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Application of the Fuzzy Multi-criteria Decision-Making Method to Identify Nonlinear Decision Models

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ABSTRACT

A large number of multi-criteria methods have been developed to deal with different kinds of problems. Most of them use a linear aggregation, what is the cause of many shortcomings in solving decision problems. This paper presents how to identify nonlinear multi-criteria decision-making models with using the new fuzzy method: the Characteristic Objects Method $(COMET)$. In this approach, models are constructed on the basis of characteristic objects and fuzzy rules. Thereby, the $COMET$ method is free of rank reversal phenomenon, which is one of the most commonly indicated shortcoming of the multi-criteria decision-making methods. This study introduces the concepts of characteristic objects and way of their determination. Subsequently, the paper presents approach to construct the fuzzy rule base and the entire model. Finally, the theoretical nonlinear problem is presented to verify the developed approach and to demonstrate its effectiveness.

General Terms:

Decision support

Keywords:

Multi-criteria Making-decision Method, Rank Reversal, Decision Making, Characteristic Objects, COMET Method

1. INTRODUCTION

In the complex world, the human decisions are not always correct and successful [1]. It is mostly caused by the fact that many decision problems involve a large number of conflicted objectives [2, 3]. This problem relates primarily to nonlinear problems, which are more difficult to solve than linear problems [4]. The multi-criteria decision-making methods were created to cope with these problems. For instance, the commonly used methods are: Simple Additive Weighting (SAW) [9, 10, 11, 12, 13, 14], the Analytic Hierarchy Process (AHP) [15, 16, 17, 18, 19, 20, 21, 22, 23], Technique for Order of Preference by Similarity to Ideal Solution $(TOPSIS)$ [24, 25, 26, 27, 28, 29, 30], Elimination and Choice Expressing Reality $(ELECTRE)$ family methods [31, 32, 33, 34, 35] and $PROMETHEE$ [36, 37, 38, 39]. These approaches cope well with the linear problems, however, the rank reversal phenomenon is occurred for nonlinear problems. The commonly used rank reversal definition emphasizes that this issue occurs when the rankings for the alternatives are changed with either the addition of or removal of an alternative. As the result, decision-makers cannot be sure, which ranking is correct. The process of evaluating is not connected with alternatives in the $COMET$ method, so it is completely rank reversal free approach. The evaluating model is created by using invariable points, which are called characteristic objects.
In section 2, the basic concepts and operations related to the fuzzy set theory are given. In section 3, the concept of the characteristic objects is introduced. Then, the $COMET$ method is presented, which is the new fuzzy multi-criteria decision-making method. The basis of this approach was developed by professor Piegat [4]. In section 4, the theoretical nonlinear problem is presented to verify the proposed approach. In section 5, the effectiveness of the $COMET$ method is discussed on the basis of model from previous section. The conclusions are presented in section 6.

2. FUNDAMENTAL NOTIONS AND CONCEPTS OF THE FUZZY SETS

Hereafter, the essential concepts of fuzzy sets are introduced, which are using in the COMET method. This section is focused on the fundamental ideas, i.e., fuzzy set, membership function, triangular fuzzy numbers, the support and core of a triangular fuzzy number, fuzzy rule, the rule base and T-norm operator.

Definition 1 Fuzzy set and membership function.
The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X inasmuch as crisp sets only allow full membership $(\mu_A(x) = 1)$ or non-membership at all $(\mu_A(x) = 0)$. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range, i.e., $\mu_{\tilde{A}} : X \rightarrow [0, 1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\}$, where $x \in X$, defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set [5, 6, 7].

Definition 2 Triangular fuzzy number (TFN).
A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\Re$, is said to be a triangular fuzzy number $\tilde{A}(a, m, b)$ if its membership

function has the following form (1) [1]:

$$\mu_{\tilde{A}}(x,a,m,b) = \begin{cases} 0 & x \le a \\ \frac{x-a}{m-a} & a \le x \le m \\ 1 & x = m \\ \frac{b-x}{b-m} & m \le x \le b \\ 0 & x \ge b \end{cases} \quad (1)$$

and the following characteristics (2, 3):

$$x_1, x_2 \in [a,b] \wedge x_2 > x_1 \Rightarrow \mu_{\tilde{A}}(x_2) > \mu_{\tilde{A}}(x_1) \quad (2)$$

$$x_1, x_2 \in [b,c] \wedge x_2 > x_1 \Rightarrow \mu_{\tilde{A}}(x_2) < \mu_{\tilde{A}}(x_1) \quad (3)$$

Definition 3 The support of a TFN $\tilde{A}$
This is the crisp subset of the set $\tilde{A}$ whose all elements have non-zero membership values in the set $\tilde{A}$ (4):

$$S(\tilde{A}) = \{x : \mu_{\tilde{A}}(x) > 0\} = [a,b] \quad (4)$$

Definition 4 The core of a TFN $\tilde{A}$
This is the singleton (one-element fuzzy set) with the membership value equal to one (5):

$$C(\tilde{A}) = \{x : \mu_{\tilde{A}}(x) = 1\} = m \quad (5)$$

Definition 5 The fuzzy rule
The single fuzzy rule can be based on tautology Modus Ponens [6, 8]. The reasoning process uses logical connectives $IF - THEN$, OR and AND.

Definition 6 The rule base.
The rule base consists of logical rules determining causal relationships existing in the system between fuzzy sets of its inputs and output [40].

Definition 7 T-norm operator: product.
The t-norm operator is a function T modeling the intersection operation AND of two or more fuzzy numbers, e.g. $\tilde{A}$ and $\tilde{B}$. In this paper, only product is used as t-norm operator [6, 8, 40] (6):

$$\mu_{\tilde{A}}(x) \;\; AND \;\; \mu_{\tilde{B}}(y) \; = \; \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(y) \quad (6)$$

3. THE CHARACTERISTIC OBJECTS METHOD (COMET)

The COMET method is a very simple approach, but to be able to understand better this technique, the basic knowledge on the Fuzzy Sets is necessary [4, 41]. The formal notation of COMET method is presented below in the five following subsections.

3.1 Define the space of the problem

An expert determines dimensionality of the problem by selecting number r of criteria, $C_1, C_2, ..., C_r$. Subsequently, the set of triangular fuzzy numbers for each criterion C_i is selected, i.e., $\tilde{C}_{i1}, \tilde{C}_{i2}, ..., \tilde{C}_{ic_i}$. In this way, the following result is obtained (7):

$$\begin{array}{l} C_1 = \{\tilde{C}_{11}, \tilde{C}_{12}, ..., \tilde{C}_{1c_1}\} \\ C_2 = \{\tilde{C}_{21}, \tilde{C}_{22}, ..., \tilde{C}_{2c_1}\} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ C_r = \{\tilde{C}_{r1}, \tilde{C}_{r2}, ..., \tilde{C}_{rc_r}\} \end{array} \quad (7)$$

where $c_1, c_2, ..., c_r$ are numbers of the fuzzy numbers for all criteria.

3.2 Generate the characteristic objects

The characteristic objects (CO) are obtained by using the Cartesian Product of triangular fuzzy numbers cores for each criteria as follows (8):

$$CO = C(C_1) \times C(C_2) \times ... \times C(C_r) \quad (8)$$

As the result of this, the ordered set of all CO is obtained (9):

$$\begin{array}{l} CO_1 = C(\tilde{C}_{11}), C(\tilde{C}_{21}), ..., C(\tilde{C}_{r1}) \\ CO_2 = C(\tilde{C}_{11}), C(\tilde{C}_{21}), ..., C(\tilde{C}_{r2}) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ CO_t = C(\tilde{C}_{1c_1}), C(\tilde{C}_{2c_2}), ..., C(\tilde{C}_{rc_r}) \end{array} \quad (9)$$

where t is a number of CO (10):

$$t = \prod_{i=1}^{r} c_i \quad (10)$$

3.3 Rank the characteristic objects

The expert determines the Matrix of Expert Judgment (MEJ). It is a result of comparison of the characteristic objects by the knowledge of expert. The MEJ structure is as follows (11):

$$MEJ = \begin{pmatrix} \alpha_{11} & \alpha_{12} & ... & \alpha_{1t} \\ \alpha_{21} & \alpha_{22} & ... & \alpha_{2t} \\ ... & ... & ... & ... \\ \alpha_{t1} & \alpha_{t2} & ... & \alpha_{tt} \end{pmatrix} \quad (11)$$

where α_{ij} is a result of comparing CO_i and CO_j by the expert. The more preferred characteristic object gets one point and the second object get zero point. If the preferences are balanced, the both objects get half point. It depends solely on the knowledge of the expert and can be presented as (12):

$$\alpha_{ij} = \begin{cases} 0.0, & f_{exp}(CO_i) < f_{exp}(CO_j) \\ 0.5, & f_{exp}(CO_i) = f_{exp}(CO_j) \\ 1.0, & f_{exp}(CO_i) > f_{exp}(CO_j) \end{cases} \quad (12)$$

where f_{exp} is an expert judgment function. The most important properties are described by equations (13) and (14):

$$\alpha_{ii} = 0.5 \quad (13)$$

$$\alpha_{ji} = 1 - \alpha_{ij} \quad (14)$$

On the basis of (13) and (14), the number of comparisons is reduced from t^2 cases to p cases (15):

$$p = \begin{pmatrix} t \\ 2 \end{pmatrix} = \frac{t(t-1)}{2} \quad (15)$$

Afterwards, the vertical vector of the summed Judgments (SJ) is obtained as follows (16):

$$SJ_i = \sum_{j=1}^{t} \alpha_{ij} \quad (16)$$

The last step assigns to each characteristic object the approximate value of preference. In the result, the vertical vector P is obtained, where $i-th$ row contains the approximate value of preference for CO_i. This algorithm is presented as a fragment of Matlab code:

```
1: k = length(unique(SJ));
2: P = zeros(t,1);
```

```
3: for i = 1:k
4:     ind = find(SJ == max(SJ));
5:     P(ind) = (k - i) / (k - 1);
6:     SJ(ind) = 0;
7: end
```

In line 1, the number k is obtained as a number of unique value of the vector SJ. In line 2, the vertical vector P of zeros is created (with identical size as vector SJ). In line 4, the index with maximum value from vector SJ is obtained. This index is used to assign the value of preference to adequate position in vector P (based on the principle of indifference of Laplacea). In line 6, the maximum value of the vector SJ is reset.

3.4 The rule base

Each one characteristic object and value of preference is converted to a fuzzy rule as follows, general form (17) and detailed form (18):

$$IF\ CO_i\ THEN\ P_i \tag{17}$$

$$IF\ C(\tilde{C}_{1i})\ AND\ C(\tilde{C}_{2i})\ AND\ ...\ THEN\ P_i \tag{18}$$

In this way, the complete fuzzy rule base is obtained (19):

$$\begin{array}{llll} IF & CO_1 & THEN & P_1 \\ IF & CO_2 & THEN & P_2 \\ ... & ... & ... & ... \\ IF & CO_t & THEN & P_t \end{array} \tag{19}$$

3.5 Inference and final ranking

The each one alternative is a set of crisp number, which corresponding with criteria $C_1, C_2, ..., C_r$. It can be presented as follows (20):

$$A_i = \{a_{1i}, a_{2i}, ..., a_{ri}\} \tag{20}$$

where condition (21) must be satisfied.

$$\begin{array}{l} a_{1i} \in [C(\tilde{C}_{11}), C(\tilde{C}_{1c_1})] \\ a_{2i} \in [C(\tilde{C}_{21}), C(\tilde{C}_{2c_2})] \\ .... \\ a_{ri} \in [C(\tilde{C}_{r1}), C(\tilde{C}_{rc_r})] \end{array} \tag{21}$$

Each one alternative activates the specified number of fuzzy rules, where for each one is determined the fulfillment degree of the conjunctive complex premise. Fulfillment degrees of all activated rules sum to one. The preference of alternative is computed as sum of the product of all activated rules, as their fulfillment degrees, and their values of the preference. The final ranking of alternatives is obtained by sorting the preference of alternatives.

4. EXPERIMENT AND RESULTS

In most cases of used $MCDM$ methods, decision-makers cannot be sure whether the resulting ranking is correct. Moreover, if decision-makers get two conflicted rankings, one of them is certainly incorrect. This issue lies in heart every multi-criteria decision-making methods. The theoretical nonlinear problem is given to examine the effectiveness of proposed approach, because the strong reference is needed to do it. Therefore, the mathematical formula will be used instead of expert. This assumption eliminates purely human errors, and allows to validate the method. Lets consider a nonlinear problem with only two criteria. The solution will be the three-dimensional surface, which can be presented in an illustration. The reference formula of this problem is presented as (22):

$$f(X, Y) = 1 - (X - 0.5)^2 - (Y - 0.5)^2 \tag{22}$$

where the variable X is the first criterion and variable Y is the second criterion . The both variables are considered in the interval from zero to one. Figure 1 presents shape of this function.
The space of the problem is defined using two criterion (23), where each of them is presented as the set of five equal division triangular fuzzy numbers (see figure 2 and figure 3).

$$\begin{array}{l} C_1 = \{\tilde{C}_{11}, \tilde{C}_{12}, \tilde{C}_{13}, \tilde{C}_{14}, \tilde{C}_{15}\} \\ C_1 = \{\tilde{C}_{21}, \tilde{C}_{22}, \tilde{C}_{23}, \tilde{C}_{24}, \tilde{C}_{25}\} \end{array} \tag{23}$$

The characteristic objects are obtained on the basis these triangular fuzzy numbers and the following formula (24):

$$CO = C(C_1) \times C(C_2) \tag{24}$$

On this basis, 25 characteristic objects are obtained, which equally divide the space of the problem. Figure 4 shows all characteristic objects and their distribution.
In the next step, the 25 characteristic objects are ranked, where the MEJ matrix is created on the basis equations (12) and (22). The

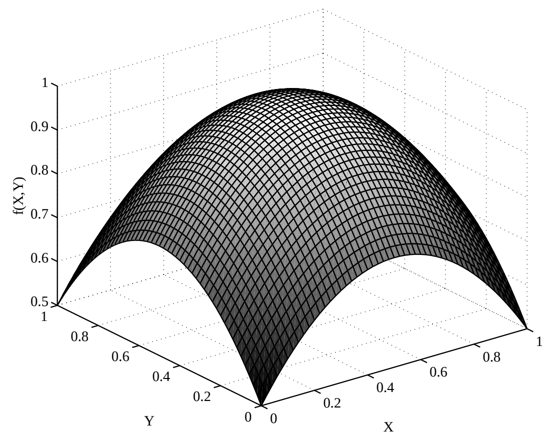

Fig. 1. The shape of the reference function.

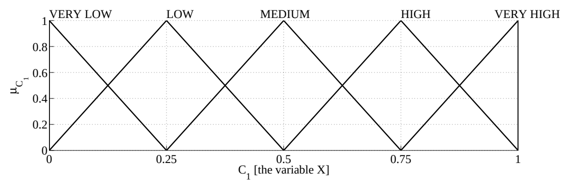

Fig. 2. Membership functions for the criterion C_1.

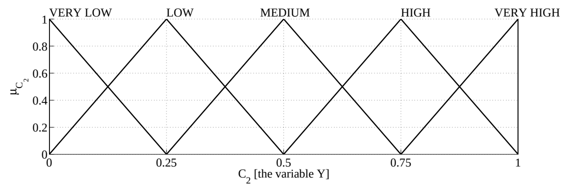

Fig. 3. Membership functions for the criterion C_2.

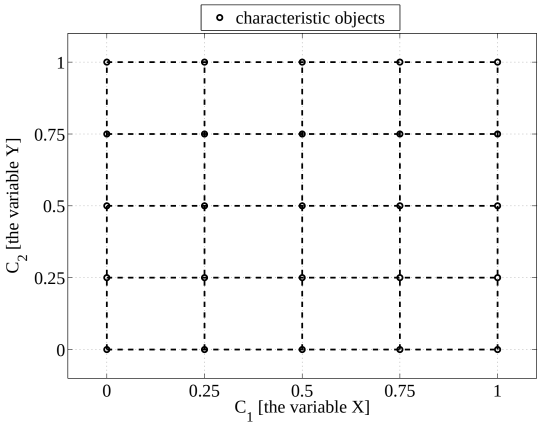

Fig. 4. Two-dimensional domain of the theoretical problem, with marked characteristic objects.

Table 1. Preference values of the characteristic objects

CO_i	$C(C_1)$	$C(C_2)$	P_i	CO_i	$C(C_1)$	$C(C_2)$	P_i
1	0.00	0.00	0.0	14	0.50	0.75	0.8
2	0.00	0.25	0.2	15	0.50	1.00	0.4
3	0.00	0.50	0.4	16	0.75	0.00	0.2
4	0.00	0.75	0.2	17	0.75	0.25	0.6
5	0.00	1.00	0.0	18	0.75	0.50	0.8
6	0.25	0.00	0.2	19	0.75	0.75	0.6
7	0.25	0.25	0.6	20	0.75	1.00	0.2
8	0.25	0.50	0.8	21	1.00	0.00	0.0
9	0.25	0.75	0.6	22	1.00	0.25	0.2
10	0.25	1.00	0.2	23	1.00	0.50	0.4
11	0.50	0.00	0.4	24	1.00	0.75	0.2
12	0.50	0.25	0.8	25	1.00	1.00	0.0
13	0.50	0.50	1.0	X	X	X	X

value of preference is determined as the vector P. The summary of this step is presented in table 1.

Finally, each one characteristic object (from table 1) is converted to a fuzzy rule. In this way, the complete fuzzy rule base is created as following:

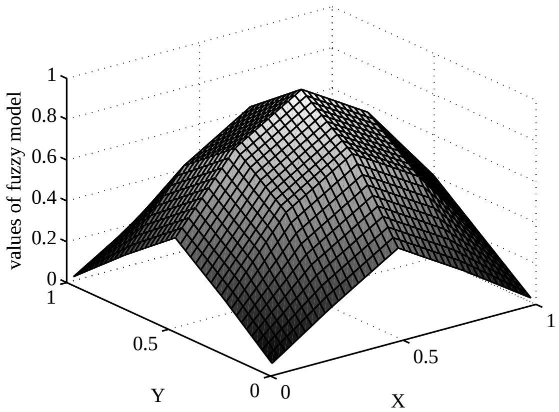

Fig. 5. The shape of the fuzzy model.

$$
\begin{array}{l}
IF\ (C_1 \sim 0.00)\ AND\ (C_2 \sim 0.00)\ THEN\ P \sim 0.0 \\
IF\ (C_1 \sim 0.00)\ AND\ (C_2 \sim 0.25)\ THEN\ P \sim 0.2 \\
IF\ (C_1 \sim 0.00)\ AND\ (C_2 \sim 0.50)\ THEN\ P \sim 0.4 \\
IF\ (C_1 \sim 0.00)\ AND\ (C_2 \sim 0.75)\ THEN\ P \sim 0.2 \\
IF\ (C_1 \sim 0.00)\ AND\ (C_2 \sim 1.00)\ THEN\ P \sim 0.0 \\
IF\ (C_1 \sim 0.25)\ AND\ (C_2 \sim 0.00)\ THEN\ P \sim 0.2 \\
IF\ (C_1 \sim 0.25)\ AND\ (C_2 \sim 0.25)\ THEN\ P \sim 0.6 \\
IF\ (C_1 \sim 0.25)\ AND\ (C_2 \sim 0.50)\ THEN\ P \sim 0.8 \\
IF\ (C_1 \sim 0.25)\ AND\ (C_2 \sim 0.75)\ THEN\ P \sim 0.6 \\
IF\ (C_1 \sim 0.25)\ AND\ (C_2 \sim 1.00)\ THEN\ P \sim 0.2 \\
IF\ (C_1 \sim 0.50)\ AND\ (C_2 \sim 0.00)\ THEN\ P \sim 0.4 \\
IF\ (C_1 \sim 0.50)\ AND\ (C_2 \sim 0.25)\ THEN\ P \sim 0.8 \\
IF\ (C_1 \sim 0.50)\ AND\ (C_2 \sim 0.50)\ THEN\ P \sim 1.0 \\
IF\ (C_1 \sim 0.50)\ AND\ (C_2 \sim 0.75)\ THEN\ P \sim 0.8 \\
IF\ (C_1 \sim 0.50)\ AND\ (C_2 \sim 1.00)\ THEN\ P \sim 0.4 \\
IF\ (C_1 \sim 0.75)\ AND\ (C_2 \sim 0.00)\ THEN\ P \sim 0.2 \\
IF\ (C_1 \sim 0.75)\ AND\ (C_2 \sim 0.25)\ THEN\ P \sim 0.6 \\
IF\ (C_1 \sim 0.75)\ AND\ (C_2 \sim 0.50)\ THEN\ P \sim 0.8 \\
IF\ (C_1 \sim 0.75)\ AND\ (C_2 \sim 0.75)\ THEN\ P \sim 0.6 \\
IF\ (C_1 \sim 0.75)\ AND\ (C_2 \sim 1.00)\ THEN\ P \sim 0.2 \\
IF\ (C_1 \sim 1.00)\ AND\ (C_2 \sim 0.00)\ THEN\ P \sim 0.0 \\
IF\ (C_1 \sim 1.00)\ AND\ (C_2 \sim 0.25)\ THEN\ P \sim 0.2 \\
IF\ (C_1 \sim 1.00)\ AND\ (C_2 \sim 0.50)\ THEN\ P \sim 0.4 \\
IF\ (C_1 \sim 1.00)\ AND\ (C_2 \sim 0.75)\ THEN\ P \sim 0.2 \\
IF\ (C_1 \sim 1.00)\ AND\ (C_2 \sim 1.00)\ THEN\ P \sim 0.0
\end{array}
$$

In the result of the investigation, the fuzzy reference model is identified for the theoretical multi-criteria decision-making problem defined as equation (22). The surface of this model is presented in figure 5.

5. DISCUSSION OF THE RESULTS

In the result of section 4, the fuzzy rule base was obtained, which represented the identified fuzzy model. On the first sight, there are quite a lot of differences between surfaces presented in figure 1 and figure 5, e.g., difference target sets or smoothness. However, for the decision-making support, a model must properly compare two alternatives. Therefore, the test sample of alternatives should be drawn to estimate percent of the correct comparisons between alternatives. The Sample size is determined by the assumed maximum error (we assume 0.001) and the confidence level, i.e., 0.9995.

For this data, it has to draw 2,706,848 test pairs [42], which round up to tens of thousands (2,750,000 samples). For each one pair is checked which one is better using the fuzzy model and equation (22). If answer from the fuzzy model is correct, then a test pair is marked as success and otherwise as defeat of classification by using $COMET$ method (success 1, defeat 0). The summed number of success divided on the number of samples is the effectiveness of the identification. In this way, the effectiveness of the fuzzy modeling is estimated at 97.78%. This result is very well as multi-criteria decision-making method.

6. CONCLUSIONS

In this paper, the traditional fuzzy modeling have been extended to the support of the multi-criteria decision-making problems. The concepts of the new fuzzy method have been presented. The COMET method is completely free from the rank reversal phenomenon, because all preferences are based on characteristic objects. Therefore, if decision-makers add or remove any number of alternatives, then the assessing of alternatives are invariable. The theoretical nonlinear problem is used to verify the developed approach and to demonstrate its effectiveness. The result show that the proposed method provides us with a useful way to deal with multi-criteria decision-making problems.

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