International Journal of Computer Applications (0975 8887)
Volume 88 - No. 16, February 2014

Robust Fractional-order Controller using Bode's Ideal Transfer Function for Power Plant Gas Turbine

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ABSTRACT

The field of application of control methodologies to gas turbine holds tremendous research potential. This paper, presents the fractional-order (FO) robust controller design for the fuel-speed loop of a gas turbine. The aim of the controller is to maintain the turbine speed, against the plant gain variation and disturbance. To the best of our knowledge this is probably the first effort to propose the design of a fractional-order controller for the speed control of a power plant gas turbine. Nowadays the application of fractional-order (FO) modeling and control is the most appreciated area for research. The Fractional Calculus field has originated from the fundamental area of fractional calculus which is the mathematical branch dealing with differentiation and integration with arbitrary order of the operation. On the other hand, FO controllers have proved their efficacy over the conventional integer-order (IO) controllers by providing more flexibility in the design and also by guaranteeing a more robust closed-loop configuration. The proposed FO controller is designed with the concept of Bode's ideal loop transfer function. Simulation studies clearly shows that the proposed FO controller makes the closed loop system more robust against the plant uncertainties and disturbances as compared to the integer order PID controller.

General Terms:

Gas Turbine Control

Keywords:

Gas turbine, Fractional-order control, Bode's ideal loop transfer function, Robust control.

1. INTRODUCTION

In recent years, gas turbines are popularly used for power generation and the worldwide installations are also day by day increasing . In aeronautical industry as well as mechanical drivers for large pumps and compressors, gas turbine role is very significant. So it is essential to know the dynamic behaviors and to understand the nature of gas turbine. Accurate model which clearly shows the complete dynamics is required, and which also help us to design efficient controller for regulating gas turbine variables [9]. Studies on the gas turbine control have been a subject of interest, since gas-turbine engines have been widely adopted as peak load candidates for electricity generation [2]. Especially, the compactness, multiple fuel applications, the fully automatic start-up function and the fast run-up characteristic of gas turbine systems have made them particularly suitable for peak-load and standby power supply purposes [1]. Start-up and shutdown procedures are the most challenging problems for control applications to develop new control algorithms. At the same time, many variables must be monitored and controlled to ensure safety of operation. Gas turbines usually have the five controllers namely start controller, speed controller, load controller, turbine's maximum temperature limit controller and turbine's mechanical load limit controller [2] [3].
In dynamic analysis of combined cycle plants, twin shaft gas turbine model, combustion turbine model, biomass-based gas turbine plant and even in micro turbine power generation the transfer function model has been used [9]. Basically this model has speed, exhaust temperature are controlled variable and the manipulated variable are fuel flow and inlet guide vane signal respectively as shown in Fig.1.
To design controller for gas turbines, various methods have been investigated by researchers. Most of these works have been applied for jet turbines, such as robust controller by [11]. However, the area of control of gas turbine is still in its infancy. Typically, employment of conventional PID controllers for the control of gas turbine is widely followed. Some implementation of modern model based control techniques like Adaptive control, MPC [8], Fuzzy control [16], PID [18], H-infinity control [1] etc are also found in the literature. However, looking at the pace of advancement in the field of control systems theory, these efforts are not sufficient to exploit the features of advanced control theory to achieve desirable operation of a gas turbine. This is probably the first effort to propose the design of a fractional-order controller for the speed control of a power plant gas turbine. Recently, sudden increase in research activities are found in the area of fractional-order (FO) modeling and control. These fields has originated from the fundamental area of fractional calculus, a branch of mathematics dealing with derivatives and integrals with arbitrary non-integer order (real or complex) [13, 14]. FO models have been found to provide a more realistic and compact representation to real world and man-made systems. On the other hand, FO controllers have proved their efficacy over the conventional integer-order (IO) controllers by providing more flexibility in the design and also by guaranteeing a more robust closed-loop performance. The proposed robust FO controller is designed using the concept of Bode's ideal loop transfer func-

tion [12]. This paper is organized as follows. In Section 2, after describing the basic of gas turbine, the transfer function model of the gas turbine between the speed and fuel flow is driven. The history and the application of fractional-calculus is explained in Section 3. The brief introduction to fractional-order controller and the concept of Bode's ideal transfer function is presented in in Section 4. Section 5, gives the design procedure of robust controller and its implementation. The simulation results are discussed in Section.6. Finally, Section. 7 highlights the concluding remarks.

2. GAS TURBINE MODEL

Gas turbines are generally comprised of compressor, combustion chamber, turbine, fuel system, inlet guide vane, positioner and the control unit as shown in Fig. 1. Where the gas pressure (usually air) is initially increased in compressor (in multi-stage compressors up to 12 times) and the pressured gas is heated in combustion chamber. Then the gas is injected with high pressure and temperature to the turbine and the thermal energy of the gas is converted in to mechanical energy [5]. The gas turbines which consume natural gas, diesel, biomass gas etc [1]. have been selected as an optimal choice in power plants. Although a large amount of the input energy to these turbines has been wasted through exhaust, it can be compensated by passing the gas via a Heat Recovery Steam Generator (HRSG) to run a steam turbine or for other purposes [5]. In power plant the accurate modeling and robust control is the most important requirement so as to achieve the stabilize frequency and electrical voltages when their is variations of electrical load in transmission and distribution systems. Various mathematical and thermodynamic models have been proposed for gas turbines. They include, simple and applied models such as the Rowen model for a gas power plant [4] and for a combined cycle power plant [6], the aerothermodynamics model, the computational model, and the thermodynamic model [9]. The history of gas turbine modeling shows the simplified mathematical model consists of a set of algebraic equations and related temperature, speed and acceleration controllers is provided in [4, 6]. Then it is modified by adding the influence of variable inlet guide vanes (VIGV) and this frequency-domain model is validated. The transfer function block diagram given in [1] of heavy duty gas turbine plant is designed, calculated and verified the system gains, coefficients and time constant by test and actual field experience accumulated from numerous installations in many different applications [4]. The Rowen model, whose parameters are identified using real data of power plant gas turbine by [1] is considered to design robust fractional-order controller. It includes combustion chamber and a multistage axial flow compressor connected to a multistage expansion turbine which drives an electric generator. A general arrangement of heat recovery power plant in a combined cycle is shown in Fig. 1. All the variables T_{amb} (Ambient temperature), IGV (inlet guide vane position), F (fuel flow), T_x (exhaust gas temperature), N (turbine speed) and P (produced power) are measurable. The value of these variables has been sampled in 1 s intervals, during the time interval in which the turbine has commenced loading up to reaching the nominal load. The simplified representation of a gas turbine system with individual transfer functions block between the important control parameter is shown in Fig.2. Where $G_1(s)$, $G_2(s)$ and $G_4(s)$ to $G_7(s)$ have been derived using ARX procedure explained by [8]. The linear model is used to designed robust FO controller. The linear transfer functions which relate the control parameters of a gas turbine are written below and they are given in the paper [1].

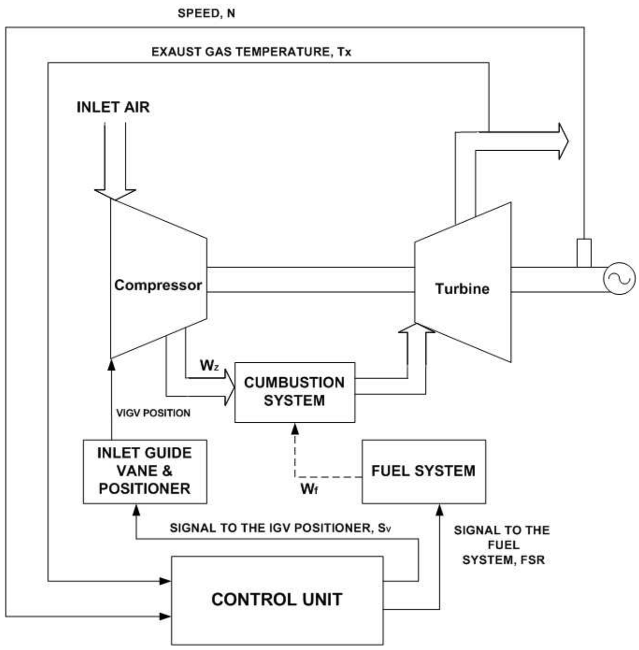

Fig. 1. A general arrangement of combined cycle power plant [1].

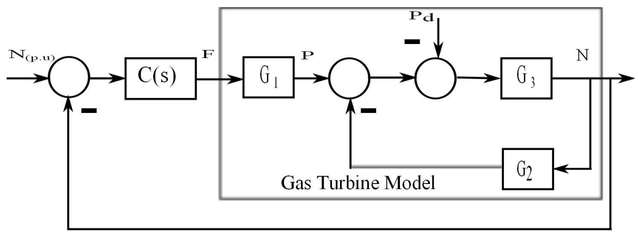

Fig. 2. Simplified representation of a gas turbine model.

$$G_1 = \frac{P}{F(s)} = \frac{0.3827s^2 + 0.8935s + 0.2562}{s^2 + 1.3331s + 0.2015}, \quad (1)$$

$$G_2 = \frac{P}{N(s)} = \frac{-0.212s^2 - 0.4496s - 0.05068}{s^2 + 1.3331s + 0.2015}, \quad (2)$$

$$G_4 = \frac{T_x(s)}{N(s)} = \frac{21.98s^2 + 207.6s + 327.2}{s^2 + 3.266s + 0.9384} \quad (3)$$

$$G_5 = \frac{T_x(s)}{T_{amb}(s)} = \frac{0.7975s^2 + 0.8849s - 1.42}{s^2 + 3.266s + 0.9384}, \quad (4)$$

$$G_6 = \frac{T_x(s)}{F(s)} = \frac{79.19s^2 + 344.5s + 372.3}{s^2 + 3.266s + 0.9384}, \quad (5)$$

$$G_7 = \frac{T_x(s)}{IGV(s)} = \frac{-119s^2 + 312.3s - 148.6}{s^2 + 3.266s + 0.9384}. \quad (6)$$

The transfer function $G_3(s)$ has been considered as [4]

$$G_3(s) = \frac{1}{\tau_r s}, \quad (7)$$

where τ_r is the rotor time constant ($\tau_r = 18.5$ sec for GE9001E model). The parameters F and IGV are control inputs which are determined by the controller and the parameters P_d (demand power) and T_{amb} which are considered as disturbances as shown in Fig.2. From the considered 4 inputs and 3 outputs parameters the complete model of the system is written

$$\begin{bmatrix} N \\ P \\ T_x \end{bmatrix} = T. \begin{bmatrix} P_d \\ T_{amb} \\ F \\ IGV \end{bmatrix} \tag{8}$$

where T is the matrix transfer function of the system with 3 *4 dimensions and its elements are obtained as a function of G_1 to G_7.

$$T = \begin{bmatrix} \frac{-G_3}{1-G_2G_3} & 0 & \frac{G_1G_3}{1-G_2G_3} & 0 \\ \frac{-G_2G_3}{1-G_2G_3} & 0 & \frac{G_1}{1-G_2G_3} & 0 \\ \frac{-G_3G_4}{1-G_2G_3} & G_5 & \frac{G_1G_3G_4}{1-G_2G_3} + G_6 & G_7 \end{bmatrix} \tag{9}$$

$$N = T_{13} = \left[\frac{G_1G_3}{1 - G_2G_3} \right] .F \tag{10}$$

The linear transfer function model of the gas turbine which relate the output speed(N) and the input fuel flow(F) (speed loop) is taken for the design of robust FO controller

$$G(s) \equiv \frac{7.08s^5 + 25.95s^4 + 28.17s^3 + 9.639s^2 + 0.955s}{342.3s^6 + 915s^5 + 757.8s^4 + 196.4s^4 + 16.82s^2 + 0.18889s} \tag{11}$$

3. FRACTIONAL CALCULUS

Fractional calculus can be defined as a more generalization of derivatives and integrals to non-integer orders. It is not a new mathematical tool and the idea of its application to control theory was described in [13]. FC allows a more compact representation and problem solution for many systems. For realization of fractional-order controllers (FOC), one needs to approximate the fractional-order models. These systems have memory or hereditary properties, while the IO systems have limited memory. For practical implementation of the FO models, we required to find the discrete approximation of FO models. So it is very important to approximately describe the FO systems using a finite difference equations. For this purpose, rational approximations are often used mainly in continuous-time domain. In practice, direct discrete approximation is more preferred. The fundamental fractional operator is ${}_aD_t^\alpha$, where a and t are the limits and α, $(\alpha \epsilon R)$ is the order of the operation. There are several definitions of fractional derivatives (FD) see [13]. Three commonly used by engineers and physicists are the Grunwald−Letnikov (GL) definition (12), Riemann−Liouville (RL) definition (13) and Caputo FD definition which are discussed in [13] [14].

(1) **The GL FD** is defined as

$$ {}_aD_t^\alpha f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^j \begin{pmatrix} a \\ b \end{pmatrix} f(t - jh), \tag{12}$$

where a and t are limits $(t > a)$, $\binom{a}{b}$ are the binomial coefficients.

(2) **The RL FD** is defined as

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \tag{13}$$

for $n - 1 < \alpha < n$, $n \in \mathbb{N}$ and $\Gamma(\cdot)$ is the gamma function, and $f(t)$ is locally integrable.

(3) **The Caputo FD** is defined as

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \tag{14}$$

where, $f^{(n)}(\tau)$ is the n^{th} derivative of $f(t)$ and $n - 1 < \alpha < n, n \in \mathbb{N}$.

Note that, the Caputo FD definition (14) is more restrictive than the RL FD, as in this case $f(t)$ has to be *n-times* differentiable. Fortunately, the technique of Laplace Transform can be extended to fractional derivatives. So we, can use it for the analysis of linear FO systems. The Laplace transform of the Caputo FD in (14), under zero initial conditions, for order $r \in \mathbb{R}$ is given by

$$\mathcal{L}[{}_0D_t^\alpha f(t)] = s^\alpha F(s), \tag{15}$$

where, $F(s) = \mathcal{L}[f(t)]$ and $\mathcal{L}[\cdot]$ denotes the Laplace transform.

4. FRACTIONAL ORDER CONTROL

The idea of fractional order operators is as old as the idea of integer order ones. But from the last decades the use of fractional order operators and operations has become more and more popular among many research areas [13, 17, 18]. The theoretical and practical interest of these operators is nowadays well established, and its applicability to science and engineering can be considered as an emerging new topic. They can be thought of as somehow ideal, in fact the useful tools for both the description of a more complex reality and the enlargement of the practical applicability of the common integer order operators. The fractional integro-differential operators (fractional calculus) are specially interesting in automatic control and robotics [14]. Maybe the first mention of the interest of considering a fractional integro-differential operator in a feedback loop, though without using the term "fractional", was made by, and next in a more comprehensive way in [12]. The first application of fractional calculus in control was started with the frequency-based methods. The frequency response and the transient response of the non-integer integral (in fact Bodes ideal transfer function) and its application to control systems were introduced by [19], and more recently in [18]. A robustness constraint is considered for forcing the phase of the open-loop system to be flat at the gain crossover frequency.

4.1 Bode's Ideal Transfer Function

Design of a feedback amplifier was come up with a feedback loop. Here a crucial issue is the performance of the closed loop should be invariant to changes in the amplifier gain and this problem was addressed by [12, 19], which is the ideal cutoff characteristic and also known as ideal loop transfer function. In his study [12] on design of feedback amplifiers he has suggested an ideal shape of the open-loop transfer function of the form:

$$G(s) = \left(\frac{\omega_c}{s} \right)^\alpha, \alpha \in R \tag{16}$$

where ω_c is the gain crossover frequency, that is $|G(j\omega c)| = 1$. The parameter α is the slope of ideal cut-off characteristics, on log-log scale and may assume integer as well non-integer values. In fact the transfer function $G(s)$ is a fractional-order differentiator for $\alpha < 0$ and a fractional-order integrator for $\alpha > 0$. The Bode diagrams of $G(s)(1 < \alpha < 2)$ are very simple. The amplitude curve is a straight line of constant slope of $-20\alpha dB/dec$, and the constant phase margin is

$$\varphi_m = \pi - \frac{\alpha\pi}{2}.$$

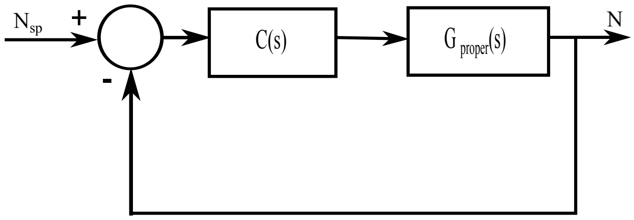

Fig. 3. Closed -loop configuration.

The Nyquist plot is a straight line through the origin giving a phase margin invariant to gain changes. General characteristics are:-

$$G_{ref}(s) = \frac{k_r}{s^{\alpha}}, 1 < \alpha < R.$$

$$|G(j\omega c)| = 1$$

Magnitude of $-20\alpha dB/dec$,

$$\omega_{gc} = (k_r)^{\frac{1}{\alpha}},$$

the crossover frequency is a function of K_r. Phase angle curve is a horizontal line at $-\frac{\alpha\pi}{2}$, the Nyquist curve is straight line argument at $-\frac{\alpha\pi}{2}$. Let us consider the unit feedback system with Bode's ideal transfer function $C(s)$ inserted in the forward path as shown in Fig.3. This choice of $C(s)$ gives a closed-loop system with the desirable property of being insensitive to gain changes. If the gain changes the crossover frequency ω_{gc} will vary but the phase margin of the system remains PM = $-\frac{\alpha\pi}{2}$ rad, independently of the value of the gain. This frequency characteristic is very interesting in terms of robustness of the system to parameter uncertainties. Clearly, this ideal system is a fractional integrator. In fact, the fractional integrator can be used as an alternative reference system for control, considering its own properties [14]. The proposed robust fractional-order controller for the speed-fuel loop of gas turbine is designed using the ideal loop transfer function features.

5. ROBUST CONTROL DESIGN

Gas turbine and its transfer function model is explained in section.2. Since the turbine speed changes cause the frequency to deviate. Here aim is to design a robust controller which will take care of plants uncertainty. The mathematical model, between the fuel-speed loop is coming strictly proper in nature, and cannot be used directly for the design of FO controller. So we have identified its equivalent reduced-order 'proper' (numerator and denominator polynomials having same degree) integer order transfer function model using the step response and frequency response data as shown in Fig.4 and Fig.5 respectively.
The frequency range considered was $\omega = [10^{-3}, 10^{1}]$rad/sec.

$$G_{prop}(s) \equiv \frac{0.0002889s^2 + 0.02939s + 0.01289}{s^2 + 0.2071s + 0.02549} \quad (17)$$

Also, it is observed from the phase plot that the plant under consideration has a phase margin (PM) of 92.1^0 at a gain crossover frequency (ωgc) of 0.0625 rad/sec. The FO controller is designed to obtain the loop transfer function $C(s)\ G_{proper}(s)$ as Bode's ideal integrator, that is,

$$C(s)G_{prop}(s) = \frac{k_c}{s^{\alpha}}$$

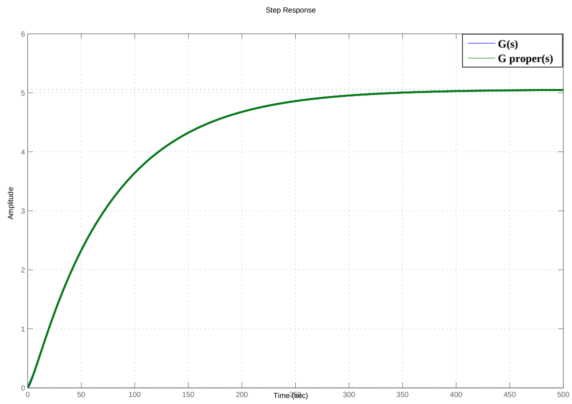

Fig. 4. Step response of the open loop system and its proper approximation.

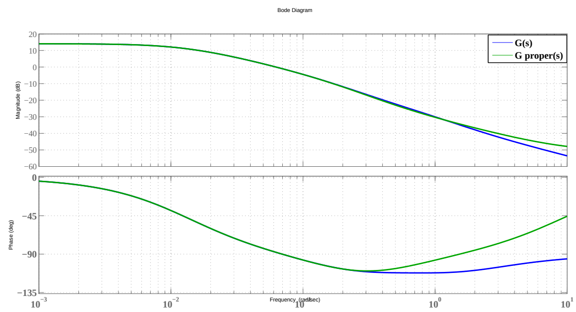

Fig. 5. Bode plots of the open loop system and its proper approximation.

where $\alpha\ \varepsilon\ R^{+}$ (positive real).The resulting closed loop system has

$$GM = \infty, PM = \pi(1 - \frac{\alpha}{2}), \omega gc = (k_c)^{\frac{1}{\alpha}}.$$

In this paper, we wish to have a PM of 120^o at 0.0625 rad/sec, which gives

$$k_c = 0.1575, \alpha = 0.667$$

Thus, substituting the value of $k_c$and α the fractional-order controller obtained is

$$C(s) = \frac{0.1575(0.0002889s^2 + 0.02939s + 0.01289)}{s^{0.667}(s^2 + 0.2071s + 0.02549)}. \quad (18)$$

The closed loop system has infinite GM and PM, it is independent of process gain k_c. Also, the gain crossover frequency depends only on k_c. As a consequence, the feedback system is robust and therefore insensitive to changes in k_c. So here in addition to the conventional rational integer-order (IO) transfer function of gas turbine, there is an additional fractional-order integrator with integration order equal to 0.667.
Fig. 6 shows the Bode plot of the forward path transfer function $C(s)G(s)$, that is nothing but FO controller applied to the original gas turbine model. It confirms that the given FO controller has achieved the desired closed-loop phase margin of 120^o.

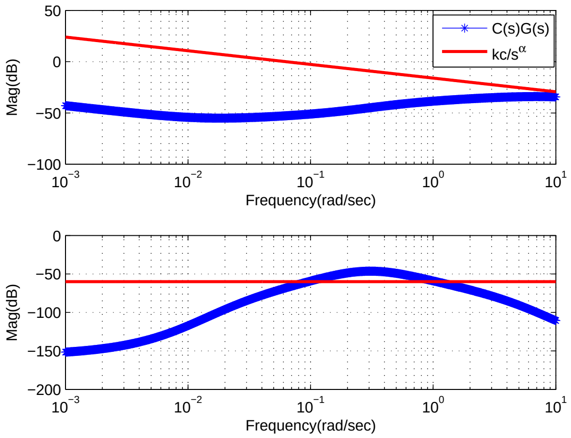

Fig. 6. Bode plots with designed FO controller

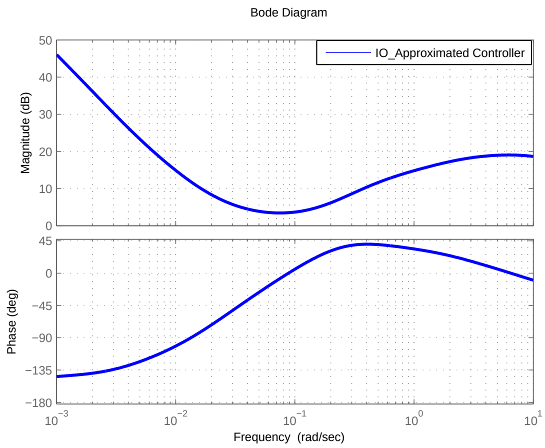

Fig. 7. Bode plots with IO Approximated controller

5.1 Controller realization

FO controller obtained using ideal loop transfer function concept is not realizable / implementable directly. This is because the FO operator (integrator) present in its denominator has an infinite memory element. This trivial issue of implementation of FO controllers is solved by using a finite memory approximation of the FO integrator [14]. There are many techniques available in the literature which gives the continuous and discrete approximations of FO operators. The most popular among these is the Oustaloup's Recursive Approximation (ORA) [14] which approximates an FO operator as a chain of first-order filters within a specified frequency band. Here we use the modified ORA which is claimed to give the best approximation as compared to its counterparts. The resulting approximation of the FO controller is an integer-order transfer function with numerator and denominator having polynomials of degree fifteen. For the validation, the frequency response of integer-order approximated controller is observed from Fig.7 and the Bode plot of IO controller and the plant is shown in Fig.8.

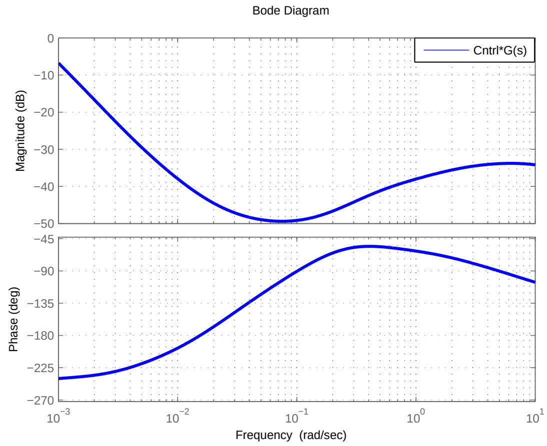

Fig. 8. Bode plots with IO Approximated controller and plant

Table 1. Comparison of open loop and closed loop response specification

Specifications	Open loop	FO Controller	PID Controller
Rise time(sec)	168	9.4	38.9
Settling time(sec)	301	130	225

Table 2. Comparison of open loop and closed loop response specification for robust case

Specification	Open loop	FO Controller	PID Controller
Settling time(sec)	301	76.5	200
Rise time(sec)	168	0.6	39.8

6. SIMULATION RESULTS

The robust fractional-order controller designed for the transfer function model of fuel-speed loop of the gas turbine in power plant is simulated using MATLAB/SIMULINK for unit step and pulse type reference signal. The performance of the FO controller is verified with the original strictly proper transfer function model $G(s)$ of the power plat gas turbine. Further, to test its efficacy, its performance is compared with that of the PID controller which is tuned with MATLAB tuning block. A set point change in speed (p.u.) is successfully tracked by the closed-loop system. Fig.9 shows the response of proposed FO controller, which results into little overshoot but settles fast compared to PID see Table no.1. The settling time reduction achieved is more than 50%, giving faster closed-loop response. The most important requirement is, the controller should take care of plant uncertainty. The proposed FO controller is very robust in nature and makes the closed-loop system insensitive to plant gain variation. Fig.10 the response of FO and PID controller with 10% change in the gas turbine model parameters. The effect of disturbances, that is demand power is considered for the gas turbine power plant. Simulation result Fig.11 shows the excellent performance under the consideration of disturbances. Table no.1, 2,3 gives the comparison of the time domain specification. Finally, the fractional-order controller is also tested for pulse type of reference signal and the simulation results are satisfactory.

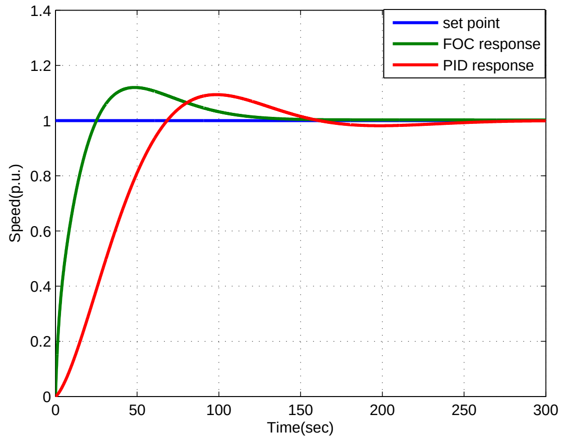

Fig. 9. Closed loop response of FO and PID controller for step input.

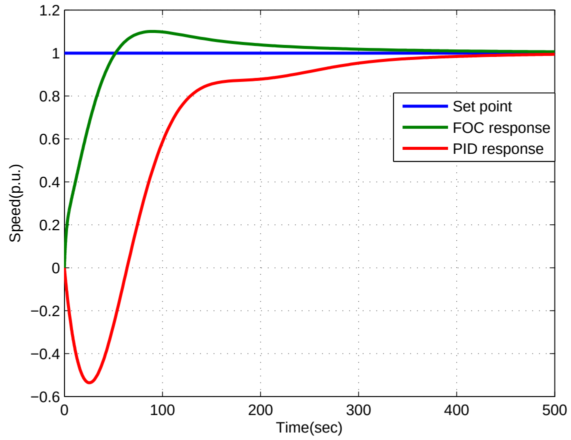

Fig. 11. Closed loop response of FO and PID controller for step input with disturbances.

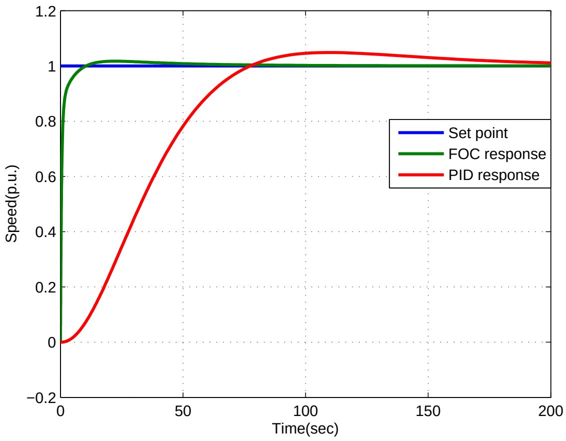

Fig. 10. Robust closed loop response of FO and PID controller for step input with gain variation.

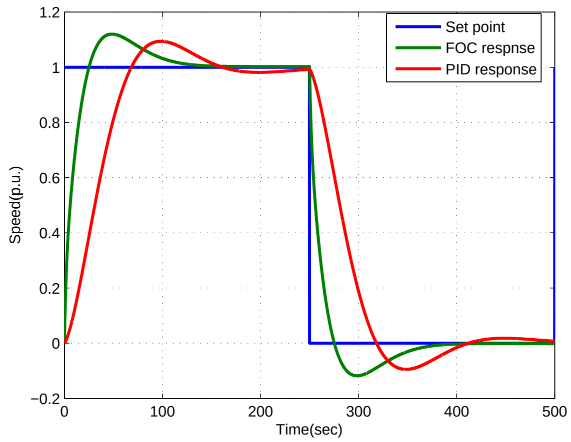

Fig. 12. Closed loop response of FO and PID controller for pulse input.

Table 3. Comparison of open loop and closed loop response specification with disturbances

Specification	Open loop	FO Controller	PID Controller
Settling time(sec)	301	400	440
Rise time(sec)	168	23	105

7. CONCLUSION

This paper, presents the first effort to propose fractional-order (FO) robust controller design for the fuel-speed loop of a gas turbine. The proposed FO controller is designed with the concept of Bode's ideal loop transfer function. The simulation results proves the superiority of the proposed controller over the well known PID controller in terms of closed-loop response, robustness to parametric changes and also in presence of power demand type disturbance. As the controller is simulated for the original fuel-speed loop transfer function model, which confirms the potential of FO controller. Simulation studies clearly shows that the proposed FO controller makes the closed loop system more robust against the plant uncertainties and disturbances as compared to the integer order PID controller.

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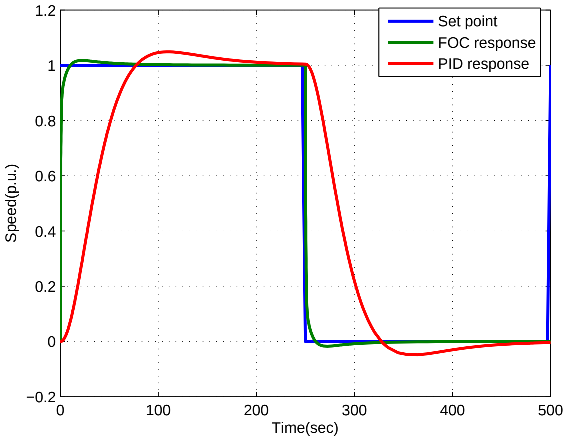

Fig. 13. Robust closed loop response of FO and PID controller for pulse input with gain variation.

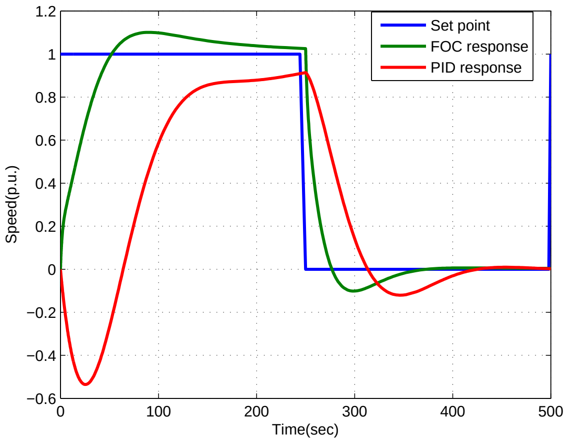

Fig. 14. Closed loop response of FO and PID controller for pulse input with disturbances.

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