Two Heterogeneous Servers Limited Capacity Markovian Queueing System Subjected to Varying Catastrophic Intensity

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ABSTRACT
In this paper we consider a limited capacity Markovian queueing system with two heterogeneous servers subjected to varying catastrophic intensity. The transient solution of the model has been obtained and various measures of performance have been computed numerically with the help of simulation technique. The steady state solution of the system has also been provided.

Keywords
Transient Solution, Varying Catastrophic Intensity, Simulation, Markovian Queueing System, Steady State Solution

1. INTRODUCTION
The single server Markovian queueing system has been the object of systematic and through investigation for a long time. In real life it is not necessary that a queueing system should have only one server. Practically they may have more than one server identical or non-identical in their functioning. Heterogeneity of service is a common feature of many real multi server queueing situations. The heterogeneous service mechanism allows the customers to receive different types of services. Queueing model with heterogeneous server were rarely treated in research. A Markovian queueing system with balking and two heterogeneous servers has been discussed by Singh [1970]. In recent year the attention has been focused on queueing model with catastrophes. A large number of research papers are available for queueing model under the influence of catastrophe see. e.g. Chao (1995), Kumar and Arivudainambi (2000), Jain and Kumar (2005a, b, c; 2006), [Crescenzo et al. (2003), and Jain and Kanethia (2006)], Brockwell et al., (1982) and Bartoszynski et al., (1989)]. Recently, the queueing system with two heterogeneous servers subject to catastrophes has been discussed by Kumar, B.K., Madheswari, Pavai S. and Venkatakrishnan, K.S., (2007). In the above mentioned work, all the researchers' have used the assumption that the occurrence of catastrophe destroys all the customers in a queuing system. But it is not always the case. So necessary amendment is incorporated in the paper of Jain and Bura [2010] in the form of varying catastrophic intensity to destroy a finite number of customers at a time. As a result of varying intensity of catastrophe the number of customers destroyed instantaneously on the occurrence of a catastrophe varies between 1 and N not necessary all in the queueing system. Although result has been reported on queueing models with two heterogeneous servers subjected to catastrophes, but no work has been found in the literature which studies queueing model with two heterogeneous servers subjected to varying catastrophic intensity. The catastrophic intensity may follow any appropriate distribution but in this paper we have considered discrete uniform distribution and modified binomial distribution. The concept of varying catastrophic intensity has numerous applications in a wide variety of areas such as computer communications, agriculture and biosciences etc.

2. QUEUEING MODEL
The queueing model investigated in this chapter is based on the following assumptions:-

(i) The customers arrive in the system one by one in accordance with a Poisson Process in a single queue with rate $\lambda > 0$.

(ii) There are two servers: server 1 and server 2. The service times of the customers are independently identically exponentially distributed with rates μ_1 and μ_2 respectively.

(iii) A customer who arrives, when there are zero customers in the system, joins the server 1with probability p and the server 2 with probability 1- p.

(iv) The queue discipline is first- come- first- served.

(v) Initially, there are zero customers in the system.

(vi) When the system is not empty, catastrophes occur according to a Poisson process with rate ξ and intensity C_r, $(r=1,2,3,....,N), \sum_{r=1}^{N} C_r = 1$. It depends upon the intensity of the catastrophe that how many customers are destroyed instantaneously.

(vii) The capacity of the system is limited to N. i.e., if at any instant there are N customers in the system, then the customers arriving in the duration for which the system remains in state N are not permitted to join the queue and considered lost for the system with probability one.

(viii) In case the catastrophe leaves only one customer in the system then the probability of his being served by server 1 is p and that of server 2 is (1 - p).

Define

$P_n(t)$= The probability that there are n customers in the system at time t.

$P_{1,0}(t)$= The probability that there is one customer in the system and he is being served by the server 1.

$P_{0,1}(t)$= The probability that there is one customer in the system and he is being served by the server 2.

The differential- difference equations governing the system are:

$$P_0^{/}(t) = -\lambda P_0(t) + \mu_1 P_{1,0}(t) + \mu_2 P_{0,1}(t) + \xi \sum_{n=1}^{N} \sum_{r=n}^{N} c_r P_n(t) \tag{1}$$

$$P_{1,0}^{/}(t) = -(\lambda + \mu_1 + \xi) P_{1,0}(t) + \lambda p P_0(t) + \mu_2 P_2(t) + \xi p \sum_{r=1}^{N-1} c_r P_{(1+r)}(t) \tag{2}$$

$$P_{0,1}^{/}(t) = -(\lambda + \mu_2 + \xi) P_{0,1}(t) + \lambda(1-p) P_0(t) + \mu_1 P_2(t) + \xi(1-p) \sum_{r=1}^{N-1} c_r P_{(1+r)}(t) \tag{3}$$

$$P_2^{/}(t) = -(\lambda + \mu_1 + \mu_2 + \xi) P_2(t) + \lambda P_{1,0}(t) + \lambda P_{0,1}(t) + (\mu_1 + \mu_2) P_3(t) + \xi \sum_{r=1}^{N-2} c_r P_{(2+r)}(t) \tag{4}$$

$$P_n^{/}(t) = -(\lambda + \mu_1 + \mu_2 + \xi) P_n(t) + \lambda P_{(n-1)}(t) + (\mu_1 + \mu_2) P_{(n+1)}(t) + \xi \sum_{r=1}^{N-n} c_r P_{(n+r)}(t), \quad n = 3, 4, 5, \ldots, N-1 \tag{5}$$

$$P_N^{/}(t) = -(\mu_1 + \mu_2 + \xi) P_N(t) + \lambda P_{(N-1)}(t) \tag{6}$$

Taking, Laplace Transform of equations (1) to (6) w.r.t.'t', we have

$$sP_0^*(s) = 1 - \lambda P_0^*(s) + \mu_1 P_{1,0}^*(s) + \mu_2 P_{0,1}^*(s) + \xi \sum_{n=1}^{N} \sum_{r=n}^{N} c_r P_n^*(s) \tag{7}$$

$$sP_{1,0}^*(s) = -(\lambda + \mu_1 + \xi) P_{1,0}^*(s) + \lambda p P_0^*(s) + \mu_2 P_2^*(s) + \xi p \sum_{r=1}^{N-1} c_r P_{[1+r]}^*(s) \tag{8}$$

$$sP_{0,1}^*(s) = -(\lambda + \mu_2 + \xi) P_{0,1}^*(s) + \lambda(1-p) P_0^*(s) + \mu_1 P_2^*(s) + \xi(1-p) \sum_{r=1}^{N-1} c_r P_{[1+r]}^*(s) \tag{9}$$

$$sP_2^*(s) = -(\lambda + \mu_1 + \mu_2 + \xi) P_2^*(s) + \lambda P_{1,0}^*(s) + \lambda P_{0,1}^*(s) + (\mu_1 + \mu_2) P_3^*(s) + \xi \sum_{r=1}^{N-2} c_r P_{[2+r]}^*(s) \tag{10}$$

$$sP_n^*(s) = -(\lambda + \mu_1 + \mu_2 + \xi) P_n^*(s) + \lambda P_{(n-1)}^*(s) + (\mu_1 + \mu_2) P_{(n+1)}^*(s) + \xi \sum_{r=1}^{N-n} c_r P_{[n+r]}^*(s) \tag{11}$$

$$sP_N^*(s) = -(\mu_1 + \mu_2 + \xi) P_N^*(s) + \lambda P_{(N-1)}^*(s) \tag{12}$$

Where

$$P_n^*(s) = \int_0^{\infty} e^{-st} P_n(t)\,dt \quad \text{and} \quad P_0(0) = 1$$

Solving equations (11) and (12) recursively, we have

$$P_n^*(s) = \rho^{-N} \left\{ \rho^n + \sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]} \sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]} \prod_{j=1}^{i-1} \left(\sum_{l_j=(i-j)}^{l_{j-1}-1} \right) \prod_{m=0}^{i-1} \left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]} \right) \eta\, \rho^{L_i+n} D_i \right\} P_N^*(s)$$

$$n = 3, 4, \ldots\ldots, N-1 \tag{13}$$

Where

$$\rho = \left(\frac{\lambda}{s+\mu_1+\mu_2+\xi}\right), \quad \eta = \prod_{j=1}^{i}\left(\frac{s+\xi\left(1-\sum_{r_j=1}^{k_j} C_{r_j}\right)}{s+\mu_1+\mu_2+\xi}\right)^{l_{(i-j)}-l_{(i-(j-1))}}, \quad [k] \rightarrow \text{An integral function.}$$

$$\prod_{j}^{i} = 1 \text{ and } \sum_{j}^{i} = 0 \text{ for } i < j, \; k_0 = 0,$$

$$A_m = \frac{N-n-(i-m)l_0 + \sum_{a=1}^{i-m-1} l_a - k_m\, l_{(i-m)} - \sum_{b=1}^{m-1}\left(k_{(m-b)} - k_{(m-(b-1))}\right) l_{(i+b-m)}}{l_0 - l_{(i-m)}}$$

$$L_i = \sum_{j=1}^{i}\left(l_{(i-j)} - l_{(i-(j-1))}\right)k_j, \quad l_j = \begin{cases} 0 & \text{if } j=i \\ l_j & \text{if } 1 \le j < i \end{cases} \quad \text{and} \quad D_i = \prod_{j=1}^{i}\binom{N-n-L_i - l_{(i-(j-1))}}{l_{(i-j)} - l_{(i-(j-1))}}$$

Now, the probabilities $P_0^*(s), P_{0,1}^*(s), P_{1,0}^*(s)$, and $P_2^*(s)$ remain to be found. For this, we consider the equations (7) - (10). After simplification, we have

$$P_0^*(s) = \frac{\left(B_1 \rho^{-N} P_N^*(s)\right) + \left(\frac{(s+\xi)}{s}(\lambda Q_1 - R R_1 R_2)\right)}{G} \quad (14)$$

$$P_{1,0}^*(s) = \frac{\left(B_2 \rho^{-N} P_N^*(s)\right) + \frac{\lambda(s+\xi)}{s}(\lambda U_1 - p R R_2)}{G} \quad (15)$$

$$P_{0,1}^*(s) = \frac{\left(B_3 \rho^{-N} P_N^*(s)\right) - \frac{\lambda(s+\xi)}{s}(\lambda U_1 + (1-p) R R_1)}{G} \quad (16)$$

$$P_2^*(s) = \frac{\left(B_4 \rho^{-N} P_N^*(s)\right) - \left(\frac{\lambda^2 (s+\xi) T_1}{s}\right)}{G} \quad (17)$$

In terms of unknown $P_N^*(s)$ and the constants:

$$B_1 = \left\{\begin{array}{l} \left(\lambda\xi(c_1+c_2)Q_1 + (\mu_1+\mu_2)Q_2 - R_1 R_2\left(R(c_1+c_2) + c_1(\mu_1+\mu_2)\right)\right)P1_3^*(s) - \\ (\lambda Q_1 - \lambda R R_1 R_2)\xi\left(\rho^N \sum_{r=1}^{N-1} c_r + \left((c_1+c_2)\sum_{n=4}^{N-1} + \sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1} c_r\right) P1_n^*(s)\right) - \\ \xi\left(Q_2 - c_1 R_1 R_2\right)\left(\rho^N C_{N-2} + \sum_{r=1}^{N-3} C_r P1_{2+r}^*(s)\right) + \\ +\xi\left(\begin{array}{l}\lambda(1-p)S_2(\mu_2-\mu_1) + \lambda p S_1(\mu_1-\mu_2) \\ -R_2\, p L_1 - R_1(1-p)L_2\end{array}\right)\left(\rho^N C_{N-1} + \sum_{r=1}^{N-2} C_r P1_{1+r}^*(s)\right) \end{array}\right\}$$

$$B_2 = \left\{ \begin{array}{l} \left(\left(-\left(\lambda^2 \xi (c_1+c_2)+\lambda\mu_2(\mu_1+\mu_2)\right)\right)U_1 + R_2 \begin{pmatrix} \lambda p \left(R(c_1+c_2)+c_1(\mu_1+\mu_2)\right) \\ -(s+\lambda+\xi)(\mu_1+\mu_2)S_2 \end{pmatrix} \right) P1_3^*(s) + \\ \left((-\lambda^2 U_1 + \lambda p R R_2)\xi \left(\rho^N \sum_{r=1}^{N-1} c_r + \left((c_1+c_2)\sum_{n=4}^{N-1} + \sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1} c_r \right) P1_n^*(s) \right) \right) + \\ (-\lambda\xi\mu_2 U_1 + \lambda p \xi c_1 R_2 - (s+\lambda+\xi)\xi R_2 S_2)\left(\rho^N C_{N-2} + \sum_{r=1}^{N-3} C_r P1_{2+r}^*(s) \right) \\ +(\lambda\xi(s+\lambda+\xi)U_1 - (s+\lambda+\xi)\xi p R R_2)\left(\rho^N C_{N-1} + \sum_{r=1}^{N-2} C_r P1_{1+r}^*(s) \right) \end{array} \right\}$$

$$B_3 = \left\{ \begin{array}{l} \left(\left(\left(\lambda^2 \xi (c_1+c_2)+\lambda\mu_1(\mu_1+\mu_2)\right)\right)U_1 + R_1 \begin{pmatrix} \lambda (1-p) \left(R(c_1+c_2)+c_1(\mu_1+\mu_2)\right) \\ -(s+\lambda+\xi)(\mu_1+\mu_2)S_1 \end{pmatrix} \right) P1_3^*(s) + \\ \left((\lambda^2 U_1 + \lambda(1-p) R R_1)\xi \left(\rho^N \sum_{r=1}^{N-1} c_r + \left((c_1+c_2)\sum_{n=4}^{N-1} + \sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1} c_r \right) P1_n^*(s) \right) \right) + \\ (\lambda\xi\mu_1 U_1 + \lambda(1-p)\xi c_1 R_1 - (s+\lambda+\xi)\xi R_1 S_1)\left(\rho^N C_{N-2} + \sum_{r=1}^{N-3} C_r P1_{2+r}^*(s) \right) \\ +(-\lambda\xi(s+\lambda+\xi)U_1 - (s+\lambda+\xi)\xi(1-p) R R_1)\left(\rho^N C_{N-1} + \sum_{r=1}^{N-2} C_r P1_{1+r}^*(s) \right) \end{array} \right\}$$

$$B_4 = \left\{ \begin{array}{l} \left(\lambda^2 \xi (c_1+c_2)T_1 + \lambda(\mu_1+\mu_2)T_2 - (s+\lambda+\xi)(\mu_1+\mu_2)R_1 R_2\right) P1_3^*(s) \\ +\lambda^2 T_1 \, \xi \left(\rho^N \sum_{r=1}^{N-1} c_r + \left((c_1+c_2)\sum_{n=4}^{N-1} + \sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1} c_r \right) P1_n^*(s) \right) + \\ \xi(\lambda T_2 - (s+\lambda+\xi)R_1 R_2)\left(\rho^N C_{N-2} + \sum_{r=1}^{N-3} C_r P1_{2+r}^*(s) \right) \\ -\lambda\xi(s+\lambda+\xi)T_1 \left(\rho^N C_{N-1} + \sum_{r=1}^{N-2} C_r P1_{1+r}^*(s) \right) \end{array} \right\}$$

$$G = \left\{ \begin{array}{l} \lambda S_1(\lambda p(\mu_2-\mu_1)+(s+\lambda+\xi)R_1) + \lambda S_2(\lambda(1-p)(\mu_1-\mu_2)+(s+\lambda+\xi)R_2) + \lambda p L_1 \\ +\lambda(1-p)L_2 - (s+\lambda+\xi)R R_1 R_2 \end{array} \right\}$$

And

$$R = (s+\lambda+\mu_1+\mu_2+\xi),\ R_1 = (s+\lambda+\mu_1+\xi),\ R_2 = (s+\lambda+\mu_2+\xi),\ S_1 = (\mu_1+\xi(1-p)c_1),$$
$$S_2 = (\mu_2+\xi p c_1), L_1 = (R\mu_1-\lambda\xi c_1),\ L_2 = (R\mu_2-\lambda\xi c_1),\ T_1 = (pR_2+(1-p)R_1),$$
$$T_2 = (\mu_1 p R_2 + \mu_2(1-p)R_1), Q_1 = (S_2 R_2 + S_1 R_1),\ Q_2 = (\mu_1 S_2 R_2 + \mu_2 S_1 R_1),$$
$$U_1 = (pS_1 - (1-p)S_2)$$

Where

$$P1_n^*(s) = \left\{ \rho^n + \sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]} \sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]} \prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1} \right) \prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]} \right) \eta \ \rho^{L_i+n} D_i \right\}$$

Using normalization condition, we have

$$P_N^*(s) = \frac{\left(G-(s+\xi)\left(\lambda Q_1 - RR_1R_2 - (R+\lambda)\lambda T_1\right)\right)}{s\rho^{-N}\left\{\begin{array}{l}(B_1+B_2+B_3+B_4)\\ +G\sum\limits_{n=3}^{N}\left\{\rho^n + \sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\ \sum\limits_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\ \prod\limits_{j=1}^{i-1}\left(\sum\limits_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\end{array}\right\}} \tag{18}$$

After using (18), equations (14)-(17) becomes:

$$P_0^*(s) = \frac{\left(\begin{array}{l}\left(\begin{array}{l}\left(\lambda\xi(c_1+c_2)Q_1 + (\mu_1+\mu_2)Q_2 - R_1R_2\left(R(c_1+c_2)+c_1(\mu_1+\mu_2)\right)\right)P1_3^*(s) - \\ (\lambda Q_1 - \lambda R R_1 R_2)\xi\left(\rho^N\sum\limits_{r=1}^{N-1}c_r + \left((c_1+c_2)\sum\limits_{n=4}^{N-1} + \sum\limits_{r=3}^{N-2}\sum\limits_{n=r+1}^{N-1}c_r\right)P1_n^*(s)\right) - \\ \xi\left(Q_2 - c_1R_1R_2\right)\left(\rho^N C_{N-2} + \sum\limits_{r=1}^{N-3}C_rP1_{2+r}^*(s)\right) + \\ +\left(\begin{array}{l}\lambda(1-p)S_2(\mu_2-\mu_1) + \lambda p S_1(\mu_1-\mu_2)\\ -R_2pL_1 - R_1(1-p)L_2\end{array}\right)\left(\rho^N C_{N-1} + \sum\limits_{r=1}^{N-2}C_rP1_{1+r}^*(s)\right)\end{array}\right) * \\ \left(\left(G-(s+\xi)\left(\lambda Q_1 - RR_1R_2 - (R+\lambda)\lambda T_1\right)\right)\right) + \left((s+\xi)(\lambda Q_1 - RR_1R_2)\right) \\ * \left(\begin{array}{l}(B_1+B_2+B_3+B_4)\\ +G\sum\limits_{n=3}^{N}\left\{\rho^n + \sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\ \sum\limits_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\ \prod\limits_{j=1}^{i-1}\left(\sum\limits_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\end{array}\right)\end{array}\right)}{sG\left((B_1+B_2+B_3+B_4) + G\sum\limits_{n=3}^{N}\left\{\rho^n + \sum\limits_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\ \sum\limits_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\ \prod\limits_{j=1}^{i-1}\left(\sum\limits_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod\limits_{m=0}^{i-1}\left(\sum\limits_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\right)} \tag{19}$$

$$
P_{1,0}^{*}(s)=\frac{\left\{\begin{aligned}&\left(\left(-\left(\lambda^{2}\xi\left(c_{1}+c_{2}\right)+\lambda\mu_{2}\left(\mu_{1}+\mu_{2}\right)\right)\right)U_{1}+R_{2}\begin{pmatrix}\lambda p\left(R\left(c_{1}+c_{2}\right)+c_{1}\left(\mu_{1}+\mu_{2}\right)\right)\\-\left(s+\lambda+\xi\right)\left(\mu_{1}+\mu_{2}\right)S_{2}\end{pmatrix}\right)P1_{3}^{*}(s)+\\&\left(\left(-\lambda^{2}U_{1}+\lambda pRR_{2}\right)\xi\left(\rho^{N}\sum_{r=1}^{N-1}c_{r}+\left(\left(c_{1}+c_{2}\right)\sum_{n=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_{r}\right)P1_{n}^{*}(s)\right)\right)+\\&\left(-\lambda\xi\mu_{2}U_{1}+\lambda p\xi c_{1}R_{2}-\left(s+\lambda+\xi\right)\xi R_{2}S_{2}\right)\left(\rho^{N}C_{N-2}+\sum_{r=1}^{N-3}C_{r}P1_{2+r}^{*}(s)\right)\\&+\left(\lambda\xi\left(s+\lambda+\xi\right)U_{1}-\left(s+\lambda+\xi\right)\xi pRR_{2}\right)\left(\rho^{N}C_{N-1}+\sum_{r=1}^{N-2}C_{r}P1_{1+r}^{*}(s)\right)\end{aligned}\right\}*\left(\left(G-\left(s+\xi\right)\left(\lambda Q_{1}-RR_{1}R_{2}-\left(R+\lambda\right)\lambda T_{1}\right)\right)\right)+\left(\lambda\left(s+\xi\right)\left(\lambda U_{1}-pRR_{2}\right)*\left(\begin{aligned}&\left(B_{1}+B_{2}+B_{3}+B_{4}\right)\\&+G\sum_{n=3}^{N}\left\{\rho^{n}+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_{0}=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_{j}=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_{m}+1}^{[A_{m}]}\right)\eta\ \rho^{L_{i}+n}D_{i}\right\}\end{aligned}\right)\right)}{sG\left(\left(B_{1}+B_{2}+B_{3}+B_{4}\right)+G\sum_{n=3}^{N}\left\{\rho^{n}+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_{0}=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_{j}=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_{m}+1}^{[A_{m}]}\right)\eta\ \rho^{L_{i}+n}D_{i}\right\}\right)}
\tag{20}
$$

$$
P_{0,1}^{*}(s)=\frac{\left\{\begin{aligned}&\left(\left(\left(\lambda^{2}\xi\left(c_{1}+c_{2}\right)+\lambda\mu_{1}\left(\mu_{1}+\mu_{2}\right)\right)\right)U_{1}+R_{1}\begin{pmatrix}\lambda(1-p)\left(R\left(c_{1}+c_{2}\right)+c_{1}\left(\mu_{1}+\mu_{2}\right)\right)\\-\left(s+\lambda+\xi\right)\left(\mu_{1}+\mu_{2}\right)S_{1}\end{pmatrix}\right)P1_{3}^{*}(s)+\\&\left(\left(\lambda^{2}U_{1}+\lambda(1-p)RR_{1}\right)\xi\left(\rho^{N}\sum_{r=1}^{N-1}c_{r}+\left(\left(c_{1}+c_{2}\right)\sum_{n=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_{r}\right)P1_{n}^{*}(s)\right)\right)+\\&\left(\lambda\xi\mu_{1}U_{1}+\lambda(1-p)\xi c_{1}R_{1}-\left(s+\lambda+\xi\right)\xi R_{1}S_{1}\right)\left(\rho^{N}C_{N-2}+\sum_{r=1}^{N-3}C_{r}P1_{2+r}^{*}(s)\right)\\&+\left(-\lambda\xi\left(s+\lambda+\xi\right)U_{1}-\left(s+\lambda+\xi\right)\xi(1-p)RR_{1}\right)\left(\rho^{N}C_{N-1}+\sum_{r=1}^{N-2}C_{r}P1_{1+r}^{*}(s)\right)\end{aligned}\right\}*\left(\left(G-\left(s+\xi\right)\left(\lambda Q_{1}-RR_{1}R_{2}-\left(R+\lambda\right)\lambda T_{1}\right)\right)\right)-\lambda\left(s+\xi\right)\left(\lambda U_{1}+(1-p)RR_{1}\right)*\left(\begin{aligned}&\left(B_{1}+B_{2}+B_{3}+B_{4}\right)\\&+G\sum_{n=3}^{N}\left\{\rho^{n}+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_{0}=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_{j}=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_{m}+1}^{[A_{m}]}\right)\eta\ \rho^{L_{i}+n}D_{i}\right\}\end{aligned}\right)}{sG\left(\left(B_{1}+B_{2}+B_{3}+B_{4}\right)+G\sum_{n=3}^{N}\left\{\rho^{n}+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_{0}=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_{j}=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_{m}+1}^{[A_{m}]}\right)\eta\ \rho^{L_{i}+n}D_{i}\right\}\right)}
\tag{21}
$$

$$P_2^*(s) = \frac{\left(\begin{array}{l} \left\{\begin{array}{l} \left(\lambda^2 \xi (c_1+c_2) T_1 + \lambda(\mu_1+\mu_2) T_2 - (s+\lambda+\xi)(\mu_1+\mu_2) R_1 R_2\right) P1_3^*(s) \\ +\lambda^2 T_1\, \xi \left(\rho^N \sum_{r=1}^{N-1} c_r + \left((c_1+c_2)\sum_{n=4}^{N-1} + \sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1} c_r\right) P1_n^*(s)\right) + \\ \xi\left(\lambda T_2 - (s+\lambda+\xi) R_1 R_2\right)\left(\rho^N C_{N-2} + \sum_{r=1}^{N-3} C_r P1_{2+r}^*(s)\right) \\ -\lambda\xi(s+\lambda+\xi) T_1 \left(\rho^N C_{N-1} + \sum_{r=1}^{N-2} C_r P1_{1+r}^*(s)\right) \end{array}\right\} * \\ \left(\left(G - (s+\xi)\left(\lambda Q_1 - R R_1 R_2 - (R+\lambda)\lambda T_1\right)\right)\right) - \\ \lambda^2 (s+\xi) T_1 * \left(\begin{array}{l} (B_1+B_2+B_3+B_4) \\ +G\sum_{n=3}^{N}\left\{\rho^n + \sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]} \sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]} \prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right) \prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right) \eta\ \rho^{L_i+n} D_i\right\} \end{array}\right) \end{array}\right)}{sG\left((B_1+B_2+B_3+B_4) + G\sum_{n=3}^{N}\left\{\rho^n + \sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]} \sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]} \prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right) \prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right) \eta\ \rho^{L_i+n} D_i\right\}\right)} \tag{22}$$

$$P_n^*(s) = \left(\frac{\begin{array}{c} \left\{\rho^n + \sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]} \sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]} \prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right) \prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right) \eta\ \rho^{L_i+n} D_i\right\} * \\ \left(\left(G - (s+\xi)\left(\lambda Q_1 - R R_1 R_2 - (R+\lambda)\lambda T_1\right)\right)\right) \end{array}}{s\left((B_1+B_2+B_3+B_4) + G\sum_{n=3}^{N}\left\{\rho^n + \sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]} \sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]} \prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right) \prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right) \eta\ \rho^{L_i+n} D_i\right\}\right)}\right) \tag{23}$$

After taking Laplace inverse of (19) to (23), we can find all the probabilities.

3. SIMULATION RESULTS

We obtain numerically the various measures of performance of this model by using simulation technique. The simulation analysis of the queueing model under investigation is carried out by using a computer program written in C language. The simulation results have been shown in tables 1-6. In tables 1-3, the simulation results are obtained by assuming that the catastrophic intensity follows the uniform distribution while in tables 4-6, the simulation results are obtained by assuming that the catastrophic intensity follows the modified binomial distribution.

Table-1
Simulation results of an M/M/2/N queueing model with two heterogeneous servers subject to uniformly distributed catastrophic intensity Effect of change in mean inter arrival time $\left(\frac{1}{\lambda}\right)$, Mean service time of server 1 = 6 minutes, Mean service time of server 2 = 8 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480, N=5

Mean inter arrival time	Average delay in queue	Average No. in queue	Server 1 Utilization	Server 2 Utilization
1	12.1858	3.9533	0.9154	0.9154
2	8.8038	0.1333	0.0643	0.0669
3	6.6864	0.7411	0.3213	0.3307
4	8.7736	0.5198	0.2050	0.2056
5	4.0348	0.3452	0.2650	0.2693
6	3.1294	0.0000	0.0051	0.0089
7	2.3818	0.1015	0.1347	0.1396
8	2.2160	0.0800	0.0720	0.0745
9	2.2716	0.0629	0.2946	0.2335
10	4.6215	0.0814	0.1076	0.1081

Table-2
Simulation results of an M/M/2/N queueing model with two heterogeneous servers subject to uniformly distributed catastrophic intensity Effect of change in mean service time of server 1 $\left(\frac{1}{\mu_1}\right)$, Mean inter arrival time = 2 minutes, Mean service time of server 2 = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480, N=5

Mean service time of server 1	Average delay in queue	Average No. in queue	Server 1 Utilization	Server 2 Utilization
1	0.5793	0.1374	0.2253	0.3426
2	3.0729	0.6185	0.4241	0.4593
3	4.7255	1.1797	0.5258	0.5459
4	4.6241	0.1946	0.1213	0.1308
5	7.3049	0.9741	0.3504	0.3509
6	8.5041	1.5390	0.4739	0.4718
7	8.8002	0.8844	0.2583	0.2566
8	8.9714	1.0563	0.3111	0.3048
9	11.7780	2.9511	0.7905	0.7789
10	10.4079	1.7684	0.4886	0.4873

Table-3
Simulation results of an M/M/2/N queueing model with two heterogeneous servers subject to uniformly distributed catastrophic intensity Effect of change in mean service time of server 2 $\left(\frac{1}{\mu_2}\right)$, Mean inter arrival time = 2 minutes, Mean service time of server 1 = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480, N=5

Mean service time of server 2	Average delay in queue	Average No. in queue	Server1 Utilization	Server 2 Utilization
1	0.6742	0.0479	0.0446	0.0321
2	1.5226	0.4428	0.5231	0.4007
3	4.1287	0.2043	0.1320	0.1174
4	5.1984	0.5185	0.2159	0.2143
5	7.3049	0.9741	0.3504	0.3509
6	7.7834	0.7722	0.2613	0.2628
7	8.1628	1.0034	0.3231	0.3290
8	11.6083	2.9928	0.7859	0.8010
9	10.2921	2.6730	0.7819	0.7713
10	8.9250	0.6030	0.1918	0.1989

Table-4
Simulation results of an M/M/2/N queueing model with two heterogeneous servers subject to uniformly distributed catastrophic intensity Effect of change in mean inter catastrophic time $\left(\frac{1}{\xi}\right)$, Mean inter arrival time = 2 minutes, Mean service time of server 1= 6 minutes, mean service time of server 2 = 8 minutes, simulation length= 480, N=5

Mean inter catastrophic time	Average delay in queue	Average No. in queue	Server 1 Utilization	Server 2 Utilization
101	12.2138	3.0654	0.7954	0.7882
102	9.7799	0.2531	0.1107	0.1097
103	11.6128	2.1070	0.5458	0.5415
104	9.3843	1.0284	0.3111	0.3113
105	10.2497	1.3395	0.3909	0.3812
106	9.9494	0.8560	0.2587	0.2556
107	10.4430	1.0812	0.3006	0.2922
108	9.8635	1.5633	0.4620	0.4479
109	10.1851	1.0888	0.3306	0.3355
110	12.1654	1.5348	0.4485	0.4424

Table-5
Simulation results of an M/M/2/N queueing model with two heterogeneous servers subject to modified binomially distributed catastrophic intensity.
Effect of change in mean inter arrival time $\left(\frac{1}{\lambda}\right)$, Mean service time of server 1 = 6 minutes, Mean service time of server 2 = 8 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480, N=5

Mean inter arrival time	Average delay in queue	Average No. in queue	Server 1 Utilization	Server 2 Utilization
1	11.8935	4.2028	0.9806	0.9806
2	9.0571	0.1052	0.0499	0.0505
3	6.8118	0.8650	0.3777	0.3920
4	8.7793	0.4342	0.1777	0.1792
5	4.0348	0.3452	0.2630	0.2693
6	3.1294	0.0000	0.0051	0.0089
7	2.3051	0.1043	0.1419	0.1451
8	2.2160	0.0800	0.0720	0.0745
9	2.2716	0.0629	0.2946	0.2335
10	4.4700	0.0922	0.0890	0.0896

Table-6
Simulation results of an M/M/2/N queueing model with two heterogeneous servers subject to modified binomially distributed catastrophic intensity.
Effect of change in mean service time of server 1 $\left(1/\mu_1\right)$
Mean inter arrival time = 2 minutes, Mean service time of server 2 = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480, N=5

Mean service time of server 1	Average delay in queue	Average No. in queue	Server 1 Utilization	Server 2 Utilization
1	0.5793	0.1374	0.2253	0.3426
2	3.1418	0.9103	0.4159	0.4500
3	4.7255	1.1797	0.5258	0.5459
4	4.4515	0.0916	0.0847	0.0970
5	7.3563	0.8923	0.3362	0.3395
6	8.9504	1.7259	0.5161	0.5192
7	8.6378	0.7640	0.2275	0.2256
8	8.9829	1.2061	0.3489	0.3417
9	11.8682	3.4734	0.9425	0.9284
10	10.8180	1.8999	0.5136	0.5116

Table-7
Simulation results of an M/M/2/N queueing model with two heterogeneous servers subject to modified binomially distributed catastrophic intensity.
Effect of change in mean service time of server 2 $\left(1/\mu_2\right)$
Mean inter arrival time = 2 minutes, Mean service time of server 1 = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480, N=5

Mean service time of server 2	Average delay in queue	Average No. in queue	Server 1 Utilization	Server 2 Utilization
1	0.6727	0.0640	0.0599	0.0438
2	1.5360	0.5795	0.6849	0.5215
3	4.0723	0.2473	0.1597	0.1395
4	5.2788	0.5434	0.2217	0.2210
5	7.3563	0.8923	0.3362	0.3395
6	7.6520	1.0865	0.3643	0.3662
7	8.1255	1.3601	0.4388	0.4436
8	12.1733	3.6002	0.9382	0.9542
9	10.1551	3.1477	0.9384	0.9256
10	8.6107	0.8195	0.2621	0.2627

Table-8
Simulation results of an M/M/2/N queueing model with two heterogeneous servers subject to modified binomially distributed catastrophic intensity.
Effect of change in mean inter catastrophic time $\left(1/\xi\right)$
Mean inter arrival time = 2 minutes, Mean service time of server 1= 6 minutes, mean service time of server 2 = 8 minutes, simulation length= 480, N=5

Mean inter catastrophic time	Average delay in queue	Average No. in queue	Server 1 Utilization	Server 2 Utilization
101	12.8427	3.6938	0.9433	0.9434
102	9.7839	0.1349	0.0783	0.0781
103	12.0068	2.3648	0.6054	0.6008
104	9.6638	1.0964	0.3287	0.3294
105	10.5273	1.5314	0.4582	0.4469
106	10.3545	1.1666	0.3428	0.3408
107	10.9170	1.1578	0.3221	0.3248
108	10.1586	1.9845	0.5836	0.5662
109	10.5369	0.9082	0.2831	0.2846
110	12.1190	1.1724	0.3512	0.3458

Steady State Solution

Using the property $\lim_{s\to 0} s\,P_n^*(s) = P_n$, We have from (19) to (23),

$$P_0 = \frac{\left(\begin{array}{l}\left(\begin{array}{l}\left(\lambda\left(c_1+c_2\right)Q_1+\left(\mu_1+\mu_2\right)Q_2-R_1R_2\left(R\left(c_1+c_2\right)+c_1\left(\mu_1+\mu_2\right)\right)\right)P1_3- \\ \left(\lambda Q_1-\lambda R R_1 R_2\right)\xi\left(\rho^N\sum_{r=1}^{N-1}c_r+\left(\left(c_1+c_2\right)\sum_{n=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_r\right)P1_n\right)- \\ \xi\left(Q_2-c_1R_1R_2\right)\left(\rho^N C_{N-2}+\sum_{r=1}^{N-3}C_r P1_{2+r}\right)+ \\ +\xi\left(\begin{array}{l}\lambda\left(1-p\right)S_2\left(\mu_2-\mu_1\right)+\lambda p S_1\left(\mu_1-\mu_2\right)-R_2 p L_1 \\ -R_1\left(1-p\right)L_2\end{array}\right)\left(\rho^N C_{N-1}+\sum_{r=1}^{N-2}C_r P1_{1+r}\right)\end{array}\right)* \\ \left(\left(G-\xi\left(\lambda Q_1-RR_1R_2-\left(R+\lambda\right)\lambda T_1\right)\right)\right) \\ +\left(\xi\left(\lambda Q_1-RR_1R_2\right)\right)*\left(\begin{array}{l}\left(B_1+B_2+B_3+B_4\right) \\ +G\sum_{n=3}^{N}\left\{\rho^n+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{\left[A_m\right]}\right)\eta\ \rho^{L_i+n}D_i\right\}\end{array}\right)\end{array}\right)}{G\left(\left(B_1+B_2+B_3+B_4\right)+G\sum_{n=3}^{N}\left\{\rho^n+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{\left[A_m\right]}\right)\eta\ \rho^{L_i+n}D_i\right\}\right)} \quad (24)$$

$$
P_{1,0} = \frac{
\begin{array}{l}
\left\{\begin{array}{l}
\left(\left(-\left(\lambda^{2}\xi(c_1+c_2)+\lambda\mu_2(\mu_1+\mu_2)\right)\right)U_1+R_2\begin{pmatrix}\lambda p\left(R(c_1+c_2)+c_1(\mu_1+\mu_2)\right)\\ -(\lambda+\xi)(\mu_1+\mu_2)S_2\end{pmatrix}\right)P1_3+\\
\left(\left(-\lambda^{2}U_1+\lambda pRR_2\right)\xi\left(\rho^{N}\sum_{r=1}^{N-1}c_r+\left((c_1+c_2)\sum_{n=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_r\right)P1_n\right)\right)+\\
\left(-\lambda\xi\mu_2U_1+\lambda p\xi c_1R_2-(\lambda+\xi)\xi R_2S_2\right)\left(\rho^{N}C_{N-2}+\sum_{r=1}^{N-3}C_rP1_{2+r}\right)\\
+\left(\lambda\xi(\lambda+\xi)U_1-(\lambda+\xi)\xi pRR_2\right)\left(\rho^{N}C_{N-1}+\sum_{r=1}^{N-2}C_rP1_{1+r}\right)
\end{array}\right\}*\\
\left(\left(G-\xi\left(\lambda Q_1-RR_1R_2-(R+\lambda)\lambda T_1\right)\right)\right)+\\
\left(\lambda\xi\left(\lambda U_1-pRR_2\right)*\left(\begin{array}{l}(B_1+B_2+B_3+B_4)\\ +G\sum_{n=3}^{N}\left\{\rho^{n}+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\end{array}\right)\right)
\end{array}
}{
G\left((B_1+B_2+B_3+B_4)+G\sum_{n=3}^{N}\left\{\rho^{n}+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\right)
}
\qquad (25)
$$

$$
P_{0,1} = \frac{
\begin{array}{l}
\left\{\begin{array}{l}
\left(\left(\left(\lambda^{2}\xi(c_1+c_2)+\lambda\mu_1(\mu_1+\mu_2)\right)\right)U_1+R_1\begin{pmatrix}\lambda(1-p)\left(R(c_1+c_2)+c_1(\mu_1+\mu_2)\right)\\ -(\lambda+\xi)(\mu_1+\mu_2)S_1\end{pmatrix}\right)P1_3+\\
\left(\left(\lambda^{2}U_1+\lambda(1-p)RR_1\right)\xi\left(\rho^{N}\sum_{r=1}^{N-1}c_r+\left((c_1+c_2)\sum_{n=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_r\right)P1_n\right)\right)+\\
\left(\lambda\xi\mu_1U_1+\lambda(1-p)\xi c_1R_1-(\lambda+\xi)\xi R_1S_1\right)\left(\rho^{N}C_{N-2}+\sum_{r=1}^{N-3}C_rP1_{2+r}\right)\\
+\left(-\lambda\xi(\lambda+\xi)U_1-(\lambda+\xi)\xi(1-p)RR_1\right)\left(\rho^{N}C_{N-1}+\sum_{r=1}^{N-2}C_rP1_{1+r}\right)
\end{array}\right\}*\\
\left(\left(G-\xi\left(\lambda Q_1-RR_1R_2-(R+\lambda)\lambda T_1\right)\right)\right)-\\
-\lambda\xi\left(\lambda U_1+(1-p)RR_1\right)*\left(\begin{array}{l}(B_1+B_2+B_3+B_4)\\ +G\sum_{n=3}^{N}\left\{\rho^{n}+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\end{array}\right)
\end{array}
}{
G\left((B_1+B_2+B_3+B_4)+G\sum_{n=3}^{N}\left\{\rho^{n}+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\right)
}
\qquad (26)
$$

$$P_2 = \frac{\left(\begin{array}{l}\left\{\begin{array}{l}\left(\lambda^2\xi(c_1+c_2)T_1+\lambda(\mu_1+\mu_2)T_2-(\lambda+\xi)(\mu_1+\mu_2)R_1R_2\right)P1_3 \\ +\lambda^2 T_1\,\xi\left(\rho^N\sum_{r=1}^{N-1}c_r+\left((c_1+c_2)\sum_{n=4}^{N-1}+\sum_{r=3}^{N-2}\sum_{n=r+1}^{N-1}c_r\right)P1_n\right)+ \\ \xi(\lambda T_2-(\lambda+\xi)R_1R_2)\left(\rho^N C_{N-2}+\sum_{r=1}^{N-3}C_rP1_{2+r}\right) \\ -\lambda\xi(\lambda+\xi)T_1\left(\rho^N C_{N-1}+\sum_{r=1}^{N-2}C_rP1_{1+r}\right)\end{array}\right\}* \\ \left(\left(G-\xi(\lambda Q_1-RR_1R_2-(R+\lambda)\lambda T_1)\right)\right)- \\ \lambda^2\xi T_1 * \left(\begin{array}{l}(B_1+B_2+B_3+B_4) \\ +G\sum_{n=3}^{N}\left\{\rho^n+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\end{array}\right)\end{array}\right)}{G\left((B_1+B_2+B_3+B_4)+G\sum_{n=3}^{N}\left\{\rho^n+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\right)} \tag{27}$$

$$P_n = \left(\frac{\left\{\rho^n+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}*\left(\left(G-\xi(\lambda Q_1-RR_1R_2-(R+\lambda)\lambda T_1)\right)\right)}{\left((B_1+B_2+B_3+B_4)+G\sum_{n=3}^{N}\left\{\rho^n+\sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n}\ D_i\right\}\right)}\right) \tag{28}$$

4. CONCLUSION

In this paper we consider two heterogeneous servers Markovian queueing system subjected to varying catastrophic intensity. The system size probabilities are calculated explicitly. The concept of varying catastrophic intensity has tremendous applications in vide variety of areas such as computer communications, agriculture and bio-sciences etc.

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