Error Analysis of 2-tier M-ary Star QAM Modulation in Shadowed Fading Channels

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ABSTRACT

The error performance of the 2-tier star shaped Quadrature Amplitude Modulation scheme over K and K_G fading channels are analyzed and evaluated. Novel closed form expressions for Symbol Error Rate (SER) have been derived for M-ary 2-tier circular Star QAM transmitted over the K and K_G fading channels. The expressions derived are in the form of sum of single definite integrals of hypergeometric functions which are calculated using numerical methods. The expressions are validated by extensive Monte Carlo simulation. A simple relationship between SER and bit error rate (BER) is proposed and experimentally verified. Using the expressions for SER, the optimum values of ring ratio are calculated for various values of M. It has also been established that the error performance of 2-tier Star QAM is considerably superior to that of the M-ary Square QAM for high and moderate fading.

General Terms:

Digital Modulation, Wireless Channel

Keywords:

Star Quadrature Amplitude Modulation (QAM), Rayleigh-Gamma(K), Nakagami-Gamma(K_G), Fading, Shadowing, Symbol Error Rate(SER), Bit Error Rate(BER)

1. INTRODUCTION

Various statistical models are used to study the effects of multipath fading and shadowing that severely degrade the performance of wireless communication systems. Effects of small-scale multipath fading on communication systems have been widely studied. The analysis of the effect of shadowing in wireless communication systems had been impeded by the complex mathematical form of the log-normal distribution conventionally used to quantify this phenomenon.

In relatively recent works the K and the K_G fading models have been found to be a suitable replacement of the log-normal distribution to estimate the combined effects of shadowing and small-scale fading in wireless media [1, 2, 3]. Moreover the Generalized-K distribution is also being used to model the effect of turbulence in free space optical (FSO) communication channels [4, 5]. The mathematical forms of these distributions allow closed form integration and hence system parameters can be efficiently computed using numerical techniques. Error analysis of lower order M-ary Phase Shift Keying (PSK) and square M-ary Quadrature Amplitude Modulation (MQAM) schemes for K and K_G channels is found in [3, 6, 7]. Similar analysis for higher order and complex modulation schemes are absent in literature.

M-ary QAM schemes are spectrally efficient modulation schemes which ensures higher data rates without requiring extra bandwidth. It has been pointed out in [8] that the widely used square MQAM scheme will have a high chance of false phase locking in channels where both amplitude and phase of the transmitted signal may vary considerably. The authors in [8] have contended that the M-ary Star QAM modulation scheme will be better suited for vehicular environments. In [9] it has been shown that the M-ary Star QAM scheme gives a better performance in heavy fading channels like the Rayleigh channel and also simplifies the receiver structure as automatic gain control (AGC) and carrier recovery are no longer required. As given in [10], the M-ary Star QAM overcomes the problem of high peak-to-average power ratio (PAPR) present in square and rectangular QAM schemes. In recent academic literature [11, 12, 13, 14] the Star QAM modulation has been studied for applications in optical communication. Additionally in [15] it has been pointed out that circular QAM schemes provide better performance than rectangular QAM for quantum detection.

Error analysis of Star MQAM schemes in small-scale multipath fading channels can be found in [16, 17, 18] but to the best of our knowledge error rate estimation of Star QAM in shadowed fading channels is absent in current literature. The aim of this paper is to analyse the combined effects of shadowing and multi-path fading on the performance of 2-tier M-ary Star QAM schemes. The contributions of this paper are :

i. Derivation of numerically computable expressions for average SER of Star MQAM considering the effects of both multipath fading and shadowing.
ii. Derivation of an efficient relationship between average SER and average BER for the 2-tier Star QAM.
iii. Proposing optimum values of ring ratio for different constellation sizes in composite multi-path and shadow fading channel.
iv. To compare the error performance of the Star and the Square constellations in the discussed propagation model.

The rest of the paper has been structured as follows. In Section 2 the channel models used are discussed. The theoretical derivation

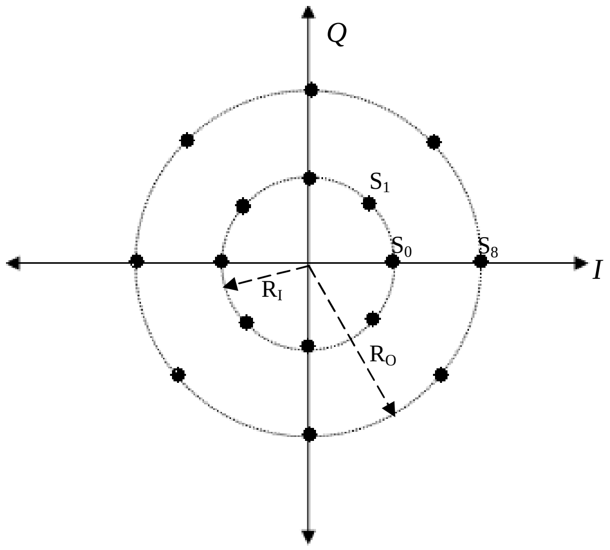

Fig. 1. Signal constellation of 2-tier 16-ary Star QAM.

of the average SER in K and K_G fading channel is shown in Section 3. The relationship between SER and bit error rate (BER) is given in Section 4. The optimum values of ring ratio for the Star QAM scheme are presented in Section 5.Numerical results and discussions are presented in Section 6; Section 7 concludes the paper.

2. SYSTEM MODEL

2.1 2-tier Star MQAM Scheme

The 2-tier M-ary Star QAM scheme, as shown in Figure 1, has the signalling points distributed over two amplitude levels R_I and R_O and on each amplitude level there are $M/2$ signalling points which are placed at a constant phase difference $(4\pi/M)$. Thus a signalling point (s_i) of a Star QAM constellation can be given as,

$$s_i = r_i.e^{j\theta_i}, \tag{1}$$

where,

$$r_i \epsilon \{R_I, R_O\}$$

and

$$\theta_i = \tfrac{4\pi i}{M}, i = 0, 1, ..., \tfrac{M}{2}.$$

The signalling points are uncoded, i.e. for the 16-QAM in Figure 1 s_0 = 0000, s_1 = 0001, s_8 = 1000 and so long. The modulated wave is baseband transmitted over the wireless channel.

2.2 Channel Model

The uncoded dual ring M-ary Star QAM signal is transmitted over a wireless channel along with AWGN. The signal in this wireless channel is corrupted due to both short-term (multipath) and long-term (shadow) fading. The short-term fading is assumed to be slow compared to the signal variation. The received baseband signal can be given by $z = sX + n$ where s is the transmitted symbol, X is a random variable following a fading statistics $f_\gamma(\gamma)$ and n is the Gaussian distributed additive noise.

When only channel degradation due to short term fading is considered, the probability density function of the signal amplitude can be given by $f_\gamma(\gamma)$ where $f_\gamma(\gamma)$ can be the Rayleigh distribution or the Nakagami-m distribution given in [19]. When long term fading is considered, the power of the envelope which is assumed to be constant for multipath fading varies according to the lognormal distribution $f_Y(y)$. Thus the joint pdf of the random variable X is given as,

$$f_X(\gamma) = \int_0^\infty f_{\gamma|Y}(\gamma|y).f_Y(y)dy. \tag{2}$$

But with lognormal distribution there is no closed form solution for $f_\gamma(\gamma)$ for the envelope of the received signal power. As shown in [1], the two parameter Gamma pdf can be used as a suitable substitute for lognormal pdf. From [20] the Gamma pdf can be given in terms of average SNR γ_0 as,

$$f_Y(y) = \frac{1}{\Gamma(c)}\gamma_0^{-c}y^{c-1}\exp\left(-\frac{y}{\gamma_0}\right); y \geq 0, \tag{3}$$

where c is the shape parameter of the Gamma distribution and $\Gamma(.)$ is the gamma function [21].

Considering the envelope of the signal to follow the Rayleigh distribution and the power of the envelope to follow the Gamma distribution given by eqn.(3), the channel as given in [20] can be modelled as the Rayleigh-Gamma or K distributed channel given as a function of SNR (γ) as,

$$f_X(\gamma) = \frac{2}{\gamma_0\Gamma(c)}\left(\frac{\gamma}{\gamma_0}\right)^{\frac{c-1}{2}} K_{c-1}\left(2\sqrt{\frac{\gamma}{\gamma_0}}\right); \gamma \geq 0, \tag{4}$$

where $K_a(.)$ is the the modified Bessel function of the second kind. The K fading channel has been generally used for RADAR and satellite applications [22, 23]. As is evident from eqn.(4), for the K distribution; the severity of the short term fading cannot be changed. Hence the K fading channel is not suitable for fading analysis where the fading characteristics vary. Thus for terrestrial applications a more general channel model is required.

The Nakagami-Gamma or K_G channel model can be used for more generalized estimation of the combined effects of short-term and long-term fading. The envelope of the received signal follows the Nakagami-m distribution and the power of the envelope follows the Gamma distribution. The K_G fading distribution function, as shown in [1, 20], can be given as a function of SNR (γ) as,

$$f_X(\gamma) = \frac{2\gamma_0}{\Gamma(c)\Gamma(m)}m^m\gamma^{m-1}(m\gamma_0\gamma)^{\frac{c-m}{2}}K_{c-m}\left(2\sqrt{\frac{m\gamma}{\gamma_0}}\right); \gamma \geq 0, \tag{5}$$

where m is the Nakagami shape parameter and $m \geq 0.5$. From [24] $K_a(.)$ is related to the Meijer-G function $G_{pq}^{mn}(.)$ by,

$$x^\mu K_\nu(x) = G_{02}^{20}\left(\frac{1}{4}x^2\middle|\begin{matrix}-\\ \frac{1}{2}\mu+\frac{1}{2}\nu, \frac{1}{2}\mu-\frac{1}{2}\nu\end{matrix}\right). \tag{6}$$

To facilitate calculations we use eqn.(6) in eqn.(5) and rewrite the K_G distribution function as

$$f_X(\gamma) = \frac{1}{\Gamma(c)\Gamma(m)}\frac{m}{\gamma_0}G_{02}^{20}\left(\frac{m\gamma}{\gamma_0}\middle|\begin{matrix}-\\ c-1, m-1\end{matrix}\right); \gamma \geq 0. \tag{7}$$

3. SER CALCULATION OF STAR MQAM IN K AND GENERALIZED-K FADING CHANNELS

Using the simple geometric method proposed by Craig in [25], closed form expressions for symbol error rate (SER) of dual ring

M-ary Star QAM in AWGN channel is derived in [16] as,

$$P_e(\gamma) = \frac{1}{2\pi}\sum_{k=1}^{4}\int_0^{\theta_k} \exp\left[-A(\theta,k).\gamma\right]d\theta, \tag{8}$$

where,

$$A(\theta,k) = \frac{\alpha_k M sin^2\phi_k}{2(1+L^2)sin^2(\phi_k+\theta)}, \tag{9}$$

and α_k, ϕ_k and θ_k are geometry dependent parameters given in [16]. M is the total number of signalling points on the constellation and L is the ring ratio [16, 17]. α_k, ϕ_k and θ_k are either constants or functions of M and L. The expression put forth in [16] can be easily computed by numerical method and gives an exact approximation of the SER for lower order M-ary Star QAM schemes.

The SER, $P_e(\gamma)$, in a fading channel characterized by the fading distribution $f_X(\gamma)$, is given by,

$$P(\gamma) = \int_0^{\infty} P_e(\gamma).f_X(\gamma).d\gamma. \tag{10}$$

3.1 SER in Rayleigh-Gamma (K) Fading Channel

The average SER in the Rayleigh-Gamma (K) channel can be obtained by substituting eqn.(4) and eqn.(8) in eqn.(10). Thus the average SER in the K channel is the solution of integral given by,

$$\begin{aligned} P_e(\gamma_0)\Big|_K =& \frac{1}{2\pi}\sum_{k=1}^{k=4}\int_0^{\theta_k}\int_0^{\infty}\frac{2}{\gamma_0\Gamma(c)}\left(\frac{\gamma}{\gamma_0}\right)^{\frac{c-1}{2}} \\ &\times K_{c-1}\left(2\sqrt{\frac{\gamma}{\gamma_0}}\right)\times \exp\left(A(\theta,k)\gamma\right)d\gamma d\theta. \end{aligned} \tag{11}$$

Using the relation [24]

$$\begin{aligned} &\int_0^{\infty} x^{\frac{m}{2}}e^{-\alpha x}K_m\left(2\sqrt{x}\right) = \\ &\qquad \frac{\Gamma(m+1)}{2\alpha}\left(\frac{1}{\alpha}\right)^{\frac{m-1}{2}}.W_{-\frac{1}{2}c,-\frac{1}{2}(c-1)}\left(\frac{1}{\alpha}\right) \end{aligned} \tag{12}$$

to compute the inner most integral of eqn.(11), the final closed form expression for SER in K fading channel is given as,

$$\begin{aligned} P_e(\gamma_0)\Big|_K =& \frac{1}{2\pi}\sum_{k=1}^{k=4}\int_0^{\theta_k}\left(\frac{1}{A(\theta,k)\gamma_0}\right) \\ &\times \exp\left(\frac{1}{2A(\theta,k)\gamma_0}\right) \\ &\times W_{-\frac{1}{2}c,-\frac{1}{2}(c-1)}\left(\frac{1}{A(\theta,k)\gamma_0}\right)d\theta, \end{aligned} \tag{13}$$

where $W_{a,b}(z)$ is the Whittaker function.

3.2 SER in Nakagami-Gamma (K_G) Fading Channel

The average SER in Nakagami-Gamma (K_G) fading channel can be computed by substituting eqn.(7) and eqn.(8) in eqn.(10). The average SER is the solution of the integral given by,

$$\begin{aligned} P_e(\gamma_0)\Big|_{K_G} =& \frac{1}{2\pi}\sum_{k=1}^{k=4}\int_0^{\theta_k}\int_0^{\infty}\frac{1}{\Gamma(c)\Gamma(m)} \\ &\times\left(\frac{m}{\gamma_0}\right)\exp\left(A(\theta,k)\gamma\right) \\ &\times G_{02}^{20}\left(\frac{m\gamma}{\gamma_0}\middle|\begin{matrix} - \\ c-1, m-1\end{matrix}\right)d\gamma d\theta. \end{aligned} \tag{14}$$

Using the relation [24]

$$\begin{aligned} &\int_0^{\infty} x^{-\rho}e^{-\beta x}G_{pq}^{mn}\left(\alpha x\middle|\begin{matrix} a_1,...,a_p \\ b_1,...,b_q\end{matrix}\right) = \\ &\qquad \beta^{\rho-1}G_{p+1q}^{mn+1}\left(\alpha x\middle|\begin{matrix}\rho, a_1,\dots,a_p \\ b_1,\dots,b_q\end{matrix}\right) \end{aligned} \tag{15}$$

for computing the inner most integral, the closed form expression for SER in K_G fading channel is given as,

$$\begin{aligned} P_e(\gamma_0)\Big|_{K_G} =& \frac{1}{2\pi}\sum_{k=1}^{k=4}\int_0^{\theta_k}\frac{1}{\Gamma(c)\Gamma(m)}.\left(\frac{m}{\gamma_0}\right) \\ &\times\frac{1}{A(\theta,k)}G_{12}^{21}\left(\frac{m}{\gamma_0 A(\theta,k)}\middle|\begin{matrix}0 \\ c-1, m-1\end{matrix}\right)d\theta. \end{aligned} \tag{16}$$

Computing the average value of SER using eqn.(16) becomes computationally expensive in MATLAB 2012 because MATLAB exports the Meijer-G function from the MuPad utility. From [21] we get the relation

$$\begin{aligned} G_{12}^{21}\left(x\middle|\begin{matrix}a \\ b,c\end{matrix}\right) =& \Gamma(b-a+1)\Gamma(c-a+1)x^{\frac{1}{2}(b+c-1)} \\ &\times e^{\frac{1}{2}x}W_{k,m}(x), \end{aligned} \tag{17}$$

where, $k = a - 0.5(b+c+1)$ and $m = 0.5(b-c)$.
Using eqn.(17) we can rewrite eqn.(16) as,

$$\begin{aligned} P_e(\gamma_0)\Big|_{K_G} =& \frac{1}{2\pi}\sum_{k=1}^{k=4}\int_0^{\theta_k}\left(\frac{m}{\gamma_0 A(\theta,k)}\right)^{\frac{1}{2}(c+m-1)} \\ &\times\exp\left(\frac{m}{2\gamma_0 A(\theta,k)}\right) \\ &\times W_{-\frac{1}{2}(c+m-1),-\frac{1}{2}(c-m)}\left(\frac{m}{A(\theta,k)\gamma_0}\right)d\theta. \end{aligned} \tag{18}$$

As expected, putting $m = 1$ in eqn.(31) we get eqn.(13). This is in conformance to the fact that the K distribution is a special case of the K_G distribution.

4. BIT ERROR RATE ANALYSIS

In [26] it is pointed out that traditional Gray's approximation underestimates the average bit error rate (BER) for Star QAM modulation. In fact Gray's approximation provides a lower bound (LB) for the BER. In this section we will derive a relation between the upper bound (UB) of BER and the SER using maximum *a priori* probability estimate.

Table 1. Average Asymptotical Optimum Ring Ratio in K_G Fading Channel.

M	L_{opt} in AWGN	Average L_{opt} in K_G
8	2.414	2.390
16	1.765	1.805
32	1.390	1.445
64	1.196	1.255

By maximum *a priori* probability (MAP) estimate the upper bound of BER is given by,

$$P_b(u) \le \frac{1}{log_2(M)} \sum_{j=1}^{M} d(c_j, c_u).A_{j|u} \tag{19}$$

where, c_u is the symbol received and c_j is the symbol decoded and

$$A_{j|u} = P(c_j isdecoded|c_u isreceived) \tag{20}$$

and d(c_j,c_u) is the Hamming distance between symbol c_j and c_u. Under conditions of severe and moderate channel degradation it can be assumed that in case of a symbol error all the other $M-1$ symbols are equi-probable receiver output. Thus the probability that symbol u is sent and j is decoded is given as,

$$A_{j|u} = \frac{SER(\gamma_0)}{M-1}, \tag{21}$$

where SER(γ_0) is the symbol error rate as a function of average input SNR (γ_0) as given in Section3. The average SER(γ_0) is independent of j . Thus eqn. (19) can be simplified as,

$$P_b(u) \le \frac{SER(\gamma_0)}{(M-1)log_2 M} \sum_{j=1}^{M} d(c_j, c_u) \tag{22}$$

Eqn. (22) can be further simplified by substituting,

$$\sum_{j=1}^{M} d(c_j, c_u) = m.2^{m-1} \tag{23}$$

where, $m = log_2(M)$, the number of bits per symbol.
Substituting equation (23) in equation (22), P_b is given as,

$$P_b \le \frac{M}{2(M-1)} SER(\gamma_0) \tag{24}$$

Thus the average BER under conditions of severe and moderate channel degradation is given by,

$$\frac{SER(\gamma_0)}{log_2(M)} \le BER \le \frac{M}{2(M-1)} SER(\gamma_0) \tag{25}$$

5. ESTIMATION OF RING RATIO (L)

The ring ratio (L) is a crucial parameter in the design of M-ary Star QAM systems. In AWGN channel the optimum value of ring ratio can be simply derived by equating the distance between a constellation point and its neighbours, i.e. from Figure 1 we equate the distance between s_0 and s_1 and s_0 and s_8. The relation can be easily computed using geometric methods as given in [27] for 16-ary Star QAM. Generalizing the expression the optimum ring ratio in AWGN is given by,

$$L_{opt} = 2.sin\left(\frac{2\pi}{M}\right) - 1 \tag{26}$$

For fading channels there is no direct formula to compute the optimum ring ratio (L). We need to employ the expressions for SER to estimate the optimum value of L. For a given value of SNR (γ), the value of L for which SER is minimum is computed programmatically. This is repeated for different values of SNR (γ). As shown in [17], the value of the optimum ring ratio assumes an asymptotic value for higher SNR. This asymptotic value is considered as the optimum value of L. Following this approach the asymptotic values L_{opt} in the K_G fading channel is computed.
The function under the integral in eqn.(31) is a hypergeometric function and hence a direct numerical search becomes computationally expensive and impractical. To overcome this a combination of numerical and empirical method is utilized to compute the asymptotic values of L_{opt}. Initially a few values of SNR are chosen over a large range (0 to 50dB). As we get an estimation of the asymptotic value of L_{opt} from this set of data, we decrease our range and take equal number of data points over that range. For example if from the first test we see that the optimum value of L starts assuming asymptotic values from 30dB, then in the next iteration we will decrease our range to 25dB to 40dB and find the variation of L for minimum SER in this range. Repeating these steps we can calculate the accurate value of L_{opt} for the K_G fading channel.
Obviously the value of L_{opt} varies with c and m. In order to facilitate system design an average value of L_{opt} has been proposed for each value of M. The values of L_{opt} for different values of c and m $(1 \le m \le 5, 1 \le c \le 5)$ are within a range of $\pm 10\%$ of the proposed mean values given in Table5.

6. RESULTS AND DISCUSSIONS

To establish the analytical results of Section3 extensive Monte Carlo simulations are preformed using MATLAB. Independent and identically distributed generalized K variates can be easily generated by extending the method shown in [28] using Gamma variates. Let G_θ be a gamma distributed random variate with the probability density function,

$$f_X(x;\theta) = \Gamma^{-1}(\theta) x^{(\theta-1)} e^x. \tag{27}$$

The generalized-K random variates with parameters m and c can then be generated by,

$$K_G = 2p\sqrt{(G_m \times Gc)}, \tag{28}$$

where,

$$p = \frac{1}{\sqrt{|c-m|+1}}. \tag{29}$$

The gamrnd(θ,1,a,b) function of the Statistical toolbox in MATLAB can be used to efficiently generate gamma variates.
The expressions derived in Section 3 are numerically evaluated by the adaptive Simpson quadrature formula using MATLAB 2012. From Fig.2 ,3, 4 and 5 it is evident that the expressions derived in Section 3 are in close agreement with the simulated results for a wide range of input SNR. From the plots it is evident that increasing c and/or m the SER decreases. This is at par with the general expectation as for higher values of c and m, shadowing and fading respectively becomes less severe.

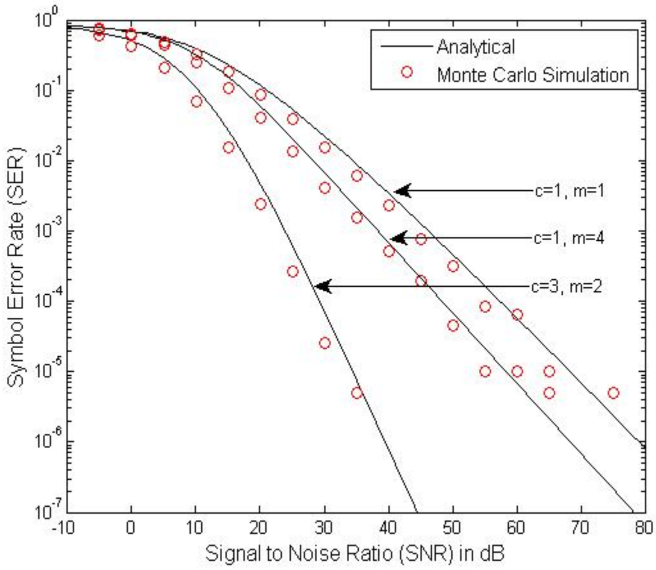

Fig. 2. Average SER vs. average SNR (γ_0) of 8-ary Star QAM in Generalized-K fading channel for several values of m and c (L=2.390).

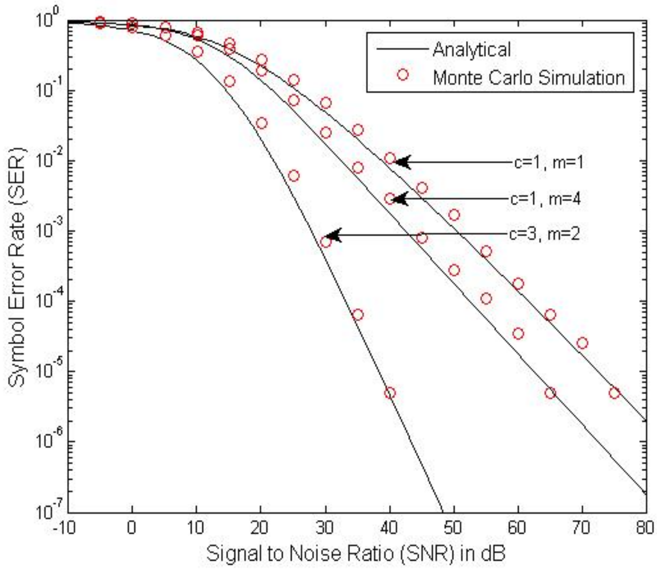

Fig. 4. Average SER vs. average SNR (γ_0) of 32-ary Star QAM in Generalized-K fading channel for several values of m and c (L=1.445).

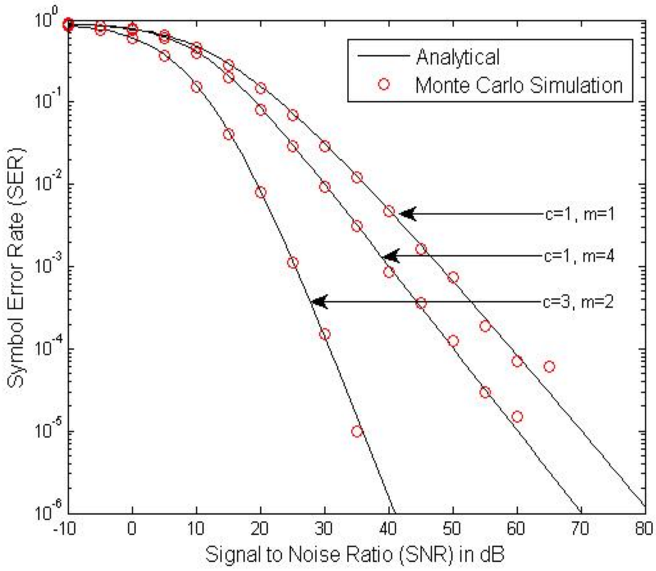

Fig. 3. Average SER vs. average SNR (γ_0) of 16-ary Star QAM in Generalized-K fading channel for several values of m and c(L=1.805).

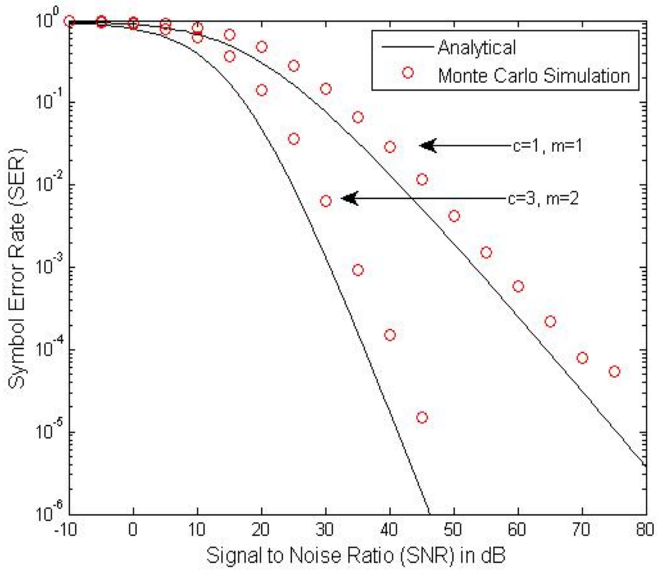

Fig. 5. Average SER vs. average SNR (γ_0) of 64-ary Star QAM in Generalized-K fading channel for several values of m and c (L=1.255).

From Figure 6 and Figure 7 it is seen that the estimation of BER using eqn.(25) is in close conformance to the Monte Carlo simulated results for high and moderate channel degradation. The importance of this result lies in the fact that BER is generally considered as the figure of merit of communication systems. To the best of the authors' knowledge, similar analysis for the circular QAM family is absent in current literature.

6.1 Comparison with Square MQAM

The SER of MQAM has been computed in [3] but the final expression in terms of the Meijer-G function is difficult to compute using MATLAB 2012. A close approximation of bit error rate (BER) for MQAM in AWGN channel has been given in [29] as,

$$BER_{MQAM}(\gamma) \approx 0.2 \exp\left(-\frac{1.6\gamma}{M-1}\right) \tag{30}$$

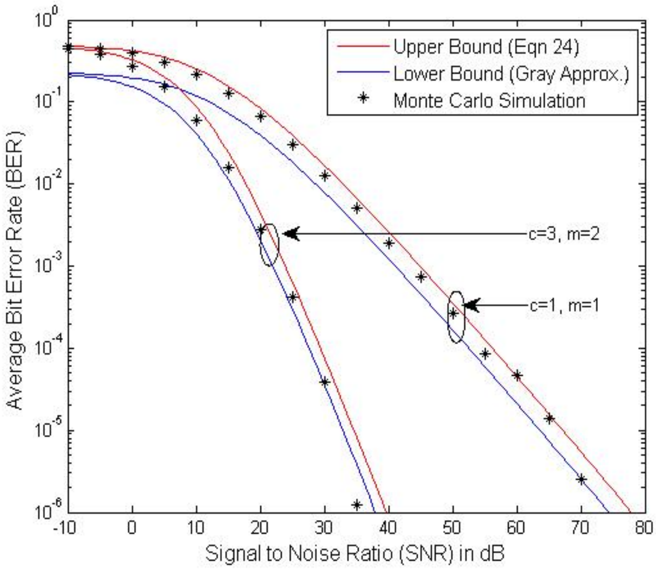

Fig. 6. UB and LB of average BER vs. average SNR (γ_0) of 16-ary Star QAM in Generalized-K fading channel for high and moderate channel degradation (L=1.805).

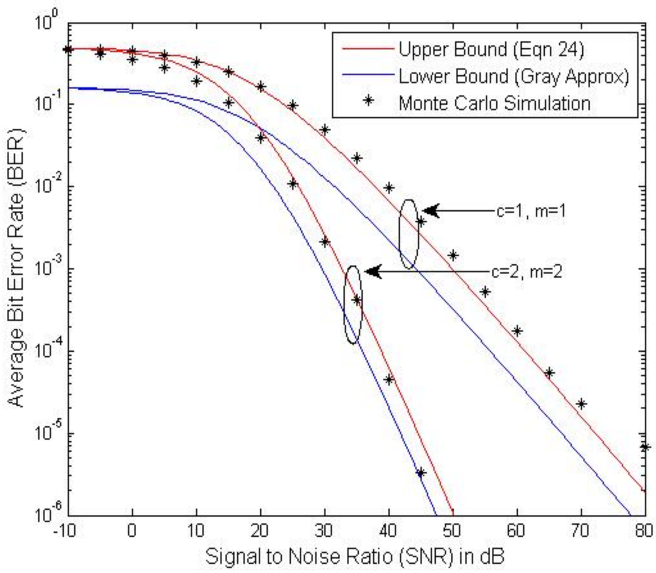

Fig. 7. UB and LB of average BER vs. average SNR (γ_0) of 64-ary Star QAM in Generalized-K fading channel for high and moderate channel degradation (L=1.255).

Following similar steps as in Section 3.2 we get the expression for BER of square MQAM in K_G fading channel as,

$$BER_{MQAM}(\gamma_0)\Big|_{K_G} = 0.4\left(m\frac{M-1}{1.6\gamma_0}\right)^{\frac{1}{2}(c+m-1)} \times \exp\left(\frac{1}{2}\frac{m(M-1)}{1.6\gamma_0}\right) \times W_{-\frac{1}{2}(c+m-1),-\frac{1}{2}(c-m)}\left(m\frac{M-1}{1.6\gamma_0}\right)d\theta. \tag{31}$$

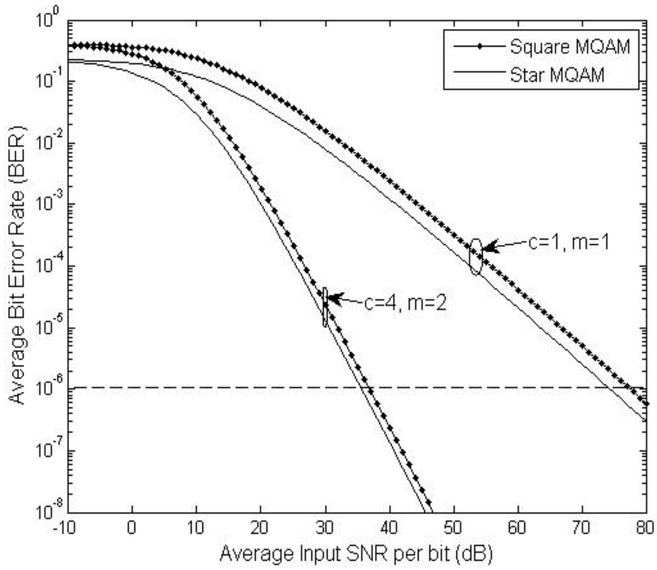

Fig. 8. Comparison of Error performance of 16-ary Star and Square QAM in K_G Fading Channel for varying channel conditions (L=1.805 for Star QAM).

The BER of Star MQAM can be estimated by the analytical expressions presented in Section 4.

In our comparitive analysis we have considered the target BER to be 10^{-6}. From Figure 8 it is seen that for $m = 1$ and $c = 1$ an advantage of 2.10dB is achieved by the use of Star MQAM. Whereas for $m = 2$ and $c = 4$ the SNR advantage over Square MQAM is 0.95 dB. From Figure 9 we find that for 64-ary schemes the advantage provided by Star MQAM over Square MQAM is 6dB for $m = 1$ and $c = 1$ and 3dB for $m = 2$ and $c = 4$.

In light of the results it is contend that the Star MQAM scheme gives a considerable advantage over the popularly used Square MQAM scheme in conditions of high and moderate fading. The advantage decrease with the decrease in the amount of fading (increase in m or c). For very high values of m and c (i.e. for light fading) the Square MQAM has a better error performance. It can be noted that the results discussed here are similar in trend as the results discussed in [9, 17] for short-term fading channels.

7. CONCLUSION

The error rate performance of coherent M-ary 2-tier Star QAM signalling under long-term fading conditions is presented in this paper. The expressions are in terms of finite summation of single definite integrals of hypergeometric functions which can be computed using numerical techniques. The paper presents a novel approach to calculating the bit error rate of the Star QAM scheme from the expressions of SER. The paper also establishes the fact that Star MQAM signalling has better error performance than Square MQAM in the K and K_G fading channels. The Star MQAM is considerably better than the Square constellation for high and moderate fading. It can also be noted that higher order Star MQAM gives greater advantage than its Square counterpart. Thus it can be inferred from our analysis that Star MQAM modulation is the better option than M-ary Square QAM for high rate data transmission over shadowed fading channels.

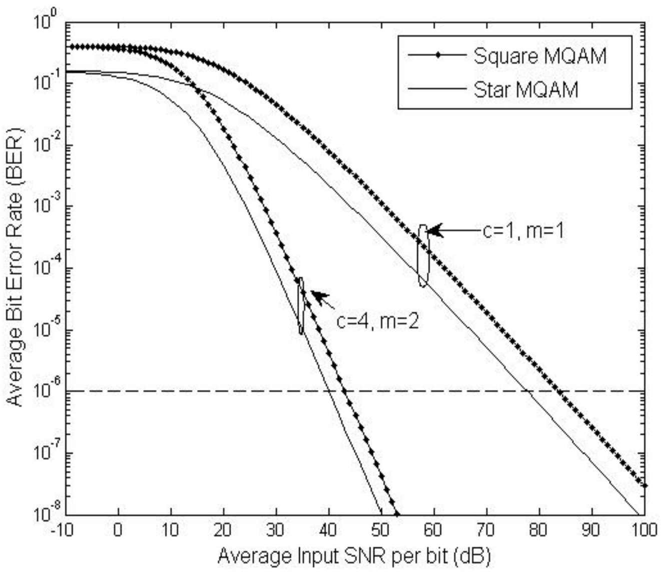

Fig. 9. Comparison of Error performance of 64-ary Star and Square QAM in K_G Fading Channel for varying channel conditions (L=1.255 for Star QAM).

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