An Enhanced Approach for Face Recognition of Newborns using HMM and SVD Coefficients

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ABSTRACT
The problems of newborn's abduction, mixing, swapping, etc. are increasing day-by-day. This problem has now reached a global aspect, as the consequences have now become critical. The researches performed to meet the challenge are very few. The growing problem motivated a need of sustainable system that can assist the hospital authorities and even the parents to keep a track of any newborn baby and his parents. The concept of biometric recognition has always proven to be a powerful tool when the identification of an individual comes into play. Therefore the face recognition among newborn is implemented in the proposed system. In order to deploy face recognition system for newborns, first a database is generated maintaining the images of a newborn and all the suspected parents. Matlab is used as a programming tool for the proposed work. The system trains the sample images of the parents using the HMM with a combination of SVD coefficients. The HMM and SVD models provided an approach to model a system that can develop a training sample of the image and can detect the image while any test sample is presented to the program. As a pre-processing method two-dimensional order static filtering is applied to the test images that improve the computational speed and accuracy of the system. Quantization SVD differentiates each image into a sequence of block and then each block of image is regarded as a numeric string. The HMM can effortlessly model the numeric block for training and recognition purpose. Matlab program generates a GUI that can train and recognize the image of a baby's parent. The proposed system is user friendly and very quick in generating results. A fast and efficient system had been developed for the purpose of face recognition among the newborns

General Terms
Face Recognition, Image Processing.

Keywords
Newborn identification, Face Recognition, Hidden Markov Model, Singular Value Decomposition

1. INTRODUCTION
Face Recognition (FR) is a field of biometrics that deals with the authentication and identification of an individual by extracting the individual's facial features. FR in new born child has emerged as a necessity due to swapping, mixing and abduction of these children. In India a problem involving the act of parents escaping and leaving their newborn child alone just after their birth is very common .These problems are now emerging as global one among various part of the nation. Humans can recognize many of faces and can easily identify frequent faces even with large. When it comes to reorganization of identities among newborns the recognition is enduring because of similar features among babies. This similarity among basic facial features turns into a reason for the successful abduction or swapping of the new born.

According to the National Centre for Missing and Exploited Children, 270 cases of newborn/infant abduction have been reported in the United States from year 1983-2010 .In medical science; different methods have been explored to identify newborns. Deoxyribonucleic Acid (DNA) typing and Human Leukocyte Antigen (HLA) typing are very efficient and accurate methods for verifying the identity of babies [1]. DNA testing has provided the biggest revolution in the identification of individuals. Still, the technology has limitations. Most genetic tests take 24-72 hours but the time taken for DNA to go from hospitals for identification and this can span as long as 14 days. Therefore DNA tests and HLA are time bound procedures, although effective but by the time that the results are back, the suspects often have been eluded [2].Newborn identification varies from hospital to hospital, but one of the most prevalent methods is the use of corresponding Identity Card (ID) bracelets between mothers and their babies. Some of the bracelets contain Radio Frequency Identification (RFID) technology to facilitate identification. Despite of the advancement in the ID technology the Infant-mother mix-ups happen more than 23,000 times each year [4]. Footprint acquisition is another method employed using a high resolution sensor (above 1500dpi), but data acquisition procedure was difficult and little time taking.

The consideration of growing demand for a highly advanced facial recognition system is essential. There is a requirement of facial recognition system that is able to recognition a newborn baby and his parents. It would be worthy to further enhance the robustness and adaptively of the existing recognition system for this purpose. The presented paper surveys the strengths, capabilities, and weaknesses of a new proposed technique based on Hidden Markov Model (HMM) and Singular Value Decomposition (SVD) Coefficients techniques for facial recognition of newborn baby and identification of his parents by comparing the two faces.

The roots of facial recognition lie in the field of biometrics. Biometric recognition is one of the most eminent techniques in day today security issues. Principal basis of FR system is the identification of characteristics within the face. Face recognition system involves two major steps a) Face Detection

b) Face Recognition
a)Face Detection: The basis of face recognition is concerning about face detection. This is a fact that detection seems quite difficult to numerous researchers these days. Segmentation problem in practical systems is basically the one in which most of the attempts go into working out this job. In

actuality, it is only a petty last few step in the major recognition based on features extracted. There are two types of face detection procedure:

1) Face detection in static images: Most Face Detection systems make an effort to extract a particular feature of the whole face, thereby removing most of the inadequate part and other areas as in an individual's head hair are nonessential for the FR task . For static images, FD is often done by running a 'window' across the image [3]. Figure 1 shows the illustration of window based face recognition.

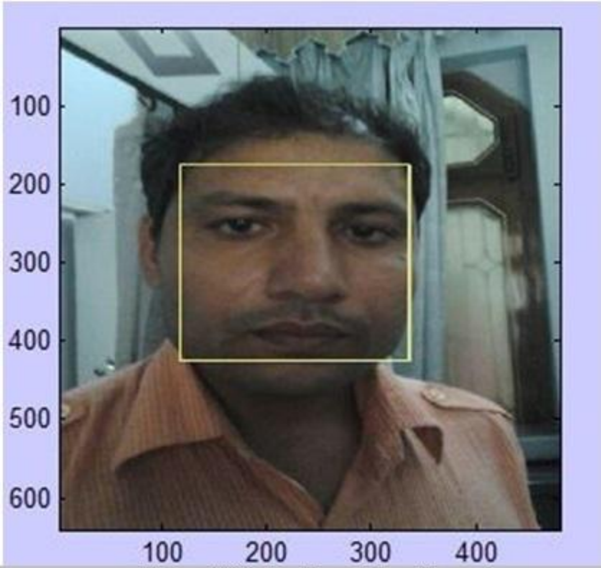

Figure 1Window Based Face Detection

2) Face Detection in Real-time: Real-time FD includes detection of a face from a sequence of frames from a video-capturing device. Real-time FD is, in reality is an effortless procedure than detecting a face in a still image whereas the hardware requirements for this type of a system are far tougher.

b) Face Recognition: Face recognition involves the implementation of various algorithms in detection of a facial feature and the matching the image with pre-stored data. In general, facial recognition can be decomposed into four [4] phases:

1.Pre-processing: This means ensuring that the image which is applied to the recognition process meets certain required standards: for such that the face is located in the centre of the image and provided part of the same; that the background satisfies certain constraints, and so on. Usually this phase is done by sampling equipment.

2. Phase segmentation or localization: is the exact location of the face or certain parts of it. This phase arises from the need to characterize, through some characteristic features, the face of a subject.

3. Feature Extraction Phase: maybe it is the core of the whole face recognition process. A feature it’s a characteristic useful for distinguish a face from another. It can be extracted from the image through different kind of processes. Usually, higher is amount of extracted features, the higher is the capacity of discrimination between similar faces.

4. Recognition Phase: Once the image is associated with an array of values, the recognition problem reduces itself to a widely studied problem in the past literature: the main part of those is then mainly related to the features extraction. The recognition problem can be divided into three phases: deciding over which features the recognition will be done; automatic extracting the chosen parameters from the face digitalized image; classifying the faces over the acquired parameters.

The presented paper surveys the strengths, capabilities, and weaknesses of a new proposed technique based on Hidden Markov Model (HMM) and Singular Value Decomposition (SVD) Coefficients techniques for facial recognition of newborn baby and identification of his parents by comparing the two faces. The elementary objective for this research paper is twofold. First, it surveys a practice for detecting and generating a database of newborn’s face precisely from still images considering the different susceptible environment that can be present in the image. Second, it routes the detected face as a previous step for helping in be familiar with the parent’s face using robust recognition techniques. Assisted by the face detection and face recognition steps a complete set for a biometric system aiming at facial recognition is thus generated. In simple words paper deals with the generation of facial recognition system using the face of newborn baby that is matched with that of parent.

2. LITERATURE REVIEW

There are many research performed for the face recognition of adults using HMM and SVD approaches but using the same technique there are very few surveys over the face recognition of the newborn babies. DNA and HLA matching techniques though are the most common methods for identifications among babies and parents but they are cost and time ineffective, therefore these are last options for matching a parent and his child. For the recognition of the newborn babies, most recent researches have been performed in the year 2013 only. Some researches based on identifications of newborns are here as:

1. Palm-Print Identification: Several methods have been imposed for the newborn baby’s detection and Anil K. Jain [5] proposed a technique based on high-resolution palm print images, for newborn identification. He used a sensor (Cross Match LSCAN 1000P), to perform preliminary studies with the goal of developing an automatic system for newborn identification. Despite the major drawback of providing only 1000ppi pictures rather than the required 1500ppi, the Cross Match Sensor (CMS) is not appropriate for newborns either, making it complicated to attain palmer pictures. However, he found that a division of the acquired images is definitely appropriate for detection.

2. Foot-print Identification: Wierschem (Wierschem, 1965) described a technique in which footprints gathered by Chicago’s hospitals (USA) were analyzed [6], including that 98% could not be used for recognition. A new investigation of the collected footprints was performed after providing training and the accurate tools to the medical team, showing that 99% allowed the newborn’s detection .But this recognition was not based on dactiloscopic ridges. Shepard et.al. (Shepard et al., 1966) collected footprints of 41 newborns, one at birth and other 4 to 6 weeks later, sending the resulting 102 impressions to the California State Justice Department of Criminal Investigation and Identification (USA) for analysis [6]. Resulting in about 20% identifiable footprints, there fingerprint experts analyzed the sample data and were only capable to identify 10 newborn babies. However it was felt that the majority of these 20 perfectly matched prints would not stand up under official inspection in the courts.

Table 1 Illustration the advantages and disadvantages of footprint identification for newborn babies.

S.No.	Advantages	Disadvantages
1	Image acquisition is fast, so reduces time for capturing footprint image.	Use of inadequate materials (ink, paper, cylinder);
2.	More reliable than other types of newborns footprints identification. Example : taking a picture of footprint	Untrained personal for footprint acquisition can lead to poor data
3.	If high resolution scanner used to scan ,greater resolution would yield better results	Baby's skin covered with an oily substance that makes data acquisition difficult.
4.	Fast and accurate identification.	Reduced thickness of the newborn epidermis, easily deforming the ridges upon contact and filling the valleys between ridges with ink and reduced size of new born ridges i.e. five time smaller than adults.

3. Ear identification: Ear identification is a long studied issue and various challenges have been acknowledged by the scientists or researchers including illumination, pose, and occlusion. Since it is hard to make the newborn babies sit stable and give good quality ear images, they can be measured as unhelpful users of biometric identification. They may display different poses, particularly if they felt uncomfortable while taking the ear image. In many cases occlusion also is an important problem because just after the birth many parents put some kind of thread or ear ring due to their old tradition. Shrikant Tiwari et al [7] had tried to use fine ear images with less covariate. In the projected framework executed by Tiwari, the biometric identification system was divided into two subsystems. These two subsystems were [7] the primary biometric system which consist of ear and the secondary biometric system [7] consisting of soft biometric qualities like weight, height, gender and blood group. Similar to ear fusion and soft [7] biometric is performed. The objective of research done by S.Tiwari was to make evident that ear with soft biometric identifiers such as blood-group weight, height, and gender could be very useful in newborn recognition. This research received the accuracy of 85.13% by implementing HAAR algorithm and after fusing it with soft biometrics it got 90.72% accuracy. But soft biometric characteristics are not as stable and consistent like the traditional biometric identifiers as an ear; they provide little knowledge about the identity of the newborn.

2.1 Previous Work on Face Recognition Using HMM and SVD

There are various approaches based on the HMM and SVD techniques. Major researches have been implemented for the facial recognition of humans using the techniques. Hossein M.Naimi and Pooya Davari (2008) [11] proposed the "A New Fast and Efficient HMM-Based Face Recognition System Using a 7-State HMM Along With SVD Coefficients" in this research the combined approach of using HMM and SVD approaches were implemented. The system was evaluated standard Olivetti Research Laboratory (ORL) face database and Yale database. Training half of the images in ORL database the research obtained 99% of the recognition rate. For the YALE database recognition rate of 97.7% was obtained. The addition of two new face regions, chin and eyebrows was done. The utilization of small number of quantized SVD coefficients as features describing blocks of face images was performed. Pre-processing system used an order-statistic filter operation. The consideration of a top-down chain of coinciding sub-image blocks was evaluated. Each face was considered as a numerical sequence that can be effortlessly implemented by HMM using quantized SVD coefficients of these blocks.

Domenico Daleno et.al (Domenico Daleno et al., 2010) tested different Hidden Markov Model [12] structure on a database (DB) obtained as a mixture of the ORL DB together with other images of persons masked wearing glasses, bandage or scarf , in order to test system consistency. The identification rate was good for all the HMM tested architectures, but the system using the HMM structure 3-6-6-6-3 gave a percentage (%) of recognition of 100%, that is to say that any of the 520 photo tested were properly acknowledged. Subsequently was made an experimental evaluation of the final results obtained with the hybrid system ANN-P2DHMM (using an HMM with structure 3-6-6-6-3) with the most important face identification algorithms projected in the literature when applied to the ORL images.

3. PROPOSED SYSTEM

The major work of the research is divided into several sections. As already discussed the proposed system uses the combination of HMM and SVD coefficients .The Matlab tool is used as the programming medium. The major factors for involving Matlab as a programming language are: (a) the user friendly GUI of the tool (b) the presence inbuilt function in the program (c) the line by error correction system including an interpreter (d) wide range of function for image processing (e) easy facilitations for the graph generation (f) lucid program constructs.

3.1 Preliminaries of Hidden Markov Model

Markov Model is used in probability theory it is a stochastic model (random) that assumes the Markov property. A random model represents a process where the state depends on previous states in a non-deterministic way. It follows the Markov property if the conditional probability distribution of upcoming states of the process (conditional on both past and present values) depends only upon the current state; that is, given the present, the next step does not depend on the past[8].

The Markov Model was developed by using the Markov property of probability theory. The model was named after a Russian mathematician Andrey Markov.

A Hidden Markov Model (HMM) is a statistical Markov model in which the system being modeled is assumed to be a Markov process with unobserved (hidden) states [9]. "A HMM can be considered the simplest dynamic Bayesian network[9]."

3.1.1 Elements of HMM:

1. The finite number of states, 'N' in the model
2. At each time 't', new state is entered based upon a transition probability distribution which depends on the previous state(Markov Property)
3. After each transition is made, an observation output symbol is produced according to a probability distribution which depends on the current state. This probability distribution is held fixed for the state regardless of when and how state is entered. Thus,

there are 'N' such observation which represents random variables in the stochastic process.

3.1.2 Mathematical Notation:

- N= │S│ is the number of all states present in the model, where S= {s1,s2,…,sN} is the set of all feasible states. The state of the model at time 't' is given by qt € S.
- M= │V│ is the number of the different observation symbols (OS), where V={V1,V2,…..VM} is the series of all possible OS vi(also called the code book of the model). The observation symbol at time't' is given by Ot € V.
- A= {aij} is the state of transition in probability matrix, given by:

$$a_{ij} = \mathrm{P}\,[\mathrm{q_t} + 1 = \mathrm{s}_j|\mathrm{q_t} = \mathrm{s_i}\,, 1 \leq \mathrm{i,j} \leq \mathrm{N}\,, 0 \leq \mathrm{a_{ij}} \leq 1$$

Equation (1)

- B={bj(k)} is the observation symbol probability of matrix, expressed by:

$$b_{\mathrm{j}}\,(\mathrm{k}) = \mathrm{P}[\,\mathrm{O_t} = \mathrm{v_k}|\mathrm{q_t} = \mathrm{s_j}\,], 1 \leq \mathrm{j} \leq \mathrm{N}, 1 \leq \mathrm{k} \leq \mathrm{M}$$

Equation (2)

- π = {π1,π2,π3,π4,……….πN} is the primary state of distribution expressed by :
  $\pi_{\mathrm{i}} = \mathrm{P}\,[\mathrm{q_1} = \mathrm{s_i}]\,, 1 \leq \mathrm{i} \leq \mathrm{N}$ Equation (3)

- In short using the Triplet HMM notation can be expressed by:
  λ= (A, B, π) Equation (4)

3.2 Singular Value Decomposition (SVD)

"SVD is a factorization of a real or compound matrix, with much useful relevance in signal processing and [10] statistics". The major application of SVD coefficients lays the decomposition of a high-dimensional data set into low dimensional subspace

3.2.1Mathematical Notation:

Suppose for any m×n matrix '**A'**, an algorithm to find matrices U, V, and W given that

A = U ∑ V$^{\mathrm{T}}$ where, Equation (5)

'U' is m×n and an orthonormal matrix

'∑' is n×n and a diagonal matrix

'V' is n×n and an orthonormal matrix

In other words U (m×m) and V (m×m) are orthogonal matrix, whereas ∑ is m×n matrix diagonal of singular values with components $\sigma_{i,j} = 0$, $i \neq j$ and $\sigma_{i,j} > 0$. Furthermore, it can be shown that there exist non-unique matrices U and V such that $\sigma_1 \geq \sigma_1 \ldots\ldots \geq 0$. The columns of the orthogonal matrices U and V are called the left and right singular vectors respectively; an important property of U and V is that they are mutually orthogonal.

3.3 Outline of Proposed System:

3.3.1Database Description:

The system obtains the images in 128×128 pixels resolution and in standard Portable Graymap Graphics (.pgm) file form. The reason for selecting PGM files as database is that the (.pgm) files are compressible and are easily converted into other file format. In the database generation process, that is used around 10 parents' and their baby's images was collected. Each parent's image had 10 sample images of their face. Similarly the baby's images samples were generated. To overcome the confusion of mixing up of the similar names of the parents, the baby name is given either in number system such as baby1, baby2, etc. or the baby's mother name is given to a female child and father's name to male newborn baby. Therefore, about 300 images samples were presented for the training purpose in the system. Fig 2 illustrates a sample of database.

Figure 2 Sample of the Database Images

3.3.2Training of Samples:

The training process involves the implementation of HMM model the flow graph of training process is shown in fig 3 each block represents the process involved in training procedure. The blocks involved in the HMM training can be explained as follows:

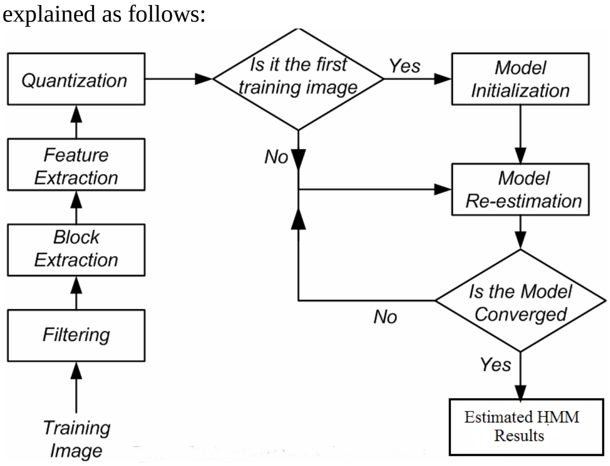

Figure 3 Flow Chart of the Sampling Process

1. Filtering: The pre-processing technique used two dimensional (2D) order-static filtering (OSF) for the proposed system.OSF are nonlinear spatial filters. It operates as follows; a sliding window slides from left to right and top to down with accordance to the steps of size one pixel, at each position the centre pixel is substituted by one of pixels of the window based on the category of filter. For instance minimum (min.), maximum (max.) and middle of pixels of the window might substitute the centre pixel. A 2D OSF, which substitute the centre element of a 56×46 pixels window with the min. element in the window, used in the proposed system. This can be represented by:

$$f^{\wedge}(x, y)\min_{(s,t)\in s_{xy}} \{g(s,t)\} \text{ Equation (6)}$$

In the equation, 'g(s, t)' is the grey level of given pixel (s, t) and s_{xy} is the window. As most of the face databases are taken over using camera flash.

2. Block Extraction and Feature Selection: A successful FR system strongly relies on the feature extraction method. One essential improvement of the given system is the employment of SVD coefficients as features as an alternative of gray-values of the pixels in the sampling windows (blocks). The illustration is shown in figure 5 .With pixels value as features explaining blocks, boosts the processing time and have an advantage to sharp computational complexity in training and FR procedure. In the research, computation of SVD coefficients of each block and using them as features is implemented.

(a) Original Image **(b) Image after applying SVD coefficients**

Figure 4 Illustration of image before and after applying SVD coefficients

3. Quantization: The SVD coefficients have naturally continuous values. SVD coefficients compose the observation vectors (OV). If they are judged in the corresponding continuous type, system meets a countless number of probable OV that can't be modeled by HMM. As a result, quantization of expressed features is done. Mainly, it is done by a process of rounding, or various other irreversible, nonlinear processes these all process leads to information loss. To show the quantization process in details, operated in the proposed system, consider a vector X=(x1,x2,…,xn) with continuous components. Suppose x_i is to be quantized into D_i distinct levels. So the differentiation relating two successive quantized values will be given as equation x.

$$\Delta_i = {(x_{imax} - x_{imin})}/{D_i} \text{ Equation (7)}$$

$x_i$max and $x_i$min are the max. and min. values that x_i acquire in all possible observation vectors respectively.Knowing Δ_i, element x_i will be exchanged with its quantized value calculated as below:

$$x_{iquantized} = \left[\frac{x_i - x_{imin}}{\Delta_i}\right] \text{ Equation (8)}$$

Therefore all the components of X will be quantized. In the end each quantized vector is linked with a label that is an integer number. So every block of image is mapped to an integer. Thus the images are now interpreted; so that use of HMM can be done for classification.

4. Training Process: Following presentation of each face image by observation vectors, they are modeled by a HMM. Five images of the identical face are used for training the related HMM and the remaining baby's images are used for testing. In the initial step U= (A,B,π) is implemented. The primary values for A and π are preset due to the left to right arrangement of the face model. The initial values for A and π are given using:

$$a_{i,i} = a_{i,i+1} = 0.5\,;\, 1 \le i \le 6 \text{ And } a_{7,7} = 1; \pi_0 = 1 \text{ Equation (9)}$$

The parameters of HMM are approximated using;

$\lambda^* = \max_{\lambda} P(O|\lambda)$.The probability of the observation 'O' is associated with image learning is maximized. This process progress into number of iterations to obtain training set of the given observation vectors.

5. Recognition and Comparison: For the proposed system the training of parents' image is employed and the test images are the samples of the images of the newborn baby.The comparison of the two images is derived through the calculates of the logarithmic sequence of the trained image of the parents, the nearest value to the selected test(baby's) image is recognized as the parents' image for the test image. Following the HMM training and learning process, each class (face) is considered to be in association with HMM. Each test image of the newborn baby encounters the block extraction, following by feature extraction and then quantization process respectively. Certainly each test image same as training images corresponds by its own observation vector. For an input face image, simple calculation of the probability of the observation vector (baby's image) given by each HMM face model is performed. A face image 'm' is recognized as face'd' if:

$$P(O^{(m)}|\lambda_d) = \max_n P(O^{(m)}|\lambda_d) \text{ Equation (10)}$$

Since the HMM recognition results completely rely over the training of the images. Thus, comparing to the training step the recognition seems to be a simple procedure. Therefore using the mentioned method the FR system can be generated for the facial recognition of an infant.

4. RESULTS

Observation of the results is paramount in order to check the efficiency of the developed system. Fig 6 illustrates the glimpse of the database including the input and output of the results obtained. While selecting an input image the developed program displays the name relevant to the highest log value calculated, nearest log value of the trained image is obtained as an output. In the fig 6 the red colored text shows the correct matching results with the output.

Typical observation of the results can be explained using the graph plot of the resultant output obtained. In the Fig.5 the typical observation of the program are demonstrated. Each graph in the figure displays the result for different input image given as an input to the developed system.

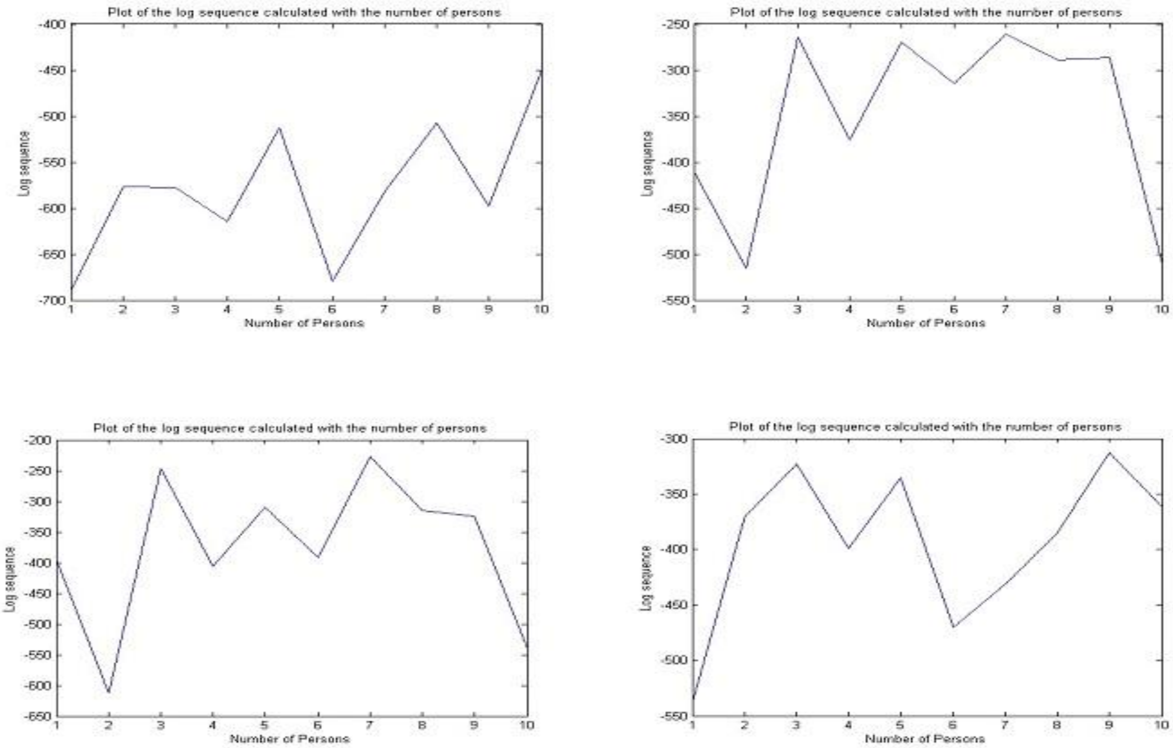

Figure 5 Results obtained for Various Images

5. CONCLUSIONS

Surveying the research based on searching the methods for identification of the infants; it can be concluded that facial identification can play a significant role and working with these methods provides for better solutions. The face recognition methodologies for newborn identification can be implemented in various hospitals and nursing homes so that a database can be generated well in advance before the problems of infant's abduction, swapping, missing etc arises. The system can be easily used by the nursing staff of the hospitals for the quick recognition of the infant and his parents; this can lead to remove the discrepancies that arise even when multiple babies born around the time period. Moreover, the Hidden Markov Model with the combination of Singular Value Decomposition techniques returns a medium to solve one of the basic problems that has been prevailing in the country since past several years. The demonstration of a fast and an efficient system is performed using these methods. The scope for future work is required in this field also.

5.1 Future Scope

In the future, the focus of the system should be over the operation of larger and much more complicated databases to test the system. For the complicated databases it is merely expected that all the database training will provide the totally expected results. Enhancement in the feature extraction and the modeling of the faces is required. The use of 2-D HMM or more complex models may improve the system performance and efficiency.

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