

On Generalized d-Closed Sets

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ABSTRACT

In this paper we present a new class of sets and functions namely gd-closed sets, gd-irresolute functions in the light of d-open sets in topological spaces. Further some of their characterizations are investigated with counter examples.

Keywords

gd-closed sets, gd-continuous functions, gd-irresolute functions.

1. INTRODUCTION

Many different forms of continuous functions have been introduced over years. Most of them involve the concept of g-closed sets, β -open sets, semiopen sets, sg-open sets, etc. In 1987, Bhattacharya and Lahiri [5] introduced the class of semi-generalized closed sets. In 1990, Arya and Nour[3] defined generalized semiclosed sets. The concept of generalized closed sets was first initiated by Levine in 1970[12]. The notion of b-open sets was defined by D. Andrijevic in 1996[2]. In this paper a new class of sets called gd-closed sets has been introduced using the concept of d-closed sets by I.Arockiarani et al[10]. Further we study the basic properties of gd-closed sets. Using this new concept of sets we have introduced new class of functions called gd-continuous and gd-irresolute functions. Some of its basic properties and composition of functions is also discussed here.

Preliminaries: We present here relevant preliminaries required for the progress of this paper

Definition 1.1: A subset A of a topological space (X, τ) is called

1. Preclosed set[15] if $\text{cl}(\text{int}(A)) \subseteq A$, preopen set if $A \subseteq \text{int}(\text{cl}(A))$
2. α -open set [17] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$, α -closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
3. Regular open set[19] if $A = \text{int}(\text{cl}(A))$, regular closed set if $A = \text{cl}(\text{int}(A))$
4. Semiopen set [11] if $A \subseteq \text{cl}(\text{int}(A))$, semiclosed set if $\text{int}(\text{cl}(A)) \subseteq A$.
5. Semi-pre-open set[1] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$, semi-pre-closed set if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.
6. d-open set[10] if $A \subseteq \text{scl}(\text{int}(A)) \cup \text{sint}(\text{cl}(A))$

Definition 1.2: A subset A of a topological space (X, τ) is called

1. A generalized closed set[12] (briefly g-closed) set if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
2. A generalized semiclosed set[3] (briefly gs-closed) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
3. A α -generalized closed set [13] (briefly α g-closed) if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
4. A generalized preclosed set [14] (briefly gp-closed) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
5. A generalized pre regular closed set[9] (briefly gpr-closed) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open
6. A α -regular generalized closed set[12] (briefly α gr-closed) if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open
7. A regular generalized closed set [18] (briefly rg-closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open.

2. GENERALIZED D-CLOSED SETS

Definition 2.1: A subset A of a space X is called Generalized d-closed set if $\text{dcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open. The class of all generalized d-closed sets is denoted by GDC(X).

Definition 2.2:

- i. The gd-closure of a subset A of X is denoted by $\text{gdcl}(A)$ is the smallest gd-closed set containing A.
- ii. The gd-interior of a subset A of X is denoted by $\text{gdint}(A)$ is the largest gd-open set contained in A.

Proposition 2.3[9]: The intersection of a open set and d-open set is d-open set.

Proposition 2.4:

- i. Every closed set is gd-closed set.
- ii. Every d-closed set is gd-closed set.

Proof:

- i. Let A be closed set such that $A \subseteq U$ where U is open in (X, τ) . Since A is closed $\text{cl}(A) = A \subseteq U$. But every closed set is d-closed. Hence A is gd-closed.
- ii. Let A be d-closed set such that $A \subseteq U$ where U is open in (X, τ) . Since A is d-closed $\text{dcl}(A) = A \subseteq U$. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.5: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. $A = \{a, c\}$ is gd-closed but not d-closed or closed.

Proposition 2.6: Every α -closed set is gd-closed set.

Proof: Let A be α -closed set such that $A \subseteq U$ where U is open in (X, τ) . Since A is α -closed $\alpha \text{cl}(A) = A \subseteq U$. But every α -closed set is d-closed. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.7: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$. $A = \{b\}$ is gd-closed but not α -closed.

Proposition 2.8: Every preclosed set is gd-closed set.

Proof: Let A be preclosed set such that $A \subseteq U$ where U is open in (X, τ) . Since A is preclosed set $\text{pcl}(A) = A \subseteq U$. But every preclosed set is d-closed set. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.9: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. $A = \{a, c\}$ is gd-closed set but not preclosed set.

Proposition 2.10: Every semiclosed set is gd-closed set.

Proof: Let A be a semiclosed set such that $A \subseteq U$ where U is open in (X, τ) . Since A is semiclosed set $\text{scl}(A) = A \subseteq U$. But every semiclosed set is d-closed set. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.11: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$. $A = \{b\}$ is gd-closed set but not semiclosed set.

Proposition 2.12: Every g-closed set is gd-closed set.

Proof: Let A be a g-closed set then $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) . Since every closed set is d-closed set we have $\text{dcl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.13: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{a, b\}\}$. $A = \{b\}$ is gd-closed set but not g-closed set.

Proposition 2.14: Every α g-closed set is gd-closed set.

Proof: Let A be a α g-closed set then $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) . Since every α -closed set is d-closed set we have $\text{dcl}(A) \subseteq \alpha \text{cl}(A) \subseteq U$. Hence A is d-closed. The converse of the above result need not be true may seen by the following example.

Example 2.15: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{b, c\}\}$. $A = \{b\}$ is gd-closed set but not α g-closed set.

Proposition 2.16: Every gp-closed set is gd-closed set.

Proof: Let A be a gp-closed set then $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) . Since every pre closed set is d-closed set we have $\text{dcl}(A) \subseteq \text{pcl}(A) \subseteq U$. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.17: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. $A = \{a\}$ is gd-closed set but not gp-closed set.

Proposition 2.18: Every gpr-closed set is gd-closed set.

Proof: Let A be a gpr-closed set then $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) . Every regular open is open hence $A \subseteq U$ and U is open. Since every pre closed set is d-closed set we have $\text{dcl}(A) \subseteq \text{pcl}(A) \subseteq U$. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.19: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. $A = \{a\}$ is gd-closed set but not gpr-closed set.

Proposition 2.20: Every α gr-closed set is gd-closed set.

Proof: Let A be a α gr-closed set then $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) . Every regular open is open hence $A \subseteq U$ and U is open. Since every α -closed set is d-closed set we have $\text{dcl}(A) \subseteq \alpha \text{cl}(A) \subseteq U$. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.21: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. $A = \{a\}$ is gd-closed set but not α gr-closed set.

Proposition 2.22: Every rg-closed set is gd-closed set.

Proof: Let A be a rg-closed set then $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) . Every regular open is open hence $A \subseteq U$ and U is open. Since every closed set is d-closed set we have $\text{dcl}(A) \subseteq \text{cl}(A) \subseteq U$. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.23: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. $A = \{a\}$ is gd-closed set but not rg-closed set.

Proposition 2.24: Every gs-closed set is gd-closed set.

Proof: Let A be a gs-closed set then $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) . Since every semiclosed set is d-closed set we have $\text{dcl}(A) \subseteq \text{scl}(A) \subseteq U$. Hence A is gd-closed. The converse of the above result need not be true may seen by the following example.

Example 2.25: Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{b, c\}\}$. $A = \{b\}$ is gd-closed set but not gs-closed set.

From the above examples we have the following diagram

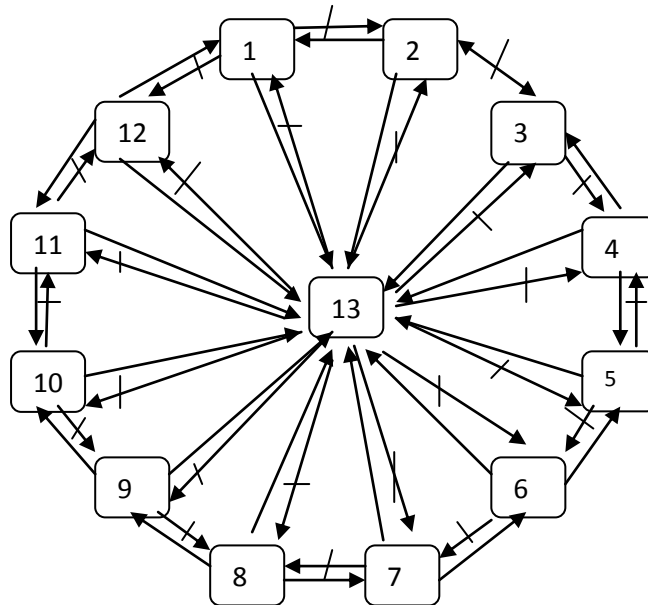


Figure. 1 Relation between generalized d-closed set and other existing closed sets

- 1.gs closed set 2.g closed set 3. α gr closed set 4. rg closed set 5. gpr closed set 6. gp closed set
 7. α g-closed set 8. α -closed set 9.semiclosed set 10.d-closed set 11.preclosed set 12.closed set
 13.gd-closed set.

Lemma 2.26: For a subset A of a space X the following hold

- $dcl(A) = A \cup (sint(cl(A)) \cap scl(int(A)))$
- $dint(A) = A \cap (scl(int(A)) \cup sint(cl(A)))$
- $dcl(X-A) = X - dint(A)$

Theorem 2.27: A set A is gd-open if and only if $F \subseteq dint(A)$ whenever F is closed and $F \subseteq A$.

Proof: Let A be gd-open and suppose that $F \subseteq A$ and F is closed. Then X-A is gd-closed set. Thus $gdcl(X-A) \subseteq X-F$. But $dcl(X-A) = X - dint(A)$. Thus $X - dint(A) \subseteq X-F$. Hence $F \subseteq dint(A)$. Conversely, let F be closed set with $F \subseteq dint(A)$ whenever $F \subseteq A$. $F \subseteq A \Rightarrow (X-A) \subseteq (X-F)$. Also $F \subseteq dint(A)$. Thus $(X - dint(A)) = dcl(X-A) \subseteq X-F$ where (X-F) is open. Thus A is gd-open.

Theorem 2.28: Let A be closed subset of (X, τ) . Then $dcl(A) - A$ does not contain any non empty closed set.

Proof: Let A be generalized d-closed and F be a non empty closed set such that $F \subseteq dcl(A) - A$. Then $F \subseteq dcl(A) \cap F \subseteq dcl(A)$ and $F \subseteq A^c$. $F \subseteq A^c \Rightarrow F^c$ is open. Then $dcl(A) \subseteq F^c \Rightarrow F \subseteq (dcl(A))^c$. $F \subseteq dcl(A) \cap (dcl(A))^c \Rightarrow F \subseteq \emptyset$ which is a contradiction. Hence $dcl(A) - A$ contains no non empty closed set.

Theorem 2.29: Let A be a gd-closed subset of (X, τ) and $A \subseteq B \subseteq dcl(A)$, then B is gd-closed.

Proof: Let $A \subseteq B \subseteq dcl(A)$ then $dcl(A) = dcl(B)$. Since A is gd-closed, $dcl(A) \subseteq G$ where $A \subseteq G$; G is open in X. Let $B \subseteq G$ and G is open in X, since A is gd-closed and since $dcl(A) = dcl(B)$, $dcl(B) \subseteq G$. Thus B is gd-closed.

Theorem 2.30: A gd-closed set A is d-closed if and only if $dcl(A)-A$ is closed

Proof: Let A be gd-closed. If A is d-closed then $dcl(A)-A = \emptyset$ is a closed set. Thus $dcl(A)-A$ is closed. Conversely, let $dcl(A)-A$ be closed. $dcl(A)-A$ is closed subset of itself hence $dcl(A)-A$ contain any non empty closed subset thus $dcl(A)-A = \emptyset$ (i.e) $A=dcl(A)$. Hence A is d-closed.

Definition 2.31: Let $B \subseteq A \subseteq X$, then we say B is gd-closed relative to A if $dcl_A(B) \subseteq U$ when $B \subseteq U$ and U is open in A

Theorem 2.32: Let $B \subseteq A \subseteq X$ where A is gd-closed and open. Then B is gd-closed relative to A if and only if B is gd-closed.

Proof: Let $B \subseteq A \subseteq X$, where A is gd-closed and open and let B is gd-closed relative to A. Since $B \subseteq A$ and A is gd-closed and open $dcl(A) \subseteq A$. Thus $dcl(B) \subseteq dcl(A) \subseteq A$.

$dcl_A(B) = A \cap dcl(B)$ so $dcl(B) = dcl_A(B) \subseteq A$. If B is gd-closed relative to A and U is a open subset of X such that $B \subseteq U$ then $B = B \cap A \subseteq U \cap A$ where $U \cap A$ is open in A. Thus B is gd-closed relative to A. $dcl(B) = dcl_A(B) \subseteq U \cap A \subseteq U$. Thus B is gd-closed in X. Conversely, Let B is gd-closed in X and U is open subset of A such that $B \subseteq U$. Let $U = V \cap A$ for some open subset V of X. As $B \subseteq V$ & B is gd-closed in X, $dcl(B) \subseteq V$. Thus $dcl_A(B) = dcl(B) \cap A \subseteq V \cap A = U$. Hence B is gd-closed relative to A.

Corollary 2.33: Let A be a gd-closed set which is also open then $A \cap F$ is gd-closed whenever F is d-closed.

Proof: Let A be a gd-closed and open then $dcl(A) \subseteq A$. But $A \subseteq dcl(A)$ thus $A = dcl(A)$. Hence $A \cap F$ is d-closed. Every d-closed is gd-closed. $A \cap F$ is gd-closed.

Remark 2.34: Intersection of two gd-closed sets need not be gd-closed.

Example 2.35: Let $X = \{a, b, c\}, \tau = \{X, \emptyset, \{a\}\}$, $A = \{a, b\}$ which is gd-closed set. $B = \{a, c\}$ which is gd-closed set. But $A \cap B = \{a\}$ is not gd-closed.

Remark 2.36: Union of two gd-closed sets need not be gd-closed.

Example 2.37: Let $X = \{a, b, c\}, \tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$, $A = \{a\}$ which is gd-closed set. $B = \{c\}$ which is gd-closed set. But $A \cup B = \{a, c\}$ is not gd-closed.

3. gd-Continuous and gd-Irresolute Functions

Definition 3.1: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called gd-continuous if the pre image of every closed set of Y is gd-closed in X.

Definition 3.2: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called gd-irresolute if the pre image of every gd-closed set of Y is gd-closed in X.

Definition 3.3: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ be d-irresolute if the pre image of every d-open set in Y is d-open in X.

Definition 3.4: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called d-continuous if the inverse image of every closed set in Y is d-closed in X

Theorem 3.5: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be d-continuous, then f is gd-continuous but not conversely.

Proof: Let V be a open set in Y. Since f is d-continuous then $f^{-1}(V)$ is d-open in X. But every d-open set is gd-open in X hence $f^{-1}(V)$ is gd-open in X for $V \in Y$. Thus f is gd-continuous.

Example 3.6: Let $X = \{a, b, c\}, \tau = \{X, \emptyset, \{a\}, \{a, b\}\}$, $Y = \{a, b, c\}, \sigma = \{\{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be identity map. f is gd-continuous but not d-continuous because $f^{-1}(\{b\}) = \{b\} \notin DO(X)$.

Theorem 3.7: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a continuous function. If f is gd-continuous then for each point $x \in X$ and for each open set V in Y containing $f(x)$, there exists a gd-open set U containing x such that $f(U) \subseteq V$

The proof follows immediately.

Theorem 3.8: If the bijective map $f : (X, \tau) \rightarrow (Y, \sigma)$ is d-irresolute and open, then f is gd-irresolute.

Proof: Let V be gd-closed in Y and let $f^{-1}(V) \subseteq U$ where U is open in X. Hence $V \subseteq f(U)$ holds. Since $f(U)$ is open and V is sgd-closed in Y $dcl(A) \subseteq f(U)$. Therefore $f^{-1}(dcl(V)) \subseteq U$ and since f is d-irresolute and $dcl(V)$ is d-closed in Y, $f^{-1}(dcl(V))$ is d-closed set in X. Thus $dcl(f^{-1}(V)) \subseteq dcl(f^{-1}(dcl(V))) = f^{-1}(dcl(V)) \subseteq U$. Hence $f^{-1}(V)$ is gd-closed and thus f is gd-irresolute.

Remark 3.9: The concept of d-irresoluteness and gd-irresoluteness are independent.

Example 3.10: Let $X = \{a, b, c\}, \tau = \{X, \emptyset, \{a\}, \{a, b\}\}$, $Y = \{a, b, c\}, \sigma = \{\{X, \emptyset, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be identity map. Here f is d-irresolute but not gd-irresolute since $V = \{c\} \in GDO(Y)$ but $f^{-1}(\{c\}) = \{c\} \notin GDO(X)$.

Example 3.11: Let $X = \{a, b, c\}, \tau = \{X, \emptyset, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = f(c) = b, f(b) = c$. Here f is gd-irresolute but not d-irresolute, since $V = \{b\} \in DC(X)$ but $f^{-1}(\{b\}) = \{a, c\} \notin DC(X)$.

Theorem 3.12: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map from a topological space (X, τ) into a topological space (Y, σ) , then the following are equivalent.

- a) f is gd-continuous.
- b) The inverse image of each open set in Y is gd-open in X.

The proof follows immediately from the definition.

Theorem 3.13: Let $f:(X, \tau) \rightarrow (Y, \sigma)$ and $g:(Y, \sigma) \rightarrow (Z, \mu)$ be any two functions then

1. $\text{gof}:(X, \tau) \rightarrow (Z, \mu)$ is gd-continuous if g is continuous and f is gd-continuous.
2. $\text{gof}:(X, \tau) \rightarrow (Z, \mu)$ is gd-irresolute if g is gd-irresolute and f is gd-irresolute.
3. $\text{gof}:(X, \tau) \rightarrow (Z, \mu)$ is gd-continuous if g is gd-continuous and f is gd-irresolute.

Proof:

1. Let V closed set in (Z, μ) . $g^{-1}(V)$ is closed in (Y, σ) , since g is continuous. Thus $f^{-1}(g^{-1}(V))$ is gd-closed in (X, τ) since f is gd-continuous. Thus gof is gd-continuous.
2. Let V be gd-closed set in (Z, μ) , since $g^{-1}(V)$ is gd-closed in (Y, σ) . Since f is gd-irresolute, it is gd-closed in (X, τ) . Thus gof is gd-irresolute.
3. Let V be closed set in (Z, μ) . Thus $g^{-1}(V)$ is gd-closed in (Y, σ) . Since f is gd-irresolute $f^{-1}(g^{-1}(V))$ is gd-closed in (X, τ) . Thus gof is gd-continuous.

Definition 3.14: A topological space X is called a $d-T_{\frac{1}{2}}$ space if every gd-closed set is closed.

Definition 3.15 [13]: A topological space X is called a $T_{\frac{1}{2}}$ space if every g-closed sets is closed.

Definition 3.16 [7]: A topological space is called a αT_b space if every α g-closed set is closed.

Proposition 3.18:

1. Every $d-T_{\frac{1}{2}}$ space is a $T_{\frac{1}{2}}$ space
2. Every $d-T_{\frac{1}{2}}$ space is a αT_b space.

Example 3.18: Let $X=\{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then the collection is $T_{\frac{1}{2}}$ and αT_b but not $d-T_{\frac{1}{2}}$ since $A=\{b\}$ is gd-closed but not closed.

Theorem 3.19: Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be gd-irresolute function then f is d-irresolute function if X is $d-T_{\frac{1}{2}}$ space.

Proof: Let V be d-closed set in Y . Since V is gd-closed in Y and f is gd-irresolute, $f^{-1}(V)$ is gd-closed in X . But X is

$d-T_{\frac{1}{2}}$ so $f^{-1}(V)$ is closed in X . Every closed is d-closed hence f is d-irresolute.

Theorem 3.20: Let X and Z be any topological spaces and Y be a $d-T_{\frac{1}{2}}$ space then the composition

1. $\text{gof}:(X, \tau) \rightarrow (Z, \mu)$ is gd-continuous if g is gd-continuous and f is gd-continuous.
2. $\text{gof}:(X, \tau) \rightarrow (Z, \mu)$ is d-continuous if g is gd-continuous and f is d-continuous.
3. $\text{gof}:(X, \tau) \rightarrow (Z, \mu)$ is gd-continuous if g is gd-irresolute and f is gd-continuous.
4. $\text{gof}:(X, \tau) \rightarrow (Z, \mu)$ is d-continuous if g is gd-irresolute and f is d-continuous.

Proof:

1. Let U be closed set in Z , then $g^{-1}(U)$ is gd-closed in Y . But Y is a $d-T_{\frac{1}{2}}$ space thus $g^{-1}(U)$ is closed in Y . Since f is gd-continuous $f^{-1}(g^{-1}(U))$ is gd-closed in X . Thus gof is gd-continuous.
2. Let U be closed set in Z , then $g^{-1}(U)$ is gd-closed in Y . But Y is a $d-T_{\frac{1}{2}}$ space thus $g^{-1}(U)$ is closed in Y . Since f is d-continuous $f^{-1}(g^{-1}(U))$ is d-closed in X . Thus gof is d-continuous.
3. Let U be gd-closed set in Z , then $g^{-1}(U)$ is gd-closed in Y . But Y is a $d-T_{\frac{1}{2}}$ space thus $g^{-1}(U)$ is closed in Y . Since f is gd-continuous $f^{-1}(g^{-1}(U))$ is gd-closed in X . Thus gof is gd-continuous.
4. Let U be gd-closed in Z , then $g^{-1}(U)$ is gd-closed in Y . But Y is a $d-T_{\frac{1}{2}}$ space thus $g^{-1}(U)$ is closed in Y . Since f is d-continuous $f^{-1}(g^{-1}(U))$ is d-closed in X . Thus gof is d-continuous in X .

3. CONCLUSION

In general topology g-closed sets has a major role. Since its inception several weak forms of g-closed sets have been introduced in general topology. The present paper investigated in new weak form of g closed sets namely gd-closed sets and functions namely gd-irresolute functions in the light of d-open sets in topological spaces. Some of its basic properties and composition of functions is also discussed. Many examples had been given to justify the results.

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