

Minimal Total Dominating Functions of Corona Product Graph of a Path with a Star

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ABSTRACT

Graph theory is one of the most flourishing branches of modern mathematics and computer applications. Domination in graphs has been studied extensively in recent years and it is an important branch of graph theory. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al. [6,7]. Recently dominating functions in domination theory have received much attention. In this paper we present some results on minimal signed dominating functions and minimal total signed dominating functions of corona product graph of a path with a star.

Keywords

Corona Product, signed dominating function, Total signed dominating function

1. INTRODUCTION

Domination Theory has a wide range of applications to many fields like Engineering, Communication Networks, Social sciences, linguistics, physical sciences and many others. Allan, R.B. and Laskar, R.[1], Cockayne, E.J. and Hedetniemi, S.T. [2] have studied various domination parameters of graphs.

Recently, dominating functions in domination theory have received much attention. The concepts of total dominating functions and minimal total dominating functions are introduced by Cockayne et al. [3]. Jeelani Begum, S. [8] has studied some total dominating functions of Quadratic Residue Cayley graphs.

Frucht and Harary [5] introduced a new product on two graphs G_1 and G_2 , called corona product denoted by $G_1 \odot G_2$. The object is to construct a new and simple operation on two graphs G_1 and G_2 called their corona, with the property that the group of the new graph is in general isomorphic with the wreath product of the groups of G_1 and of G_2 .

The authors have studied some dominating functions of corona product graph of a cycle with a complete graph [9] and published papers on minimal dominating functions, some variations of Y – dominating functions and Y – total dominating functions [10,11,12].

In this paper we study the concept of total dominating functions of corona product graph of a path with a star and some results on minimal total dominating functions are obtained.

2. CORONA PRODUCT OF P_n AND $K_{1,m}$

The **corona product** of a path P_n with star $K_{1,m}$ is a graph obtained by taking one copy of a n – vertex path P_n and n copies of $K_{1,m}$ and then joining the i^{th} vertex of P_n to every vertex of i^{th} copy of $K_{1,m}$ and it is denoted by $P_n \odot K_{1,m}$.

We require the following theorem whose proof can be found in Siva Parvathi, M. [8].

Theorem 2.1: The degree of a vertex v_i in $G = P_n \odot K_{1,m}$ is given by

$$d(v_i) = \begin{cases} m + 3, & \text{if } v_i \in P_n \text{ and } 2 \leq i \leq (n-1), \\ m + 2, & \text{if } v_i \in P_n \text{ and } i = 1 \text{ or } n, \\ m + 1, & \text{if } v_i \in K_{1,m} \text{ and } v_i \text{ is in first partition,} \\ 2, & \text{if } v_i \in K_{1,m} \text{ and } v_i \text{ is in second partition.} \end{cases}$$

3. TOTAL DOMINATING SETS AND TOTAL DOMINATING FUNCTIONS

The concepts of total dominating functions and minimal total dominating functions are introduced by Cockayne et al. [4]. In this section we prove some results related to total dominating functions of the graph $G = P_n \odot K_{1,m}$. First let us recall some definitions.

Definition: Let $G(V,E)$ be a graph without isolated vertices. A subset T of V is called a **total dominating set** (TDS) if every vertex in V is adjacent to at least one vertex in T .

If no proper subset of T is a total dominating set, then T is called a **minimal total dominating set** (MTDS) of G .

Definition: The minimum cardinality of a MTDS of G is called a **total domination number** of G and is denoted by $\gamma_t(G)$.

Theorem 3.1: The total domination number of $G = P_n \odot K_{1,m}$ is n .

Proof: Let T denote a total dominating set of G . Let T contain the vertices of the path P_n .

By the definition of the graph G , every vertex in P_n is adjacent to all vertices of associated copy of $K_{1,m}$. That is the vertices in P_n dominate the vertices in all copies of $K_{1,m}$ respectively. Further these vertices being in P_n , they dominate among themselves. Thus T becomes a TDS of G . Obviously this set is minimum.

Therefore $\gamma_t(G) = n$

Theorem 3.2: Let T be a MTDS of $G = P_n \odot K_{1,m}$. Then a function $f: V \rightarrow [0, 1]$ defined by

$$f(v) = \begin{cases} 1, & \text{if } v \in T, \\ 0, & \text{otherwise.} \end{cases}$$

becomes a MTDF of $G = P_n \odot K_{1,m}$

Proof: Consider the graph $G = P_n \odot K_{1,m}$ with vertex set V .

Let T be a MTDS of G such that it contain all the vertices of P_n .

The summation value taken over $N(v)$ of $v \in V$ is as follows:

Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G .

Then $N(v)$ contains $m + 1$ vertices of $K_{1,m}$ and two vertices of P_n in G .

$$\text{So } \sum_{u \in N(v)} f(u) = 1 + 1 + \left[\underbrace{0 + \dots + 0}_{(m+1)\text{-times}} \right] = 2.$$

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G .

Then $N(v)$ contains $m + 1$ vertices of $K_{1,m}$ and one vertex of P_n in G .

$$\text{So } \sum_{u \in N(v)} f(u) = 1 + \left[\underbrace{0 + \dots + 0}_{(m+1)\text{-times}} \right] = 1.$$

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G .

Then $N(v)$ contains m vertices of $K_{1,m}$ and one vertex of P_n in G .

$$\text{So } \sum_{u \in N(v)} f(u) = 1 + \left[\underbrace{0 + \dots + 0}_{m\text{-times}} \right] = 1.$$

Case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G .

Then $N(v)$ contains one vertex of $K_{1,m}$ and one vertex of P_n in G .

$$\text{So } \sum_{u \in N(v)} f(u) = 1 + 0 = 1.$$

Therefore for all possibilities,

$$\text{we get } \sum_{u \in N(v)} f(u) \geq 1, \quad \forall v \in V.$$

This implies that f is a TDF.

Now we check for the minimality of f .

Define $g: V \rightarrow [0, 1]$ by

$$g(v) = \begin{cases} r, & \text{for } v = v_k \in T, \\ 1, & \text{for } v \in T - \{v_k\}, \\ 0, & \text{otherwise.} \end{cases}$$

where $0 < r < 1$.

Since strict inequality holds at the vertex $v_k \in T$, it follows that $g < f$.

Case (i): Let $v \in P_n$ be such that $d(v) = m + 3$ in G .

Sub case 1: Let $v_k \in N(v)$.

Then

$$\sum_{u \in N(v)} g(u) = r + 1 + \left[\underbrace{0 + \dots + 0}_{(m+1)\text{-times}} \right] = r + 1 > 1.$$

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = 1 + 1 + \left[\underbrace{0 + \dots + 0}_{(m+1)\text{-times}} \right] = 2.$$

Case (ii): Let $v \in P_n$ be such that $d(v) = m + 2$ in G .

Sub case 1: Let $v_k \in N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = r + \left[\underbrace{0 + \dots + 0}_{(m+1)\text{-times}} \right] = r < 1.$$

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = 1 + \left[\underbrace{0 + \dots + 0}_{(m+1)\text{-times}} \right] = 1.$$

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G .

Sub case 1: Let $v_k \in N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = r + \left[\underbrace{0 + \dots + 0}_{m\text{-times}} \right] = r < 1.$$

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = 1 + \left[\underbrace{0 + \dots + 0}_{m\text{-times}} \right] = 1.$$

Case (iv): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G .

Sub case 1: Let $v_k \in N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = r + 0 = r < 1.$$

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = 1 + 0 = 1.$$

This implies that $\sum_{u \in N(v)} g(u) < 1$, for some $v \in V$.

So g is not a TDF.

Since g is taken arbitrarily, it follows that there exists no $g < f$ such that g is a TDF.

Therefore f is a MTDF. ■

Theorem 3.3: A function $f: V \rightarrow [0, 1]$ defined by $f(v) = \frac{1}{q}$, $\forall v \in V$ is a TDF of $G = P_n \odot K_{1,m}$ if $q \leq 2$. It is a MTDF if $q = 2$.

Proof: Consider the graph $G = P_n \odot K_{1,m}$ with vertex set V .

Let f be a function defined as in the hypothesis.

Case I: Suppose $q < 2$.

Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G .

$$\text{So } \sum_{u \in N(v)} f(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+3)\text{-times}} = \frac{m+3}{q} > 1,$$

since $q < 2$ and $m \geq 2$.

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G .

$$\text{So } \sum_{u \in N(v)} f(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+2)\text{-times}} = \frac{m+2}{q} > 1,$$

since $q < 2$ and $m \geq 2$.

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G .

$$\text{So } \sum_{u \in N(v)} f(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+1)\text{-times}} = \frac{m+1}{q} > 1,$$

since $q < 2$ and $m \geq 2$.

Case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G .

$$\text{So } \sum_{u \in N(v)} f(u) = \frac{1}{q} + \frac{1}{q} = \frac{2}{q} > 1, \text{ since } q < 2.$$

Therefore for all possibilities,

$$\text{we get } \sum_{u \in N(v)} f(u) > 1, \forall v \in V.$$

This implies that f is a TDF.

Now we check for the minimality of f .

Define $g: V \rightarrow [0,1]$ by

$$g(v) = \begin{cases} r, & \text{for } v = v_k \in V, \\ \frac{1}{q}, & \text{otherwise.} \end{cases}$$

where $0 < r < \frac{1}{q}$.

Since strict inequality holds at a vertex v_k of V , it follows that $g < f$.

Case (i): Let $v \in P_n$ be such that $d(v) = m + 3$ in G .

Sub case 1: Let $v_k \in N(v)$.

$$\begin{aligned} \text{Then } \sum_{u \in N(v)} g(u) &= r + \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+2)\text{-times}} \\ &< \frac{1}{q} + \frac{m+2}{q} = \frac{m+3}{q} > 1, \end{aligned}$$

since $q < 2$ and $m \geq 2$.

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+3)\text{-times}} = \frac{m+3}{q} > 1,$$

since $q < 2$ and $m \geq 2$.

Case (ii): Let $v \in P_n$ be such that $d(v) = m + 2$ in G .

Sub case 1: Let $v_k \in N(v)$.

$$\begin{aligned} \text{Then } \sum_{u \in N(v)} g(u) &= r + \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+1)\text{-times}} \\ &< \frac{1}{q} + \frac{m+1}{q} = \frac{m+2}{q} > 1, \end{aligned}$$

since $q < 2$ and $m \geq 2$.

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+2)\text{-times}} = \frac{m+2}{q} > 1,$$

since $q < 2$ and $m \geq 2$.

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G .

Sub case 1: Let $v_k \in N(v)$.

$$\begin{aligned} \text{Then } \sum_{u \in N(v)} g(u) &= r + \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{m\text{-times}} \\ &< \frac{1}{q} + \frac{m}{q} = \frac{m+1}{q} > 1, \end{aligned}$$

since $q < 2$ and $m \geq 2$.

Sub case 2: Let $v_k \notin N(v)$.

So
$$\sum_{u \in N(v)} g(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+1)\text{-times}} = \frac{m+1}{q} > 1,$$

since $q < 2$ and $m \geq 2$.

Case (iv): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G .

Sub case 1: Let $v_k \in N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = r + \frac{1}{q} < \frac{2}{q}.$$

Since $q < 2$, it follows that $\frac{2}{q} > 1$.

Sub case 2: Let $v_k \notin N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = \frac{1}{q} + \frac{1}{q} = \frac{2}{q} > 1,$$
 since $q < 2$.

Hence, it follows that $\sum_{u \in N(v)} g(u) > 1, \quad \forall v \in V.$

Thus g is a TDF.

This implies that f is not a MTFD.

Case II: Suppose $q = 2$.

Case 5: Let $v \in P_n$ be such that $d(v) = m + 3$ in G .

So

$$\sum_{u \in N(v)} f(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+3)\text{-times}} = \frac{m+3}{q} = \frac{m+3}{2} = 1 + \frac{m+1}{2} > 1.$$

Case 6: Let $v \in P_n$ be such that $d(v) = m + 2$ in G .

So

$$\sum_{u \in N(v)} f(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+2)\text{-times}} = \frac{m+2}{q} = \frac{m+2}{2} = 1 + \frac{m}{2} > 1.$$

Case 7: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G .

So

$$\sum_{u \in N(v)} f(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+1)\text{-times}} = \frac{m+1}{q} = \frac{m+1}{2} > 1,$$

since $m \geq 2$.

Case 8: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G .

So
$$\sum_{u \in N(v)} f(u) = \frac{1}{q} + \frac{1}{q} = \frac{2}{q} = \frac{2}{2} = 1.$$

Therefore for all possibilities,

we get
$$\sum_{u \in N(v)} f(u) \geq 1, \quad \forall v \in V.$$

This implies that f is a TDF.

Now we check for the minimality of f .

Define $g : V \rightarrow [0,1]$ by

$$g(v) = \begin{cases} r, & \text{for } v = v_k \in V, \\ \frac{1}{q}, & \text{otherwise.} \end{cases}$$

where $0 < r < \frac{1}{q}$.

Since strict inequality holds at a vertex v_k of V , it follows that $g < f$.

Then as in case (i), for $v \in P_n$ be such that $d(v) = m + 3$, we get

$$\sum_{u \in N(v)} g(u) = r + \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+2)\text{-times}} > 1,$$

if $v_k \in N[v]$.

And

$$\sum_{u \in N(v)} g(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+3)\text{-times}} = \frac{m+3}{q} = \frac{m+3}{2} = 1 + \frac{m+1}{2} > 1,$$

if $v_k \notin N[v]$.

Again as in case (ii), for $v \in P_n$ be such that $d(v) = m + 2$,

We have
$$\sum_{u \in N(v)} g(u) = r + \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+1)\text{-times}} > 1,$$

if $v_k \in N[v]$.

And

$$\sum_{u \in N(v)} g(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+2)\text{-times}} = \frac{m+2}{q} = \frac{m+2}{2} = 1 + \frac{m}{2} > 1,$$

if $v_k \notin N[v]$.

Again we can see as in case (iii) that for $v \in K_{1,m}$ be such that $d(v) = m + 1$,

$$\sum_{u \in N(v)} g(u) = r + \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{m\text{-times}} > 1,$$

if $v_k \in N[v]$.

And
$$\sum_{u \in N(v)} g(u) = \underbrace{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q}}_{(m+1)\text{-times}} = \frac{m+1}{q} = \frac{m+1}{2} > 1,$$

if $v_k \notin N[v]$.

Similarly we can show as in case (iv) that $v \in K_{1,m}$ be such that $d(v) = 2$,

$$\sum_{u \in N(v)} g(u) = r + \frac{1}{q} < \frac{2}{q} = \frac{2}{2} = 1, \text{ if } v_k \in N[v],$$

and $\sum_{u \in N(v)} g(u) = \frac{1}{q} + \frac{1}{q} = \frac{2}{q} = \frac{2}{2} = 1, \text{ if } v_k \notin N[v].$

This implies that $\sum_{u \in N(v)} g(u) < 1$, for some $v \in V$.

So g is not a TDF.

Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a TDF.

Thus f is a MTDf. ■

Theorem 3.4: A function $f: V \rightarrow [0, 1]$ defined by $f(v) = \frac{p}{q}, \forall v \in V$ where $p = \min(m, n)$ and $q = \max(m, n)$ is a TDF of $G = P_n \odot K_{1,m}$ if $\frac{p}{q} \geq \frac{1}{2}$. Otherwise it is not a TDF. Also it becomes a MTDf if $\frac{p}{q} = \frac{1}{2}$.

Proof: Consider the graph $G = P_n \odot K_{1,m}$ with vertex set V .

Let $f: V \rightarrow [0, 1]$ be defined by $f(v) = \frac{p}{q}, \forall v \in V$, where $p = \min(m, n)$ and $q = \max(m, n)$.

Clearly $\frac{p}{q} > 0$.

Case 1: Let $v \in P_n$ be such that $d(v) = m + 3$ in G .

$$\text{So } \sum_{u \in N(v)} f(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+3)\text{-times}} = (m+3) \frac{p}{q}.$$

Case 2: Let $v \in P_n$ be such that $d(v) = m + 2$ in G .

$$\text{So } \sum_{u \in N(v)} f(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+2)\text{-times}} = (m+2) \frac{p}{q}.$$

Case 3: Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G .

$$\text{So } \sum_{u \in N(v)} f(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+1)\text{-times}} = (m+1) \frac{p}{q}.$$

Case 4: Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G .

$$\text{So } \sum_{u \in N(v)} f(u) = \frac{p}{q} + \frac{p}{q} = 2 \left(\frac{p}{q} \right).$$

From the above four cases, we observe that f is a TDF if

$$\frac{p}{q} \geq \frac{1}{2}.$$

Otherwise f is not a TDF.

Case 5: Suppose $\frac{p}{q} > \frac{1}{2}$.

Clearly f is a TDF.

Now we check for the minimality of f .

Define $g: V \rightarrow [0, 1]$ by

$$g(v) = \begin{cases} r, & \text{if } v = v_1 \in V, \\ \frac{p}{q}, & \text{otherwise.} \end{cases}$$

where $0 < r < \frac{p}{q}$.

Since strict inequality holds at a vertex v_k of V , it follows that $g < f$.

Case (i): Let $v \in P_n$ be such that $d(v) = m + 3$ in G .

Sub case 1: Let $v_k \in N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = r + \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+2)\text{-times}} < \frac{p}{q} + (m+2) \frac{p}{q} = (m+3) \frac{p}{q} > 1,$$

since $\frac{p}{q} > \frac{1}{2}$ and $m \geq 2$.

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+3)\text{-times}} = (m+3) \frac{p}{q} > 1,$$

since $\frac{p}{q} > \frac{1}{2}$ and $m \geq 2$.

Case (ii): Let $v \in P_n$ be such that $d(v) = m + 2$ in G .

Sub case 1: Let $v_k \in N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = r + \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+1)\text{-times}} < \frac{p}{q} + (m+1) \frac{p}{q} = (m+2) \frac{p}{q} > 1,$$

since $\frac{p}{q} > \frac{1}{2}$ and $m \geq 2$.

Sub case 2: Let $v_k \notin N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+2)\text{-times}} = (m+2) \frac{p}{q} > 1,$$

since $\frac{p}{q} > \frac{1}{2}$ and $m \geq 2$.

Case (iii): Let $v \in K_{1,m}$ be such that $d(v) = m + 1$ in G .

Sub case 1: Let $v_k \in N(v)$.

$$\text{Then } \sum_{u \in N(v)} g(u) = r + \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{m\text{-times}}$$

$$\left\langle \frac{p}{q} + m \frac{p}{q} = (m+1) \frac{p}{q} \right\rangle > 1,$$

since $\frac{p}{q} > \frac{1}{2}$ and $m \geq 2$.

Sub case 2: Let $v_k \notin N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+1)\text{-times}} = (m+1) \frac{p}{q} > 1,$$

since $\frac{p}{q} > \frac{1}{2}$ and $m \geq 2$.

Case (iv): Let $v \in K_{1,m}$ be such that $d(v) = 2$ in G .

Sub case 1: Let $v_k \in N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = r + \frac{p}{q} < 2 \left(\frac{p}{q} \right).$$

Since $\frac{p}{q} > \frac{1}{2}$ and $m \geq 2$, it follows that $2 \left(\frac{p}{q} \right) > 1$.

Sub case 2: Let $v_k \notin N(v)$.

Then
$$\sum_{u \in N(v)} g(u) = \frac{p}{q} + \frac{p}{q} = 2 \left(\frac{p}{q} \right) > 1,$$
 since $\frac{p}{q} > \frac{1}{2}$.

Hence it follows that $\sum_{u \in N(v)} g(u) > 1, \forall v \in V$.

Thus for all possibilities, we get that g is a TDF.

This implies that f is not a MTDF.

Case 6: Suppose $\frac{p}{q} = \frac{1}{2}$.

As in case 1 and 2, we have that

$$\sum_{u \in N(v)} f(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+3)\text{-times}} = (m+3) \left(\frac{p}{q} \right) > 1,$$

since $m \geq 2$ and $\frac{p}{q} = \frac{1}{2}$.

And
$$\sum_{u \in N(v)} f(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+2)\text{-times}} = (m+2) \left(\frac{p}{q} \right) > 1,$$

since $m \geq 2$ and $\frac{p}{q} = \frac{1}{2}$.

Again as in case 3 and 4, we have that

$$\sum_{u \in N(v)} f(u) = \underbrace{\frac{p}{q} + \frac{p}{q} + \dots + \frac{p}{q}}_{(m+1)\text{-times}} = (m+1) \left(\frac{p}{q} \right) > 1,$$

since $m \geq 2$ and $\frac{p}{q} = \frac{1}{2}$.

And
$$\sum_{u \in N(v)} f(u) = \frac{p}{q} + \frac{p}{q} = 2 \frac{p}{q} = 2 \frac{1}{2} = 1$$

Therefore for all possibilities,

we get
$$\sum_{u \in N(v)} f(u) \geq 1, \forall v \in V.$$

This implies that f is a TDF.

Now we check for the minimality of f .

Define $g : V \rightarrow [0,1]$ by

$$g(v) = \begin{cases} r, & \text{if } v = v_k \in V, \\ \frac{p}{q}, & \text{otherwise.} \end{cases}$$

where $0 < r < \frac{p}{q}$.

Since strict inequality holds at a vertex v_k of V , it follows that $g < f$.

Then we can show as in case (i) and (ii) of case 5 that

$$\sum_{u \in N(v)} g(u) > 1, \text{ if } v \in P_m \text{ and } v_k \in N(v) \text{ or } v_k \notin N(v).$$

Again as in case (iii) of case 5, we can show that

$$\sum_{u \in N(v)} g(u) > 1, \text{ if } v \in K_{1,m} \text{ and } v_k \in N(v) \text{ or } v_k \notin N(v).$$

As in case (iv) of case 5, we can show that

$$\sum_{u \in N(v)} g(u) = r + \frac{p}{q} < 2 \frac{p}{q} = 2 \frac{1}{2} = 1,$$

if $v \in K_{1,m}$ and $v_k \in N(v)$.

And
$$\sum_{u \in N(v)} g(u) = \frac{p}{q} + \frac{p}{q} = 2 \frac{p}{q} = 2 \frac{1}{2} = 1,$$

if $v \in K_{1,m}$ and $v_k \notin N(v)$.

This implies that $\sum_{u \in N(v)} g(u) < 1$, for some $v \in V$.

So g is not a TDF.

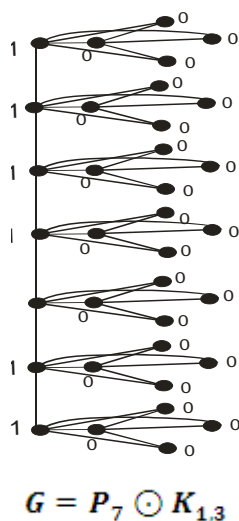
Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a TDF.

Thus f is a MTDF.

4. CONCLUSION

It is interesting to study the total dominating functions of the corona product graph of a path with a star. This work gives the scope for the study of convexity of these minimal total dominating functions and the authors have also studied this concept.

5. ILLUSTRATION



The function f takes the value 1 for the vertices of P_7 and the value 0 for the vertices in each copy of $K_{1,3}$.

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