Strong Split Geodetic Number of a Graph

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ABSTRACT

A set $S \subseteq V(G)$ is a strong split geodetic set of G, if S is a geodetic set and $\langle V-S\rangle$ is totally disconnected. The strong split geodetic number of a graph G, is denoted by $g_{ss}(G)$, is the minimum cardinality of a strong split geodetic set of G. In this paper we investigate many bounds on strong split geodetic number in terms of elements of G and covering number of G, further the relationship between strong split geodetic number and split geodetic number.

Keywords:

Cartesian product, Distance, Edge covering number, Split geodetic number, Vertex covering number.

1. INTRODUCTION

In this paper we follow the notations of [1]. As usual $n = |V|$ and $m = |E|$ denote the number of vertices and edges of a graph G respectively.
The graphs considered here have at least one component which is not complete or at least two non trivial components.
The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u-v$ path in G. It is well known that this distance is a metric on the vertex set $V(G)$. For a vertex v of G, the eccentricity $e(v)$ is the distance between v and a vertex farthest from v. The minimum eccentricity among the vertices of G is radius, rad G, and the maximum eccentricity is the diameter, diam G. A $u-v$ path of length d(u, v) is called a $u-v$ geodesic. We define $I[u, v]$ to the set (interval) of all vertices lying on some $u-v$ geodesic of G and for a nonempty subset S of $V(G)$, $I[S] = \bigcup_{u,v\in S} I[u, v]$.
A set S of vertices of G is called a geodetic set in G if $I[S] = V(G)$, and a geodetic set of minimum cardinality is a minimum geodetic set. The cardinality of a minimum geodetic set in G is called the geodetic number of G, and we denote it by $g(G)$.
Split geodetic number of a graph was studied by in [4]. A geodetic set S of a graph $G = (V, E)$ is a split geodetic set if the induced subgraph $\langle V-S\rangle$ is disconnected. The split geodetic number $g_s(G)$ of G is the minimum cardinality of a split geodetic set. Now we define strong split geodetic number of a graph. A set S' of vertices of $G = (V, E)$ is called the strong split geodetic set if the induced subgraph $\langle V-S'\rangle$ is totally disconnected and a strong split geodetic set of minimum cardinality is the strong split geodetic number of G and is denoted by $g_{ss}(G)$.
A vertex v is an extreme vertex in a graph G, if the subgraph induced by its neighbors is complete. A vertex cover in a graph G is a set of vertices that covers all edges of G. The minimum number of vertices in a vertex cover of G is the vertex covering number $\alpha_0(G)$ of G. An edge cover of a graph G without isolated vertices is a set of edges of G that covers all the vertices of G. The edge covering number $\alpha_1(G)$ of a graph G is the minimum cardinality of an edge cover of G.
For any undefined term in this paper, see [1] and [2].

2. PRELIMINARY NOTES

We need the following results to prove further results.

THEOREM 2.1. [3] *Every geodetic set of a graph contains its extreme vertices.*

THEOREM 2.2. [3] *For any path P_n, with n vertices, $g(P_n) = 2$.*

THEOREM 2.3. [3] *For integers $r, s \geq 2$, $g(K_{r,s}) = min\{r, s, 4\}$.*

THEOREM 2.4. [3] *Let G be a connected graph of order at least 3. If G contains a minimum geodetic set S with a vertex x such that every vertex of G lies on some $x-w$ geodesic in G for some $w \in S$, then $g(G) = g(G \times K_2)$.*

THEOREM 2.5. [2] *For any graph G, $\alpha_0 + \beta_0 = \alpha_1 + \beta_1$.*

PROPOSITION 2.6. *For any graph G, $g_s(G) \leq g_{ss}(G)$.*

PROPOSITION 2.7. *For any tree T of order n and number of cut vertices c_i then the number of end edges is $n - c_i$.*

3. MAIN RESULTS

THEOREM 3.1. *Let T be a tree that has at least three internal vertices. If T has k end-vertices, then $g_{ss}(T) = k + \lceil \frac{n-(k+1)}{2} \rceil$.*

Proof. Let $F = \{v_1, v_2, ..., v_k\}$ be the set of all end vertices in T, $|F| = k$. Consider $S = F \cup H$, where $H \subseteq V(T) - F$, such that H contains a vertex of maximum degree and a minimum set of alternating vertices in $V - F$, $|H| = \lceil \frac{n-(k+1)}{2} \rceil$. Now S be the minimal set of vertices which covers all the vertices in T. Clearly set of vertices of a subgraph $\langle V - S \rangle$ is totally disconnected, then by the above argument S is a minimal strong split geodetic set of T. Clearly it follows that, $|S| = |F \cup H| = k + \lceil \frac{n-(k+1)}{2} \rceil$. Therefore $g_{ss}(T) = k + \lceil \frac{n-(k+1)}{2} \rceil$.

COROLLARY 3.2. *For any path P_n, $n \geq 5$, $g_{ss}(P_n) = 2 + \lceil \frac{n-3}{2} \rceil$.*

Proof. Proof follows from the above theorem.

THEOREM 3.3. *For cycle C_n of order $n > 3$*

$$g_{ss}(C_n) = \begin{cases} \frac{n}{2} & \text{if n is even} \\ \frac{n+1}{2} & \text{if n is odd.} \end{cases}$$

Proof. Let $n > 3$, we have the following cases.
Case 1: Let n be even.
Consider $\{v_1, v_2, ..., v_n, v_1\}$ be a cycle with n vertices where n is even, let $S = \{v_1, v_3, ..., v_n\}$ be the set of alternating vertices which covers all the vertices of C_n and for any $v_i \in V - S$, $degv_i = 0$. Clearly S forms minimal strong split geodetic set of C_n, it follows that $|S| = \frac{n}{2}$. Therefore $g_{ss}(C_n) = \frac{n}{2}$.
Case 2: Let n be odd.
Consider $\{v_1, v_2, ..., v_n\}$ be a cycle with n vertices where n is odd, let $S = (v_1, v_n) \cup \{v_3, v_5, ..., v_{n-2}\}$ which covers all the vertices of C_n and for any $v_i \in V - S$, $degv_i = 0$. Clearly S forms minimal strong split geodetic set of C_n, it follows that $|S| = \frac{n+1}{2}$. Therefore $g_{ss}(C_n) = \frac{n+1}{2}$.

COROLLARY 3.4. *For any cycle C_n of order $n > 3$, $g_{ss}(C_n) = \alpha_0(C_n)$.*

Proof. We have the following cases.
Case 1: Let n be even.
Let $n > 3$ be the number of vertices which is even and α_0 is the vertex covering number of C_n. We have by Case 1 of Theorem 3.3, $g_{ss}(C_n) = \frac{n}{2}$. Also for even cycle, vertex covering number is $\alpha_0(C_n) = \frac{n}{2}$. Hence $g_{ss}(C_n) = \alpha_0(C_n)$.
Case 2: Let n be odd.
Let $n > 3$ be the number of vertices which is odd and α_0 is the vertex covering number of C_n. We have by Case 2 of Theorem 3.3, $g_{ss}(C_n) = \frac{n+1}{2}$. Also for odd cycle, vertex covering number is $\alpha_0(C_n) = \frac{n+1}{2}$. Hence $g_{ss}(C_n) = \alpha_0(C_n)$.

THEOREM 3.5. *For the wheel $W_n = K_1 + C_{n-1}$ $(n \geq 6)$,*

$$g_{ss}(W_n) = \begin{cases} \frac{n+2}{2} & \text{if n is even} \\ \frac{n+1}{2} & \text{if n is odd.} \end{cases}$$

Proof. Let $W_n = K_1 + C_{n-1}(n \geq 6)$ and let $V(W_n) = \{x, u_1, u_2, ..., u_{n-1}\}$, where $deg(x) = n - 1 > 3$ and $deg(u_i) = 3$ for each $i \in \{1, 2, ..., n-1\}$. We have the following cases
Case 1. Let n be even. Consider geodesic $P : \{u_1, u_2, u_3\}$, $Q : \{u_3, u_4, u_5\}$,...,$R : \{u_{2n-1}, u_{2n}, u_{2n+1}, x\}$. It is clear that the vertices $u_2, u_4, ..., u_{2n}$ lies on the geodesics P, Q,..., R. Also $S = \{u_1, u_3, u_5, ..., u_{2n-1}, u_{2n+1}, x\}$ is a minimal strong split geodesic set such that $V - S$ is totally disconnected and it has $\frac{n}{2} + 1$ vertices.
Hence $g_{ss}(W_n) = \frac{n+2}{2}$.

Case 2. Let n be odd. Consider geodesic $P : \{u_1, u_2, u_3\}$, $Q : \{u_3, u_4, u_5\}$,..., $R : \{u_{2n-1}, u_{2n}, u_{2n+1}, x\}$. It is clear that the vertices $u_2, u_4 ... u_{2n}$ lies on the geodesic P, Q,...,R. Also $S = \{u_1, u_3, u_5, ..., u_{2n-1}, u_{2n+1}, x\}$ is a minimal strong split geodesic set such that $V - S$ is totally disconnected and it has $\frac{n-1}{2} + 1$ vertices.
Hence $g_{ss}(W_n) = \frac{n+1}{2}$.

COROLLARY 3.6. *For the wheel $W_n = K_1 + C_{n-1}$ $(n \geq 6)$,*

$$g_{ss}(W_n) = \begin{cases} \frac{\Delta+\delta}{2} & \text{if n is even} \\ \frac{\Delta+\delta-1}{2} & \text{if n is odd.} \end{cases}$$

Proof. Let $W_n = K_1 + C_{n-1}(n \geq 6)$ and let $V(W_n) = \{x, u_1, u_2, ..., u_{n-1}\}$, where $deg(x) = n-1 > 3$ and $deg(u_i) = 3$ for each $i \in \{1, 2, ..., n-1\}$. Maximum degree(Δ) of W_n is $n-1$ and minimum degree(δ) of W_n is 3.
We have the following cases
Case 1: Let n be even. We have from Case 1 of Theorem 3.5
$g_{ss}(W_n) = \frac{n+2}{2}$
$\Rightarrow g_{ss}(W_n) = \frac{(n-1)+3}{2}$
$\Rightarrow g_{ss}(W_n) = \frac{\Delta+\delta}{2}$.
Case 2: Let n be odd. We have from Case 2 of Theorem 3.5
$g_{ss}(W_n) = \frac{n+1}{2}$
$\Rightarrow g_{ss}(W_n) = \frac{(n-1)+3-1}{2}$
$\Rightarrow g_{ss}(W_n) = \frac{\Delta+\delta-1}{2}$.

THEOREM 3.7. *Let G be a connected graph of order n and diameter d. Then $g_{ss}(G) \leq n - d + 2$, except for tree.*

Proof. Let u and v be vertices of G for which $d(u, v) = d$ and let $u = v_0, v_1, ..., v_d = v$ be the $u - v$ path of length d. Now let $S = V(G) - \{v_1, v_2, ..., v_{d-1}\}$. Then $I[S] = V(G), V - (S \cup \{v_i\})$ is totally disconnected and thus $g_{ss}(G) \leq |S| + 1 = n - d + 2$.

THEOREM 3.8. *For any tree T with at least three internal vertices and order n, diameter d. Then $g_{ss}(G) \leq n - d + k$, where k be the number of end vertices.*

Proof. Let u and v be vertices of G for which $d(u, v) = d$ and let $u = v_0, v_1, ..., v_d = v$ be the $u - v$ path of length d. Now let $S = V(G) - \{v_1, v_2, ..., v_{d-1}\}$. Then $I[S] = V(G)$, $V - (S \cup \{v_2, v_3, ..., v_{k-2}\})$ is totally disconnected and thus $g_{ss}(G) \leq |S| + k - 1 = n - d + k$.

THEOREM 3.9. *For any integers* $r, s \geq 2$ $g_{ss}(K_{r,s}) = min\{r, s\}$.

Proof. Let $G = K_{r,s}$, such that $U = \{u_1, u_2, ..., u_r\}$, $W = \{w_1, w_2, ..., w_s\}$ are the partite sets of G, where $r \leq s$ and also $V = U \cup W$.
Consider $S = U$, for every w_k, $1 \leq k \leq s$ lies on the $u_i - u_j$ geodesic for $1 \leq i \neq j \leq r$. Since $V - S$ is totally disconnected, we have S is a strong split geodetic set of G.
Let $X = \{u_1, u_2, ..., u_{r-1}\}$ be any set of vertices such that $|X| < |S|$, then X is not a geodetic set of G, since $u_r \notin I[X]$. It is clear that S is a minimum strong split geodetic set of G. Hence $g_{ss}(K_{r,s}) = |S| = r$.

THEOREM 3.10. *For any connected graph G of order n,* $g_s(G) + g_{ss}(G) < 2n$.

Proof. Suppose $S = \{v_1, v_2, ..., v_n\} \subseteq V(G)$ be the set of vertices which covers all the vertices in G and $V - S$ is disconnected. Then S is a minimal split geodetic set of G. Further if the subgraph $\langle V - S \rangle$ contains the set of vertices v_i, $1 \leq i \leq n$, such that $degv_i = 0$. Then S itself is an strong split geodetic set of G. Otherwise, $S^{'} = S_1 \cup I$, where $S_1 \subseteq S$ and $I \subseteq V(G) - S$ is the minimum set of alternate vertices, $S^{'}$ forms a minimal strong split geodetic set of G. Since $V - S^{'}$ contains isolated vertices, it follows that $|S| \cup |S^{'}| < 2n$. Therefore, $g_s(G) + g_{ss}(G) < 2n$.

The following corollaries are immediate consequence of above Theorem and Theorem 2.5.

COROLLARY 3.11. *For any connected graph G of order n,* $g_s(G) + g_{ss}(G) < 2(\alpha_0(G) + \beta_0(G))$.

COROLLARY 3.12. *For any connected graph G of order n,* $g_s(G) + g_{ss}(G) < 2(\alpha_1(G) + \beta_1(G))$.

4. ADDING AN END EDGE

For an edge $e = (u, v)$ of a graph G with $deg(u) = 1$ and $deg(v) > 1$, we call e an end-edge and u an end-vertex.

THEOREM 4.1. $G^{'}$ *be the graph obtained by adding an end edge* (u, v) *to a cycle* $C_n = G$ *of order* $n > 3$*, with* $u \in G$ *and* $v \notin G$*. Then*

$$g_{ss}(G^{'}) = \begin{cases} \frac{n+2}{2} & \text{for even cycle} \\ \frac{n+3}{2} & \text{for odd cycle.} \end{cases}$$

Proof. Let $\{u_1, u_2, ..., u_n, u_1\}$ be a cycle with n vertices. Let $G^{'}$ be the graph obtained from $G = C_n$ by adding an end-edge (u, v) such that $u \in G$ and $v \notin G$.
We have the following cases.
Case 1: Let G be an even cycle.
Let $S = \{v, u_i\} \subseteq V(G^{'})$, where $v \notin G$ is an end vertex of $G^{'}$ and u_i is an antipodal vertex of u. Consider $S^{'} = S \cup H$, where $H \subseteq V(G^{'}) - S$ is a minimum set of non-adjacent vertices, $|H| = \frac{n}{2} - 1$. Now $S^{'}$ be the minimal set of vertices which covers all the vertices of $G^{'}$. Clearly for any $u_i \in V - S^{'}$, $degu_i = 0$, by the above argument it follows that $S^{'}$ is a minimal strong split geodetic set of $G^{'}$. Clearly $|S^{'}| = |S \cup H| = 2 + \frac{n}{2} - 1 = \frac{n+2}{2}$. Therefore $g_{ss}(G^{'}) = \frac{n+2}{2}$.
Case 2: Let G be an odd cycle.
(a) When $n = 5$
Let $S = \{v, a, b\}$ be a geodetic set, where $v \notin G$, is an end-vertex of $G^{'}$ and $a, b \in G$, such that $2d(u, a) = d(u, b)$ and $d(a, b) = 2$. Thus $I[S] = V(G^{'})$ and $V - S$ is an induced subgraph which has two components. Let $S^{'} = S \cup H$ where $H \subseteq V - S$ such that H contains minimum alternate vertices from both the components having $\frac{n-3}{2}$ vertices. Clearly $S^{'}$ forms the minimal strong split geodetic set of $G^{'}$, since $V - S^{'}$ forms an independent set. Clearly $|S^{'}| = |S \cup H| = 3 + \frac{n-3}{2} = \frac{n+3}{2}$. Therefore $g_{ss}(G^{'}) = \frac{n+3}{2}$.
(b) When $n > 5$
Let $S = \{v, a, b\}$ be a geodetic set where $v \notin G$ is an end-vertex of $G^{'}$ and $a, b \in G$, such that $d(u, a) = d(u, b)$ and $d(a, b)$ is the diameter of G. Thus $I[S] = V(G^{'})$ and $V - S$ is an induced subgraph which has two components. Let $S^{'} = S \cup H$ where $H \subseteq V - S$ such that H contains minimum alternate vertices from both the components having $\frac{n-3}{2}$ vertices. Clearly $S^{'}$ forms the minimal strong split geodetic set of $G^{'}$, since $V - S^{'}$ forms an independent set. Clearly $|S^{'}| = |S \cup H| = 3 + \frac{n-3}{2} = \frac{n+3}{2}$. Therefore $g_{ss}(G^{'}) = \frac{n+3}{2}$.

THEOREM 4.2. *Let* $G^{'}$ *be the graph obtained by adding end edge* (u_i, v_i)*,* $i = 1, 2, ..., n$*, to each vertex of* $G = C_n$ *of order* $n > 3$ *such that* $u_i \in G$*,* $v_j \notin G$*. Then*

$$g_{ss}(G^{'}) = \begin{cases} k + \frac{n}{2} & \text{for even cycle} \\ k + \frac{n+1}{2} & \text{for odd cycle.} \end{cases}$$

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Proof. Let $G = C_n = \{u_1, u_2, ..., u_n, u_1\}$ be a cycle with n vertices. Let $G^{'}$ be the graph obtained by adding an end-edge (u_i, v_i), $i = 1, 2, ..., n = k$ to each vertex of G such that $u_i \in G$,$v_i \notin G$.
Case 1: Let G be an even cycle.
Let $F = \{v_1, v_2, ..., v_k\}$ is the k number of end-vertices of $G^{'}$ and $H \subseteq V(G^{'}) - F$ is an even cycle. Let $S = F \cup H_1$, where $H_1 \subseteq H$ such that $H_1 \notin E(H)$. Now S be the minimal set of vertices which covers all the vertices in $G^{'}$. Clearly for any $u_i \in G^{'}$, $deg(u_i) = 0$. Then by the above argument S is the minimal strong split geodetic set of $G^{'}$, it follows that $|S| = |F \cup H_1| = k + \frac{n}{2}$. Therefore $g_{ss}(G^{'}) = k + \frac{n}{2}$.
Case 2: Let G be odd cycle.
Let $F = \{v_1, v_2, ..., v_k\}$ is the k number of end-vertices of $G^{'}$ and $H \subseteq V(G^{'}) - F$ is an odd cycle. Let $S = F \cup (u_1, u_n) \cup H_1$, where $H_1 \subseteq H$ such that $H_1 \notin E(H)$. Now S be the minimal set of
vertices which covers all the vertices in $G^{'}$. Clearly for any $u_i \in G^{'}$, $deg(u_i) = 0$. Then by the above argument S is the minimal strong split geodetic set of $G^{'}$, it follows that $|S| = |F \cup (u_1, u_n) \cup H_1| = k + 2 + \frac{n-3}{2}$. Therefore $g_{ss}(G^{'}) = k + \frac{n+1}{2}$.

5. CARTESIAN PRODUCT

The cartesian product of the graphs H_1 and H_2, written as $H_1 \times H_2$, is the graph with vertex set $V(H_1) \times V(H_2)$, two vertices u_1, u_2 and v_1, v_2 being adjacent in $H_1 \times H_2$ if and only if either $u_1 = v_1$ and $(u_2, v_2) \in E(H_2)$, or $u_2 = v_2$ and $(u_1, v_1) \in E(H_1)$.

THEOREM 5.1. *For any path P_n of order n, $g_{ss}(K_2 \times P_n) = n$.*

Proof. Consider $G = P_n$. Let $K_2 \times P_n$ be graph formed from two Copies G_1 and G_2 of G. Let $V = \{v_1, v_2, ..., v_n\}$ be the vertices of G_1, $W = \{w_1, w_2, ..., w_n\}$ be the vertices of G_2 and $U = V \cup W$.
Case 1. Let n be even.
Consider $S = H_1 \cup H_2$, where $H_1 = \{v_1, v_3, v_5, ..., v_{n-1}\} \subseteq V$ having $\frac{n}{2}$ vertices, $H_2 = \{w_2, w_4, w_6, ..., w_n\} \subseteq W$ having $\frac{n}{2}$ vertices. Now S be the minimal set of vertices which covers all the vertices in $K_2 \times P_n$. Such that set of vertices of a subgraph $U - S$ is isolated, then by the above argument S is a minimal strong split geodetic set of $K_2 \times P_n$. Clearly it follows that, $|S| = |H_1 \cup H_2 = \frac{n}{2} + \frac{n}{2} = n$. Therefore $g_{ss}(K_2 \times P_n) = n$.
Case 2. Let n be odd.
Consider $S = H_1 \cup H_2$, where $H_1 = \{v_2, v_4, v_6, ..., v_{n-1}\} \subseteq V$ having $\frac{n-1}{2}$ vertices, $H_2 = \{w_1, w_3, w_5, ..., w_n\} \subseteq W$ having $\frac{n+1}{2}$ vertices. Now S be the minimal set of vertices which covers all the vertices in $K_2 \times P_n$. Such that set of vertices of a subgraph $U - S$ is isolated, then by the above argument S is a minimal strong split geodetic set of $K_2 \times P_n$. Clearly it follows that, $|S| = |H_1 \cup H_2 = \frac{n-1}{2} + \frac{n+1}{2} = n$. Therefore $g_{ss}(K_2 \times P_n) = n$.
The following Corollaries are immediate consequence of above Theorem and Theorem 2.5.

COROLLARY 5.2. *For any path P_n of order n, $g_{ss}(K_2 \times P_n) = \alpha_0 + \beta_0$.*

COROLLARY 5.3. *For any path P_n of order n, $g_{ss}(K_2 \times P_n) = \alpha_1 + \beta_1$.*

THEOREM 5.4. *For any complete graph of order n, $g_{ss}(K_2 \times K_n) = 2n - 2$.*

Proof. Let G_1 and G_2 be disjoint copies of $G = K_n, n \geq 2$. Let $V = \{v_1, v_2, ..., v_n\}$ and $W = \{w_1, w_2, ...w_n\}$ be the vertex set of G_1 and G_2 respectively and let $v_i w_i \in E(K_2 \times K_n)$ for $i \in \{1, 2, ..., n\}$. Let S be the minimum geodetic set of $K_2 \times K_n$ by Theorem 2.4 $g(K_2 \times K_n) = g(K_n) = n$. Consider $S' = S \cup H$, where $H \subseteq U - S$ having $n - 2$ vertices, since $U - S$ has two components which are complete graphs. Now S' be the minimal set of vertices which covers all the vertices in $K_2 \times K_n$, such that set of vertices of subgraph $U - S'$ are isolated, then by the above argument S' is a minimal strong split geodetic set of $K_2 \times K_n$. Clearly it follows that $|S'| = |S \cup H| = n + n - 2 = 2n - 2$.

OBSERVATION 5.5. *For any complete graph of order n, $g(K_3 \times K_n) = g(K_n)$.*

THEOREM 5.6. *For any complete graph of order n, $g_{ss}(K_3 \times K_n) = 3n - 3$.*

Proof. Let G_1 and G_2 be disjoint copies of $G = K_n, n \geq 2$. Let $X = \{x_1, x_2, ..., x_n\}$, $Y = \{y_1, y_2, ..., y_n\}$ and $Z = \{z_1, z_2, ..., z_n\}$ be the vertex set of G_1, G_2 and G_3 respectively. Let S be the minimum geodetic set of $K_3 \times K_n$ by Observation 5.5 $g(K_3 \times K_n) = g(K_n) = n$. Consider $S' = S \cup H$, where $H \subseteq V - S$ having $2n - 3$ vertices. Now S' be the minimal set of vertices which covers all the vertices in $K_3 \times K_n$, such that set of vertices of subgraph $V - S'$ are isolated, then by the above argument S' is a minimal strong split geodetic set of $K_3 \times K_n$. Clearly it follows that $|S'| = |S \cup H| = n + 2n - 3 = 3n - 3$.

THEOREM 5.7. *G' be the graph obtained by adding an end edge (u, v) to a cycle $C_n = G$ of order $n > 3$, with $u \in G$ and $v \notin G$. Then $g_{ss}(K_2 \times G') = n + 2$.*

Proof. Let $\{u_1, u_2, ..., u_n, u_1\}$ be a cycle with n vertices. Let G' be the graph obtained from $G = C_n$ by adding an end-edge (u, v) such that $u \in G$ and $v \notin G$.
We have the following cases.
Case 1: Let G be an even cycle.
Let S be the minimum geodetic set of $K_2 \times G'$, by Theorem 2.4 $g(K_2 \times G')$=$g(G') = 2$. Consider $S' = S \cup H$, where $H \subseteq V - S$ having n vertices. Now S' be the minimal set of vertices which covers all the vertices in $K_2 \times G'$, such that set of vertices of subgraph $V - S'$ are totally disconnected. Then by the above argument S' is a minimal strong split geodetic set of $K_2 \times G'$. Clearly it follows that $|S'| = |S \cup H| = 2 + n$.
Case 2: Let G be an odd cycle.
Let S be the minimum geodetic set of $K_2 \times G'$, by Theorem 2.4 $g(K_2 \times G')$=$g(G') = 3$. Consider $S' = S \cup H$, where $H \subseteq V - S$ having $n - 1$ vertices. Now S' be the minimal set of vertices which covers all the vertices in $K_2 \times G'$, such that set of vertices of subgraph $V - S'$ are totally disconnected. Then by the above argument S' is a minimal strong split geodetic set of $K_2 \times G'$. Clearly it follows that $|S'| = |S \cup H| = 3 + n - 1 = n + 2$.

6. CONCLUSION

In this paper we establish many bounds on strong split geodetic number in terms of elements of G and covering number of G, further the relationship between strong split geodetic number and split geodetic number.

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