock out period, the supplier may offer a variable price discount (Pan et. al [14,15]) on the stock out item depending on the seriousness of the back order condition.

From financial standpoint, an inventory represents a capital investment and must compete with other assets within the firm's limited capital funds. Most of the classical inventory models did not take into account the effects of inflation and time value of money. This has happened mostly because of the belief that inflation and time value of money will not influence the cost and price components i.e., the inventory control policy to any significant degree. But, during last few decades, due to high inflation and consequent sharp decline in the purchasing power of money in the developing countries like Brazil, Argentina, India, Bangladesh etc., the financial situation has been completely changed and so it is not possible to ignore the effect of inflation and time value of money any further. Following Buzacott [16] and Misra [17], several researchers (Ray and Chaudhuri [18], Sarkar et.al [19] etc.) have extended their approaches to different inventory models by considering the time value of money, different inflation

rates for the internal and external costs, finite replenishment, shortages, etc.

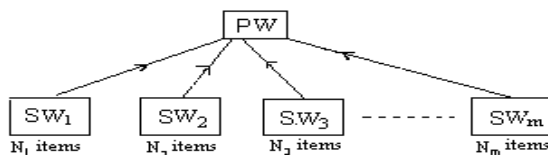
In this paper, we consider one Primary Warehouse (PW), m secondary warehouses (viz., SW₁, SW₂, ..., SW_m) and N items. The warehouses SW₁, SW₂, ..., SW_m contains N₁, N₂, ..., N_m items respectively such that N₁ + N₂ + ... + N_m = N. Items are sold from PW which is located at main market and shortages are allowed at this shop. Due to large stock and insufficient space of existing PW, excess items are stored in m secondary warehouses (viz., SW₁, SW₂, ..., SW_m) of finite capacity which are far away from PW. Here all the warehouses are of rental basis but rent of PW is greater than the rent of the secondary warehouses as they (SW's) are in far away from market place. Deterioration rate of an item is taken to be different in different warehouses. The stocks of SW's are transferred to PW under continuous release pattern and the associated transportation cost is directly varies with the distance from PW to SWs but the holding cost of an item in SWs has reverse effect with distance. In realistic retail situations, some customers wait for backlogged items in stock-out period and hence lost sale occurs. The backlogged demand is assumed to be a function of currently backlogged amount. Here, the problem is developed with fuzzy lead-time under inflation and time value of money. The fuzzy parameter is transformed into a crisp number by Graded Mean Integration Value(GMIV) method (Chen et. al.[13]). There are three scenarios in each cycle depending upon the time when new order is placed:

Scenario-I: New order is placed at the time before exhaustion of the inventory level of SW_j.

Scenario-II: New order is placed at the time when inventory level of SW_j reaches zero.

Scenario-III: New order is placed when inventory level is only at PW.

Numerical examples are used to illustrate each scenario via genetic algorithm. *Block diagram of the model is given below:*



2. ASSUMPTIONS AND NOTATIONS

The inventory model with fuzzy lead-time is developed under the following assumptions and notations:

Assumptions:

- (i) Rate of replenishment is infinite and the replenishment size is finite.
- (ii) The inventory-planning horizon is infinite and the inventory system involves n items.
- (iii) Shortages are allowed in PW and backlogged them fully.
- (iv) There is no quantity discount.
- (v) The units will be sold from primary warehouse (PW) and the space in PW will be immediately filled up by shifting same

amount from secondary warehouses (SW's) to PW.

(vi) Time tag between selling from PW and filling up its space by new units from SW's is negligible.

(vii) Wavy bar '~' represents the fuzzification of the parameter.

Notations:

For i-th items (i=1,2,...,N) and j-th secondary warehouse (j=1,2,...,m)

- (i) S_i = Total stock of the system at t=0.
- (ii) W_i = Inventory of i-th item at PW.
- (iii) W_i' = On hand inventory of the system for i-th item when the new order is placed for the next cycle.
- (iv) R_i = Shortage amount for i-th item.
- (v) t_{1i} = Time when new order is placed for the next consignment.
- (vi) t_{2i} = Time when inventory level of SW_j is zero.
- (vii) t_{3i} = Time when inventory level of PW is zero i.e., when shortage begins.
- (viii) t_{4i} = Total time period for the cycle.
- (ix) C_{1i}^P = The inventory carrying cost per unit per unit time in PW.
- (x) d_j = Distance of SW_j from PW.
- (xi) C_{1i}^{Swj} = C_{1i}^P - c' d_j = The inventory carrying cost per unit per unit time in SW_j.
- (xii) C_{2i} = Shortage cost per unit per unit time.
- (xiii) C_{3i} = Replenishment cost per cycle.
- (xiv) L_i~ = Imprecise lead-time for the i-th item, where L_i~ = (L_{1i}, L_{2i}, L_{3i}) is parabolic or triangular fuzzy number.
- (xv) q_{1i}(t) = Inventory level of secondary warehouses (SW_j) at any time t.
- (xvi) q_{2i}(t) = Inventory level of primary warehouse (PW) at any time t.
- (xvii) f(p_i, q_i) = α_i - β_i p_i + γ_i q_i(t), where q_i(t) is the inventory level at any time t.
- (xviii) θ_{1i} = Constant deterioration rate for i-th item in PW.

- (xix) θ_{2i} = Constant deterioration rate for i-th item in SW_j .
- (xx) $C_{pi} = C'_{pi} - C''_{pi} S_i^{K_1} - C'''_{pi} L_i$,
Purchasing cost per unit quantity depends upon the initial stock level S_i and lead-time L_i ($C'_{pi}, C''_{pi}, C'''_{pi}, K_1, K_2 > 0$).
- (xxi) $p_i = K' \cdot C_{pi}$, selling price per unit quantity in stock-in period.
- (xxii) π_{di} = Backorder price discount offered by the supplier per unit which is variable and lies between 0 and $(p_i - C_{pi})$.
- (xxiii) $p'_i = p_i - \pi_{di}$, Selling price per unit quantity in stock-out period.
- (xxiv) $T_{ci} = T'_{ci} + T''_{ci} S_{RPi} d_i$, transportation cost for transporting S_{RPi} units from SW_j to PW ($T'_{ci}, T''_{ci}, k > 0$) where T'_{ci} is the fixed transportation cost and T''_{ci} is the transportation cost per unit per unit distance.
- (xxv) R = Difference between inflation rate and discount rate.
- (xxvi) PW_C = Total capacity of PW .
- (xxvii) SW_{Cj} = Capacity of j-th secondary warehouse.

3. MATHEMATICAL FORMULATION

Pictorial representation of the scenarios is depicted as below:

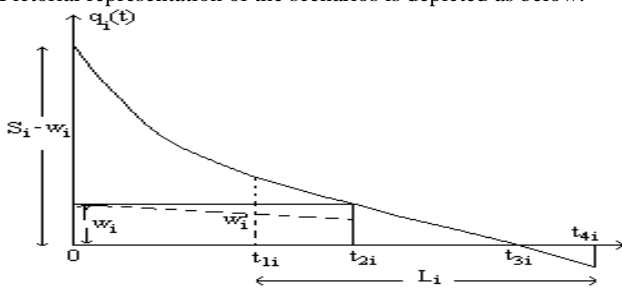


Fig-1: inventory situation in scenario-I

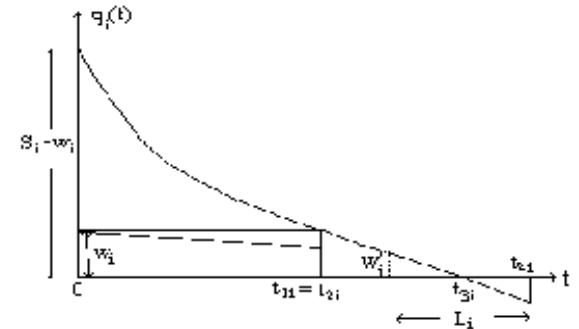
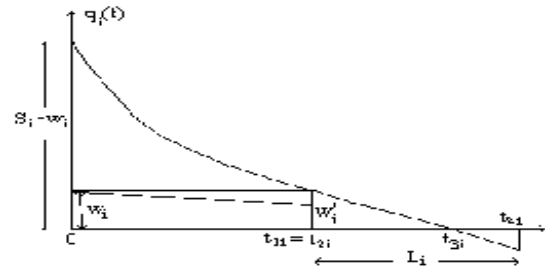


Fig-2: Inventory situation in scenario-II
Fig-3: Inventory situation in scenario-III

Formulation of scenario-I

The differential equation describing the inventory level $q_{1i}(t)$ ($i=1,2,\dots,n$) of the system is

$$\frac{dq_{1i}(t)}{dt} = -\theta_{1i} q_{1i}(t) - D_i - \theta_{2i} w_i, \quad 0 \leq t \leq t_{2i} \quad (1)$$

with boundary conditions:

$$\begin{aligned} q_{1i}(t) &= S_i - w_i & \text{at } t &= 0 \\ &= w'_i - w_i & \text{at } t &= t_{1i} \\ &= 0 & \text{at } t &= t_{2i} \\ &= -R_i & \text{at } t &= t_{4i} \end{aligned} \quad (2)$$

The differential equation describing the inventory level $q_{2i}(t)$ ($i=1,2,\dots,n$) of the system is

$$\frac{dq_{2i}}{dt} = \begin{cases} -(\theta_{2i} q_{2i} + D_i), & t_{2i} \leq t \leq t_{3i} \\ -(D_i + \delta_i q_{2i}), & t_{3i} \leq t \leq t_{4i} \end{cases} \quad (3)$$

where $\delta_i = \frac{\delta_0}{\pi_{di}}$ ($\delta_0 > 0$) and boundary conditions are:

$$\begin{aligned} q_{2i}(t) &= w_i & \text{at } t &= t_{2i} \\ &= 0 & \text{at } t &= t_{3i} \\ &= -R_i & \text{at } t &= t_{4i} \end{aligned} \quad (4)$$

where

$$D_i = \begin{cases} f(p_i, w_i), & q_i \geq w_i \\ f(p_i, q_i), & 0 \leq q_i \leq w_i \\ f(p_i, 0), & q_i < 0 \end{cases}$$

(5)

Using (2), the solution of the differential equation (1) is given by

$$q_{1i}(t) = \frac{1}{\theta_{1i}} \{f(p_i, w_i) + \theta_{2i} w_i\} \{e^{\theta_{1i}(t_2i-t)} - 1\}$$

(6)

Using (4), the solution of the differential equation (3) is given by

$$q_{2i}(t) = \begin{cases} \frac{(\alpha_i - \beta_i p_i)}{\gamma_i + \theta_{2i}} \{e^{(\gamma_i + \theta_{2i})(t_{3i}-t)} - 1\}, & t_{2i} \leq t \leq t_{3i} \\ \frac{D_i}{\delta_i} \{e^{\delta_i(t_{3i}-t)} - 1\}, & t_{3i} \leq t \leq t_{4i}^{Hi} \end{cases}$$

(7)

Using (2) and (4), equations (5) and (6) give

$$S_i = w_i + \frac{1}{\theta_{1i}} \{f(p_i, w_i) + \theta_{2i} w_i\} \{e^{\theta_{1i} t_{2i}} - 1\}$$

$$= w_i + \frac{1}{\theta_{1i}} \{\alpha_i - \beta_i p_i + (\gamma_i + \theta_{2i}) w_i\} \{e^{\theta_{1i} t_{2i}} - 1\}$$

(8)

$$w_i' = w_i + \frac{1}{\theta_{1i}} \{f(p_i, w_i) + \theta_{2i} w_i\} \{e^{\theta_{1i}(t_{2i}-t_{1i})} - 1\}$$

(9)

$$\tilde{R}_i = \frac{D_i}{\delta} \{1 - (1 - \delta \tilde{L}_i) e^{\delta(t_{3i} - t_{1i})}\}$$

(10)

$$t_{1i} = \int_{w_i-w_i}^{S_i-w_i} \frac{dq_{1i}}{\theta_{1i} q_{1i}(t) + f(p_i, q_i) + \theta_{2i} w_i}$$

$$= \frac{1}{\theta_{1i}} \log \left| \frac{\alpha_i - \beta_i p_i + (\gamma_i + \theta_{2i} - \theta_{1i}) w_i + \theta_{1i} S_i}{\alpha_i - \beta_i p_i + (\gamma_i + \theta_{2i} - \theta_{1i}) w_i + \theta_{1i} w_i} \right|$$

(11)

$$t_{2i} = \int_0^{S_i-w_i} \frac{dq_{1i}}{\theta_{1i} q_{1i}(t) + f(p_i, q_i) + \theta_{2i} w_i}$$

$$= \frac{1}{\theta_{1i}} \log \left| \frac{\alpha_i - \beta_i p_i + (\gamma_i + \theta_{2i} - \theta_{1i}) w_i + \theta_{1i} S_i}{\alpha_i - \beta_i p_i + (\gamma_i + \theta_{2i}) w_i} \right|$$

(12)

$$t_{3i} = t_{2i} + \int_0^{w_i} \frac{dq_{2i}}{\theta_{2i} q_{2i}(t) + f(p_i, q_i)}$$

$$= \frac{1}{\gamma_i + \theta_{2i}} \log \left| \frac{\alpha_i - \beta_i p_i + (\gamma_i + \theta_{2i}) w_i}{\alpha_i - \beta_i p_i} \right|$$

(13)

C_{1i}^{Swj} = Present value of holding cost for the i-th item in the j-th Secondary Warehouse (SW_j)

$$= C_{1i}^{Swj} \int_0^{t_{2i}} q_{1i}(t) e^{-Rt} dt$$

(14)

$$= \frac{C_{1i}^{Swj}}{\theta_{1i}} \{\alpha_i - \beta_i p_i + (\gamma_i + \theta_{2i}) w_i\} \left[\frac{1}{\theta_{1i} + R} (e^{\theta_{1i} t_{2i}} - e^{-Rt_{2i}}) - \frac{1}{R} (1 - e^{-Rt_{2i}}) \right]$$

C_{Hi}^P = Present value of holding cost for the i-th item in the Primary Warehouse (PW)

$$= C_{1i}^P \left[\int_0^{t_{2i}} w_i e^{-Rt} dt + \int_{t_{2i}}^{t_{3i}} q_{2i}(t) e^{-Rt} dt \right]$$

$$= C_{1i}^P \left[\frac{w_i}{R} (1 - e^{-Rt_{2i}}) + \frac{\alpha_i - \beta_i p_i}{(\gamma_i + \theta_{2i})^2 + R(\gamma_i + \theta_{2i})} e^{(\gamma_i + \theta_{2i}) t_{3i}} \left\{ e^{-(\gamma_i + \theta_{2i} + R)t_{2i}} - e^{-(\gamma_i + \theta_{2i} + R)t_{3i}} \right\} \right]$$

$$- \frac{\alpha_i - \beta_i p_i}{R(\gamma_i + \theta_{2i})} (e^{-Rt_{3i}} - e^{-Rt_{2i}})]$$

(15)

C_{Di} = Present value of the cost for the total deteriorated i-th item during (0, t_{3i})

$$\begin{aligned}
 &= p_i \left[\int_0^{t_{2i}} \theta_{1i} q_{1i}(t) e^{-Rt} dt + \int_0^{t_{2i}} \theta_{2i} w_i e^{-Rt} dt + \int_{t_{2i}}^{t_{3i}} \theta_{2i} q_{2i}(t) e^{-Rt} dt \right] \\
 &= \frac{p_i \theta_{2i} C_{Hi}^{Swj}}{C_{li}^{Swj}} + \frac{p_i \theta_{2i} C_{Hi}^P}{C_{li}^P} \\
 &= TC_i' + S_{RPi} d_j TC_i'', \quad j=1,2,\dots,m
 \end{aligned}$$

S_{RPi} = Total units of i-th item transferred from SW_j 's to PW

$$\begin{aligned}
 &= (S_i - w_i) - \int_0^{t_{2i}} \theta_{1i} q_{1i}(t) dt \\
 &= (S_i - w_i) - \{ \alpha_i - \beta_i p_i + (\gamma_i + \theta_i) w_i \} \left\{ \frac{1}{\theta_{1i}} (e^{\theta_{1i} t_{1i}} - 1) - t_{2i} \right\}
 \end{aligned}$$

\tilde{C}_{Si} = Present value of Shortage cost during (t_{3i}, t_{4i})

$$= -C_{2i} \int_{t_{3i}}^{t_{4i}} q_{2i}(t) e^{-Rt} dt$$

$$= \frac{C_{2i} D_i}{(R + \delta_i) \delta_i} \left[\{ 1 - (R + \delta_i) \tilde{L}_i \} e^{-Rt_{1i} + \delta_i (t_{3i} - t_{1i})} - e^{-Rt_{3i}} \right] - \frac{C_{2i} D_i}{R \delta_i} \left[(1 - R \tilde{L}_i) e^{-Rt_{1i}} - e^{-Rt_{3i}} \right]$$

\tilde{P}_{ci} = Present value of total purchasing cost for the i-th item during $(0, t_{4i})$

$$= C_{pi} \left\{ S_i + R_i (1 - R \tilde{L}_i) e^{-Rt_{1i}} \right\}$$

(19)

\tilde{P}_i = Selling price for the i-th item = $K_4 \tilde{C}_{Pi}$

\tilde{S}_{pi} = Present value of total sales for the i-th item during $(0, t_{4i})$

$$\begin{aligned}
 &= p_i \int_0^{t_{3i}} D_i e^{-Rt} dt + p_i R_i e^{-R \tilde{t}_{4i}} \\
 &= p_i \int_0^{t_{2i}} (\alpha_i - \beta_i p_i + \gamma_i w_i) e^{-Rt} dt + p_i \int_{t_{2i}}^{t_{3i}} \{ \alpha_i - \beta_i p_i + \gamma_i q_{2i}(t) \} e^{-Rt} dt
 \end{aligned}$$

(20)

TC_i^j = Transportation cost for transporting S_{RPi} units from SW_j to PW

where TC_i' = Fixed transportation cost for the i-th item and TC_i'' = Transportation cost per unit per unit distance.

The profit function for the i-th item is given by

$$\begin{aligned}
 PROF_i &= \text{Average profit for the i-th item during } (0, t_{4i}) \\
 &= \frac{\tilde{S}_{pi} - \tilde{P}_{ci} - C_{di} - C_{Hi}^{Swj} - C_{Hi}^P - \tilde{C}_{Si} - C_{3i} - TC_i^j}{t_{4i}}
 \end{aligned}$$

Total profit for N-items is

$$PROF = \sum_{i=1}^N PROF_i$$

Our problem is to Maximize PROF

$$\text{Subject to, } \sum_{i=1}^N a_i w_i \leq OW$$

$$\sum_{i=N_{j-1}+1}^{N_j} a_i (S_i - w_i) \leq RW_j,$$

$j=1,2,3,\dots,m$ with $N_0=0$.

$$\text{and } N_1 + N_2 + N_3 + \dots + N_m = N$$

4. GRADED MEAN INTEGRATION VALUE (GMIV) OF A FUZZY NUMBER

Let A be a fuzzy number with membership function $\mu_{\tilde{A}}(x)$ as given below:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & -\infty < x \leq a \\ L(x) & a \leq x \leq b \\ 1 & b \leq x \leq c \\ R(x) & c \leq x \leq d \\ 0 & d \leq x < \infty \end{cases}$$

where $L(x)$ is strictly increasing within $[a,b]$, $R(x)$ is strictly monotonic decreasing within $[c,d]$. Then

according to Chen and Hsien (1999), GMIV of \tilde{A} is denoted by $P(\tilde{A})$ and defined as:

$$P(\tilde{A}) = \frac{\int_0^1 x \{w L^{-1}(x) + (1-w) R^{-1}(x)\} dx}{\int_0^1 x dx} = 2 \int_0^1 x \{w L^{-1}(x) + (1-w) R^{-1}(x)\} dx$$

Where $w \in [0,1]$ a pre-assigned parameter is called degree of optimism. $w=1$ represents an optimistic viewpoint, $w=0$ represents a pessimistic viewpoint and $w=0.5$ reflects a moderately optimistic DM's view point. Using this rule GMIV of different type of fuzzy number is obtained and present in Table-1.

Table-1: GMIV formula for different fuzzy numbers with degree of optimism w

Triangular fuzzy number(TFN) (a,b,c)	Trapezoidal fuzzy number(TrFN) (a,b,c,d)
$[wa+2b+(1-w)c]/3$	$[w(a+2b)+(1-w)(d+2c)]/3$

5. GENETIC ALGORITHM

A genetic algorithm is a heuristic search process for optimization that resembles natural selection. Gas was first proposed by J. Holland. It has been applied successfully in different areas (Davis[20]). As the name suggests, GA originated from the analogy of biological evolution. GAs considers a population is a set of feasible solutions of a problem. A member of the population is called a genotype, a chromosome, a string or a permutation. A genetic algorithm contains three operators-reproduction, crossover and mutation. Initially, a population is selected and by means of above operators, the better of the population will remain, because of the survival of the fittest. The GAs procedure is shown below:

```

begin
t←0
initialize Population(t)
evaluate Population(t)
while (not terminate-condition)
{
t←t+1
select Population(t) from Population(t-1)
after (crossover and mutate) Population(t)
evaluate Population(t)
}
Print Optimum Result
end.
```

Implementing GA

It is generally accepted that a GA to solve a decision-making problem must have five basic components:

- (i) values for the parameters (population size, probabilities of applying genetic operators, etc.)
- (ii) a genetic representation for potential solutions,
- (iii) a way to create an initial population of solutions,
- (iv) an evaluation function (i.e., the environment), rating solutions in terms of their "fitness", and
- (v) genetic operators that alter the genetic composition of parents during reproduction.

We discuss these components for the system developed for the present model in the following sections.

Parameters:

Firstly, we set the different parameters on which this GA depends. These are the number of generations (MAXGEN), population size (POPSIZE), probability of crossover (PXOVER), probability of mutation (PMU). There is no clear indication as to how large a population should be. If the population is too large, there may be difficulty in storing the data, but if the population is too small, there may not be enough string for good crossovers. In our experiment, a population consists of 50 members i.e.,

POPSIZE = 50. In this case, PXOVER=0.2, PMU=0.2, MAXGEN=5000.

Chromosome representation:

An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chromosome are not effective in many highly non-linear physical problems. Since the proposed problem is highly non-linear, hence to overcome the difficulty, a real number representation is used. In this representation, each chromosome V_i consists of the genes

G_{iw_j}, G_{iS_j} and $G_{iw'_j}$ where these genes respectively denote decision variables w_i, S_i and w'_i i.e., $V_i = \{G_{iw_j}, G_{iS_j}, G_{iw'_j}, j=1,2,3,4\}$, $i= 1$ to

POPSIZE.

Initial population:

To initialize the population, we first determine the independent and dependent variables and then their boundaries. For each chromosome V_i , the gene G_{iw_j} which is independent, is randomly generated between its boundaries (LB_K, UB_K) where LB_K and UB_K are the lower and upper bounds of the decision variable and after that other genes are also generated randomly between its boundaries until it is feasible by the following algorithm.

Step-I: $G_{iw_j} = \text{Rand}(0, \frac{OW}{a_j})$, $j=1$ to 4

Step-II: $G_{iS_j} = \text{Rand}(G_{iw_j}, \frac{RW_1}{a_j})$, $j=1, 2$

and $G_{iS_j} = \text{Rand}(G_{iw_j}, \frac{RW_2}{a_j})$, $j=3, 4$

Step-III: $G_{iwj} = \begin{cases} \text{Rand}(G_{iwj}, G_{isj}), j=1 \text{ to } 4 \text{ for scenario-I} \\ \text{Rand}(0, G_{iwj}), j=1 \text{ to } 4 \text{ for scenario-III} \end{cases}$

Step-IV: If all constraints are not satisfied by V_i then go to step-I

Evaluation:

Evaluation function plays the same role in GA as that which the environment plays in natural evaluation. To this problem, the evaluation function is $\text{EVAL}(V_i) = \text{objective function value}$.

Selection:

Before selection process, all chromosomes V_i are arranged in descending order according to $\text{eval}(V_i)$. After that the roulette wheel selection process is applied on it POPSIZE times. Each time we select a single chromosome for the new population in the following way:

- (a) Calculate the fitness value $\text{eval}(V_i)$ for each chromosome V_i
- (b) Find Total fitness of the population $\sum_{i=1}^{\text{popsize}} \text{eval}(V_i)$
- (c) Calculate the probability of selection, $p_i = \text{eval}(v_i)/F$ for each chromosome V_i
- (d) Calculate the cumulative probability q_i for each chromosome $V_i : q_i = \sum_{j=1}^i p_j$
- (e) Generate a random real number r in $(0,1)$.
- (f) If $r < q_i$, then the first chromosome is V_i otherwise select the i -th chromosome V_i ($2 \leq i \leq \text{POPSIZE}$) such that $q_{i-1} \leq r < q_i$.
- (g) Repeat steps (e) and (f) POPSIZE times and obtain POPSIZE copies of chromosomes.

By this process, better chromosomes may be selected several times depending upon the generated random numbers.

Crossover operations:

The exploration and exploitation of the solution space is made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generate offspring solutions by recombining both parent solution features. After selection of chromosomes for new population, the crossover operation is applied. Here, the whole arithmetic crossover operation is used. It is done in the following way:

- (a) Firstly, we generate a random real number, r in $(0,1)$.
- (b) Secondly, we select two chromosomes V_k and V_i randomly among population for crossover if $r < \text{PXOVR}$.
- (c) Then two offsprings V_k' and V_i' are produced as follows:
 $V_k' = c * V_k + (1 - c) * V_i$

$V_i' = c * V_i + (1 - c) * V_k$ where $c \in [0,1]$.

- (d) Repeat the steps (a), (b) and (c) $\text{POPSIZE} / 2$ times.

In this problem crossovers are performed only on the gene which corresponds the variable w_i for $i=1$ to 4 and other genes are generated randomly between its boundaries. If new chromosome satisfies all the constraints then OK otherwise the previous values of the genes of the respective chromosomes are retained.

Mutation operation:

Mutation operation is used to prevent the search process from converging to local optima rapidly. Unlike crossover, it is applied to a single chromosome V_i . Here, the mutation operation is defined as follows:

- (a) Firstly, we generate a random real number r in $(0,1)$.
- (b) Secondly, we select a chromosome V_i randomly from population if $r < \text{PMU}$.

- (c) Thirdly, we select one gene G_{iwk} among $G_{iwk}, k= 1$ to 4, of the selected chromosome V_i randomly.

- (d) Then the new gene G_{iwj}^{mui} of G_{iwj} due to mutation is produced in the following way: $G_{iwj}^{mui} = \text{Rand}(LB_k, UB_k)$

- (e) Repeat the steps (a), (b), (c) and (d) POPSIZE times. In this problem mutation is performed only on the gene which corresponds the variable w_i for a randomly selected i among 0 and 4 and other corresponding genes are generated randomly between its boundaries. If new chromosome satisfies all the constraints then OK otherwise the previous values of the genes of the respective chromosomes are retained.

Termination:

If number of iteration is less than or equal to MAXGEN then the process is going on, otherwise it terminates.

6. NUMERICAL ILLUSTRATION

For numerical illustration, only four items and two rented warehouses are considered here. The non-linear programming problem (25) is solved by GA for the following numerical data shown in Tables-2a, 2b and the optimal results are shown in Table-3 and Table-4, profit values are shown for different values of γ (degree of optimism).

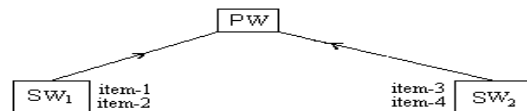


Table-2a: Parametric values

Item	C_1'	C_2'	C_3'	α_1	β_1	γ_1	C_1''	C_2''	C_3''	θ_1	θ_2	a_i
Item-1	1.4	2.8	4.6	1.5	0.6	0.3	70.0	0.03	0.05	0.1	0.1	0.4
Item-2	1.4	2.8	4.5	1.5	0.6	0.3	70.1	0.03	0.06	0.1	0.1	0.5
Item-3	1.4	2.8	4.5	1.5	0.6	0.3	70.1	0.03	0.07	0.1	0.1	0.9
Item-4	1.4	2.8	4.5	1.5	0.6	0.3	70.1	0.03	0.05	0.1	0.1	0.5

Table-2b: Parametric values

Item	L_{1i}	L_{2i}	L_{3i}	K_{1i}	T_{Ci}'	T_{Ci}''	K_i'	d_{1i}	d_{2i}	R	δ_0
Item-1	0.35	0.36	0.37	0.9	125	0.050	1.54	2.3	2.4	0.078	0.8
Item-2	0.33	0.34	0.35	0.9	126	0.051	1.54	2.3	2.4		
Item-3	0.3	0.33	0.34	0.9	124	0.052	1.54	2.3	2.4		
Item-4	0.35	0.35	0.36	0.9	126	0.052	1.54	2.3	2.4		

t_{32}	0.6780	1.7181	3.0433
t_{33}	0.8210	0.4967	0.3191
t_{34}	1.2475	0.4766	0.7744

Table-4: Effect on profit for different values of y

Scenario	Y=0.0	Y=0.25	Y=0.5	Y=0.75	Y=1.0
I	98716.62	98787.01	98845.31	98903.55	98961.79
II	90516.89	90571.66	90626.43	90681.14	90735.90
III	81014.16	81060.82	81107.81	81154.16	81200.63

Table-3: Optimal Result for y=0.5

	Scenario-I	Scenario-II	Scenario-III
PROF	98845.31	90626.43	81107.82
S_1	225.14	95.36	89.78
S_2	59.80	149.84	317.70
S_3	74.93	45.99	27.75
S_4	121.67	41.77	70.79
W_1	2.00	35.87	6.67
W_2	14.99	3.55	44.22
W_3	11.87	24.13	1.07
W_4	39.21	9.64	20.89
W_1'	36.36	35.87	0.55
W_2'	33.82	3.55	38.29
W_3'	25.37	24.13	0.35
W_4'	52.16	9.64	14.02
t_{11}	2.1802	0.6267	1.0548
t_{12}	0.02917	1.6767	2.6407
t_{13}	0.5405	0.2308	0.3187
t_{14}	0.6948	0.3648	0.6144
t_{21}	2.5823	0.6267	0.9813
t_{22}	0.5035	1.6767	2.5833
t_{23}	0.6882	0.2308	0.3067
t_{24}	0.8249	0.3648	0.5389
t_{31}	2.6058	1.0333	1.0615

7. DISCUSSION

For the input data's (given in Table-2a and -2b), optimal results in Table-3 for particular value of $y = 0.5$ shows that the maximum profit value occurs corresponding to Scenario-I. The optimal results in Table-4 shows that the profit value increases as the value of y increases and for all these values of y , Scenario-I gives the better result than other scenarios.

8. CONCLUSION

In this paper, we consider one Primary Warehouse, m secondary warehouses and N items. The warehouses SW_1, SW_2, \dots, SW_m contains N_1, N_2, \dots, N_m items respectively such that $N_1 + N_2 + \dots + N_m = N$. Items are sold from PW which is located at the market place and shortages are allowed at this shop. Due to large stock and insufficient space of existing PW, excess items are stored in m secondary warehouses (viz., SW_1, SW_2, \dots, SW_m) of finite capacity which are little away from PW. Here all the warehouses are of rental basis but rent of PW is greater than the rent of the secondary warehouses as they (SWs) are in little away from market place. Deterioration rate of an item is taken to be different in different warehouses. As the demand of an item is stock dependent, attempt is made to keep the showroom (PW) full bringing the items from SWs at the earliest though the rent at SWs are less than that at PW. Hence the stocks of SWs are transferred to PW under continuous release pattern and the associated transportation cost is directly varies with the distance from PW to SWs but the holding cost of an item in SWs has reverse effect with distance.. The backlogged demand is assumed to be a function of currently backlogged amount. Here, the problem is developed with fuzzy lead-time under inflation and time value of money. The fuzzy parameter is transformed into a crisp number by Graded Mean Integration Value (GMIV) method. There are three scenarios in each cycle depending upon the time when new order is placed. The present analysis can be applied for seasonable / fashionable goods which are marketed for a fixed time period. A soft computing method i.e., genetic algorithm for inventory control problems have been developed in most general way and this can be used to solve other single-objective inventory models in different environments.

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