Odd - Even Graceful Labeling for Different Paths using Padavon Sequence

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ABSTRACT
A function f is called an odd-even graceful labeling of a graph G if f: V(G) → {0,1,2,…,q} is injective and the induced function f^* : E(G) → { { 0,2,4,…,2q+2i/i= 1 to n} such that when each edge uv is assigned the label |f(u) – f(v)| the resulting edge labels are {2,4,6,…,2q}. A graph which admits an odd-even graceful labeling is called an odd-even graceful graph. In this paper, the odd-even gracefulness of paths $p_1, p_2, p_3, \ldots, p_{11}$ is obtained.

Keywords
Padavon sequence, vertex labeling, edge labeling, graceful labeling, odd-even graceful labeling

1. INTRODUCTION
Throughout this paper, we consider simple, finite, connected and undirected graph G = (V(G), E(G)) with p vertices and q edges. G is also caleed a (p, q) graph. A path of length n is denoted by P_{n+1}. For standard terminology and notation we follow Gross and Yellen[4].

Graph labelings is an active area of research in graph theory which has rigorous applications in coding theory, communication networks, optimal circuits layouts and graph decomposition problems. According to Beinke and Hegde[1] graph labeling serves as a frontier between number theory and the structure of graphs. For a dynamic survey of various graph labeling problems along with an extensive bibiliograph, we refer to Gallian[3].

A study of graceful graphs and graceful labeling methods was introduced by Rosa[4]. Rosa defined a β-valuation of a graph G with q edges an injection from the vertices of G to the set {0,1, …,q} such that when each edge uv is assigned the label |f(u)-f(v)|, the resulting edges are distinct. β – valuation is a function that produces graceful labeling. However the term graceful labeling was not used until Golomb studied such labeling several years later[3].

2. DEFINITIONS
Defn 2.1: Graceful graph
A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0,1,…,m} such that when each edge uv is assigned the label |f(u)-f(v)| and the resulting labels are distinct. Then the graph G is graceful.

Defn 2.2: Path
A path in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. The first vertex is called the start vertex and the last vertex is called the end or terminal vertices of the path and the other vertices in the path are internal vertices.

Defn 2.3: Odd-Even graceful graph
A function f is called an odd-even graceful labeling of a graph G if f: V(G) → {0,1,2,…,q} is injective and the induced function f^* : E(G) → { { 0,2,4,…,2q+2i/i= 1 to n} such that when each edge uv is assigned the label |f(u) – f(v)| the resulting edge labels are {2,4,6,…,2q}. A graph which admits an odd-even graceful labeling is called an odd-even graceful graph.

Defn 2.4: Padavon sequence
The padavon sequence is the sequence of integers P(n) defined by the initial values P(0) = P(1) = P(2) = 1, and the recurrence relation

$$P(n) = P(n-2) + P(n-3).$$

The first few values of P(n) are 1, 1, 1, 2, 2, 3, 4, 5, 7, 9,12, 16, 21, 28, 37, 49, 65, 86, 114, 151, 200, 265, 351, 465, 616,…

3. VARIOUS ODD-EVEN GRACEFUL LABELING:
Theorem 3.1: P_n is odd-even graceful for every integer n≥2.

Proof:

Let $V(P_2) = \{u_1, u_2\}$ where $V(P_2)$ is the vertex set of the path P_2. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$$f(u_1) = q-1;\ f(u_2) = q+1$$

The edge labeling function f* is defined as follows.

$$f^*(u_1u_2) = q$$

Figure 1 shows the method of odd-even graceful labeling of the path P_2.

This completes the proof.

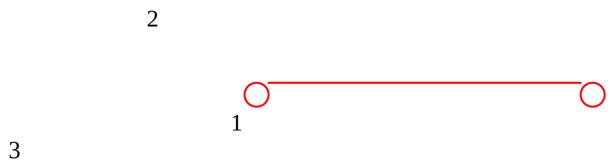

Fig 1

Theorem 3.2: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_3) = \{u_1, u_2, u_3\}$ where $V(P_3)$ is the vertex set of the path P_3. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$$f(u_1) = q\text{-}1;\ f(u_2) = q\ ;\ \ f(u_3) = q + 2$$

The edge labeling function f* is defined as follows.

$$f^*(u1u2) = q + 1 = f^*(u2u3)$$

Figure 2 shows the method of odd-even graceful labeling of the path P_3.

This completes the proof.

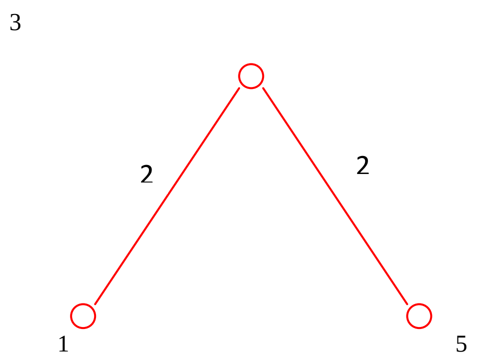

Fig 2

Theorem 3.3: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_4) = \{u_1, u_2, u_3, u_4\}$ where $V(P_4)$ is the vertex set of the path P_4. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$$f(u_1) = q\text{-}3;\ f(u_2) = q + 1\ ;\ \ f(u_3) = q + 3\ ;\ f(u_4) = q + 1$$

The edge labeling function f* is defined as follows.

$$f^*(u_1u_2) = f^*(u_3u_4) = q + 2\ ;\ \ f^*(u_2u_3) = q$$

Figure 3 shows the method of odd-even graceful labeling of the path P_4.

This completes the proof.

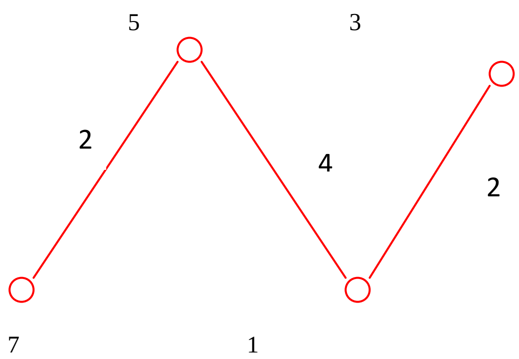

Fig 3

Theorem 3.4: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_5) = \{u_1, u_2, u_3, u_4, u_5\}$ where $V(P_5)$ is the vertex set of the path P_5. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$f(u_1) = q\text{-}1;\ f(u_2) = 3q\text{-}2\ ;\ \ f(u_3) = 2q\ \text{-}1;\ f(u_4) = q + 2\ ;$ $f(u_5) = q\ ;$

The edge labeling function f* is defined as follows.

$f^*(u_1u_2) = 2q + 2;\ f^*(u_2u_3) = q - 1;\ \ f^*(u_3u_4) = q - 3 =$ $f^*(u_4u_5)$

Figure 4 shows the method of odd-even graceful labeling of the graph P_5.

This completes the proof.

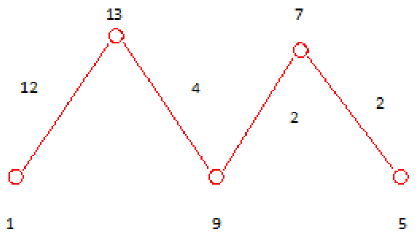

Fig 4

Theorem3.5: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_6) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ where $V(P_6)$ is the vertex set of the path P_6. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$$f(u_1) = q\text{-}5;\ f(u_2) = 3q - 1\ ;\ \ f(u_3) = q - 1\ ;\ f(u_4) = q - 3\ ;$$

$$f(u_5) = q + 1\ ;\qquad f(u_6) = q + 3$$

The edge labeling function f* is defined as follows.

$$f^*(u_1u_2) = 3q - 2;\ f^*(u_2u_3) = 2q;\ \ f^*(u_3u_4) = f^*(u_5u_6) = q\ \ -4;$$

$$f^*(u_4u_5) = \ q - 2$$

Figure 5 shows the method of odd-even graceful labeling of the path P_6.

This completes the proof.

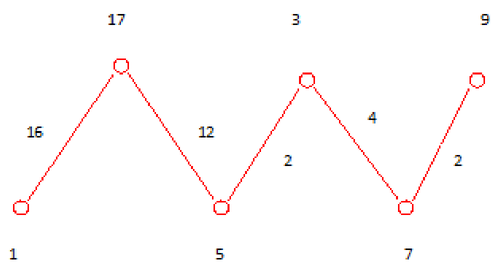

Fig 5

Theorem 3.6: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_7) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ where $V(P_7)$ is the vertex set of the path P_7. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$$f(u_1) = q - 6; f(u_2) = 4q + 1 ; f(u_3) = 2q - 1 ; f(u_4) = 2q + 3;$$

$$f(u_5) = q - 2 ; \quad f(u_6) = q; \ f(u_7) = q + 2$$

The edge labeling function f is defined as follows.*

$$f^*(u_1u_2) = 4q; f^*(u_2u_3) = 2q + 2: f^*(u_3u_4) = q - 3;$$

$$f^*(u_4u_5) = 2q - 2; \ f^*(u_5u_6) = q - 5 = f^*(u_6u_7)$$

Figure 6 shows the method of odd-even graceful labeling of the path P_7.

This completes the proof.

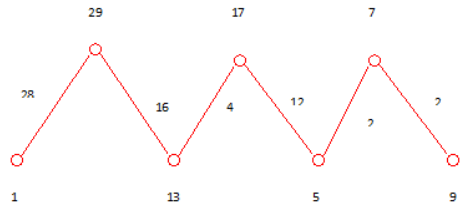

Fig 6

Theorem 3.7: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_8) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ where $V(P_8)$ is the vertex set of the path P_8. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$$f(u_1) = q-7; f(u_2) = 11q - 1 ; f(u_3) = 8q + 3 ; f(u_4) = 5q + 3 ;$$

$$f(u_5) = 4q - 1 ; \ f(u_6) = 3q + 3 ; \ f(u_7) = 3q + 1 ; f(u_8) = 3q - 1$$

The edge labeling function f* is defined as follows.

$$f^*(u_1u_2) = 11q - 1 ; f^*(u_2u_3) = 3q + 4 ; \ f^*(u_3u_4) = 2q ;$$

$$f^*(u_4u_5) = q + 4; \ f^*(u_5u_6) = q - 4; f^*(u_6u_7) = q - 6 = f^*(u_7u_8)$$

Figure 7 shows the method of odd-even graceful labeling of the path P_8.

This completes the proof.

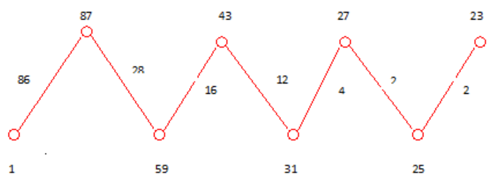

Fig 7

Theorem3.8: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_9) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9\}$ where $V(P_9)$ is the vertex set of the path P_9. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$$f(u_1) = q - 8; f(u_2) = 13q - 2 ; \ f(u_3) = 3q + 2 ; f(u_4) = q + 4 ;$$

$$f(u_5) = 5q - 4; \ f(u_6) = 4q + 1 ; \ f(u_7) = 3q - 4 ; f(u_8) = 2q + 5 ; \ f(u_9) = 2q + 3$$

The edge labeling function f* is defined as follows.

$$f^*(u_1u_2) = 13q - 3; f^*(u_2u_3) = 10q - 4: \ f^*(u_3u_4) = 2q - 2 ;$$

$$f^*(u_4u_5) = 3q + 1 ; f^*(u_5u_6) = q - 5 ; f^*(u_6u_7) = q + 3; \ f^*(u_7u_8) = q - 7 =$$

$$f^*(u_8u_9)$$

Figure 8 shows the method of odd-even graceful labeling of the path P_9.

This completes the proof.

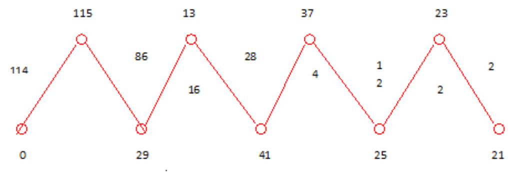

Fig 8

Theorem 3.9: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_{10}) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ where $V(P_{10})$ is the vertex set of the path P_{10}. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$$f(u_1) = q-9; f(u_2) = 20q + 1 ; \ f(u_3) = 9q - 3 ; f(u_4) = 7q + 1 ;$$

$$f(u_5) = 16q - 3; \ f(u_6) = 13q - 1 ; \ f(u_7) = 12q - 3; f(u_8) = 12q - 7 ;$$

$$f(u_9) = 11q + 1 ; \ f(u_{10}) = 11q - 1;$$

The edge labeling function f* is defined as follows.

$$f^*(u_1u_2) = 20q; f^*(u_2u_3) = 12q - 6 ; f^*(u_3u_4) = 2q - 4;$$

$$f^*(u_4u_5) = 9q - 4 ; f^*(u_5u_6) = 3q - 2 ; f^*(u_6u_7) = q + 2 ;$$

$$f^*(u_7u_8) = q - 6; f^*(u_8u_9) = q - 8 = f^*(u_9u_{10})$$

Figure 9 shows the method of odd-even graceful labeling of the path P_{10}.

This completes the proof.

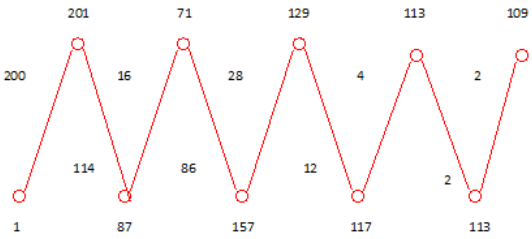

Fig 9

Theorem 3.10: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_{11}) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}\}$ where $V(P_{11})$ is the vertex set of the path P_{11}. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$f(u_1) = q-10$; $f(u_2) = 56q+1$; $f(u_3) = 38q - 1$; $f(u_4) = 28q - 5$;

$f(u_5) = 20q - 3$; $f(u_6) = 17q + 2$; $f(u_7) = 11q + 3$; $f(u_8) = 15q-4$;

$f(u_9) = 14q + 3$; $f(u_{10}) = 14q + 1$; $f(u_{11}) = 14q - 1$

The edge labeling function f* is defined as follows.

$f^*(u_1u_2) = 56q$; $f^*(u_2u_3) = 18q + 2$; $f^*(u_3u_4) = 10q + 4$;

$f^*(u_4u_5) = 8q + 2$; $f^*(u_5u_6) = 2q + 6$; $f^*(u_6u_7) = q + 5$;

$f^*(u_7u_8) = q + 1$; $f^*(u_8u_9) = q – 7$; $f^*(u_9u_{10}) = q – 9 = f^*(u_{10}u_{11})$

Figure 10 shows the method of odd - even graceful labeling of the path P_{11}.

This completes the proof.

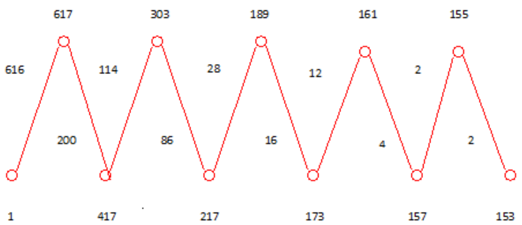

Fig 10

Theorem 3.11: P_n is odd-even graceful for every integer $n \geq 2$.

Proof:

Let $V(P_{12}) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$ where $V(P_{12})$ is the vertex set of the path P_{12}. For every vertex u_i, the odd-even graceful labeling functions $f(u_i)$ is defined as follows.

$f(u_1) = q-11$; $f(u_2) = 68q+1$; $f(u_3) = 17q - 3$; $f(u_4) = 34q - 7$;

$f(u_5) = 24q - 1$; $f(u_6) = 21q + 7$; $f(u_7) = 15q - 7$; $f(u_8) = 13q + 1$;

$f(u_9) = 12q + 1$; $f(u_{10}) = 12q - 4$; $f(u_{11}) = 11q + 7$; $f(u_{12}) = 11q + 5$;

The edge labeling function f* is defined as follows.

$f^*(u_1u_2) = 68q$; $f^*(u_2u_3) = 51q + 4$; $f^*(u_3u_4) = 17q - 4$;

$f^*(u_4u_5) = 9q + 6$; $f^*(u_5u_6) = 2q + 4$; $f^*(u_6u_7) = 7q + 2$;

$f^*(u_7u_8) = q + 4$; $f^*(u_8u_9) = q$; $f^*(u_9u_{10}) = q – 8$; $f^*(u_{10}u_{11}) = q – 10 = f^*(u_{11}u_{12})$

Figure 10 shows the method of odd - even graceful labeling of the path P_{11}.

This completes the proof.

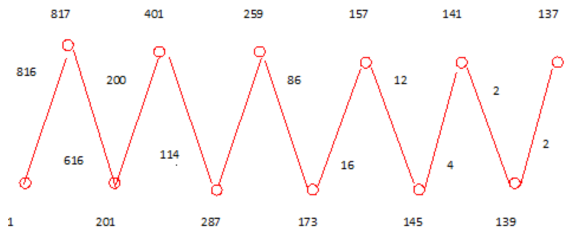

Fig 11

4. CONCLUSION

In all the above paths, we can see that the vertices are labeled with odd numbers and the edges are labeled with even numbers and hence it is an odd-even padavon graceful graph.

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