Genetic Algorithm for Constrained Optimization with Stepwise Approach in Search Interval Selection of Variables

Shekhar L. Pandharipande
Associate Professor,
Department of Chemical Engineering, LIT, RTMNU, Nagpur, India

Aarti R. Deshmukh
M.Tech, Department of Chemical Engineering, LIT, RTMNU, Nagpur, India

Rohit P. Kalnake
M.Tech, Department of Chemical Engineering, LIT, RTMNU, Nagpur, India

ABSTRACT
Genetic algorithms are evolutionary algorithms that are well suited in searching global solution to varied nature of optimization problems. The inspirations in developing GA are derived from working principle of natural genetics. The operators such as reproduction, crossover & mutation are employed similar to natural genetics. These steps involved elements of probability that make search for optimal solution random making GA stochastic & nondeterministic. There are several initiatives made by researcher in improving the search direction & making it more definitive. Present work aims at suggesting a novel stepwise approach in search interval selection of variables using Genetic algorithm. Three non-linear optimization problems are selected for numerical experimentation with comparative studies of respective solution using conventional methods and GA techniques with & without stepwise approach.
Test run are conducted with constant GA parameters and the best function values for five consecutive run are tabulated. Corresponding values of variables decide the search interval selection criteria for step 1. Five elite-GA© run are conducted for step 1 for newly defined search interval of variables. The corresponding values of the variables obtained as in step 1 decide the search interval selection for step 2. Number of steps continues till no further improvement in the function values is obtained. Based on the result of the present work it can be concluded that the optimal values obtained for all the three test problems evaluated using the stepwise approach are better than those obtained using GA without stepwise approach & conventional techniques.
The present work is demonstrative & it is felt necessary to substantiate the claim by extending this stepwise search interval approach of GA in selection of variables to other problems of optimization.

Keywords
Genetic algorithm; non-linear optimization problems; stepwise approach; search interval selection.

1. INTRODUCTION
The optimization problems are characterized by objective functions with or without constraints. In constrained optimization there are possibilities of combinations of linear & non-linear objective functions with linear & non-linear constraints. There are various methods reported, which address to these optimization problems. Each method has its limitations and can be applied to certain situations selectively. In addition to this these techniques are found to be inefficient and often arrive at relative optimum that is closest to the starting point. Genetic Algorithm has uniqueness amongst methods of optimization and has emerged as a universal tool that can be can be applied to the various problems of optimization. Where the objective function and constraints are well defined and GA reached the global optimum with high probability[1-3].

2. GENETIC ALGORITHM
Genetic Algorithms (GAs) come under the category of evolutionary algorithms with working principle based on the mechanics of natural genetics. The basic objective in natural genetics is the retention of the fit genes & discard of the redundant. New generations created by manipulating the genetic code using the tools such as selection, crossover & mutation. GA also works in similar manner with the objective to search appropriate solution for the problems involving either minimization or maximization of the objective function. GA use similar tools as selection, crossover & mutation applied to a population of binary strings generated randomly. In every generation, a new set of artificial species or strings is created using bits and pieces of the fittest among old; an occasional new part may be tried for good measure. Genetic algorithms are proven to yield robust search in complex spaces[4].
Many methods, techniques & algorithms have been developed over the decades that are reported in books[1-3] & journals for the optimization problems that are originating from the day to day life & different disciplines including engineering operations[4]. Genetic Algorithm differs from the other optimization and search procedures in following ways:

- The search is carried over a population generated randomly for combination of variables of a possible solution by manipulating their binary coded version
- GA can be seen as universal technique that can address to several types of optimization problems & handle non-linear, complex and noisy functions.
- GA performs global search & very often arrive at or near the global optimum.
- GA does not put prerequisites on function such as smoothness, derivability, and continuity.

2.1 Working principle of GAs:
GA is well suited for both *maximization* and *minimization* of an objective function. The fitness function essentially measures the “goodness” or “quality” of each candidate solution and its magnitude is proportional the fitness of objective function.

A population of genes or binary coded numbers representing the respective variables involved in an objective function is randomly generated. These are arranged to form chromosomes or strings to represent the possible combinations of variables for the solution. The fitness evaluation of each chromosome or the candidate is based on the relative values of their objective function. The stronger strings are selected & the weaker discarded. A new population is generated by combining the strings by swapping their respective parts in a pair using the crossover tool. The mutation is carried on few of the bits of the string population by changing their values from 0 into 1 and vice versa; as the case may be. The fitness test on new generation is carried out & the process is repeated for several generations. The solutions represented by such a new generation chromosomes are likely to be better in terms of their fitness values when compared with those represented by the chromosomes in the current population[4].

The steps involved in developing genetic algorithm are as follows:

2.1.1 Initialization
It is the first step in which a population of suitable pop size of binary strings of suitable chromosome length is created. All the strings are evaluated for their fitness values using specified fitness function. The objective function is interpreted in the light minimization and maximization & becomes the fitness function.

2.1.2 Reproduction
It involves selection of the chromosomes from the current population to form a mating pool for the next generation production. The selection procedure is stochastic wherein fitter chromosomes have a better chance of getting selected.

2.1.3 Crossover
This step results in creating two offspring chromosomes from each parent pair selected randomly. The two parent chromosomes selected are cut at same randomly selected crossover points to obtain two sub-strings per parent string. The second sub-string is then mutually exchanged and combined with the respective first sub-string to form two offspring chromosomes.

2.1.4 Mutation
Among the members of the population generated, randomly as many elements of the offspring are mutated with probability equal to $P_{mut.}$ This is usually very small & avoids creation of entirely different search sub-spaces. This prevents the GA search from becoming absolutely random.

The new population undergoes the fitness test. The steps are repeated & finally, the values of the variables obtained hereby represent the optimized solution.

In one generation crossover and mutation operators are applied only once. Thus generation means how many times the crossover and mutation must operate on the population. Generation is synonymous to iteration.

Unconstrained & constrained are the two broad classifications in optimization. Several optimization techniques reported to solve unconstrained problems[1]. The more complex situation are constrained optimization involving non-linear function with linear & non linear constraints[5-7] & researchers are engaged in suggesting & improving techniques in solving them.

GAs are stochastic search algorithm and can be employed to multiobjective[8-9] & both unconstrained[10-11]-constrained problems[13-17]. Several papers have been reported using GA techniques to solve test problems & problems representing engineering operations[18-24].

GA is a random search method with an element of uncertainty in moving the search direction towards global optimal value. This limitation of GA in handling problems with multi-modal minima or maxima has drawn attention of researchers[25-26]. Several initiatives in incorporating features in modification of GA have been reported[27-30].

Present work proposes stepwise approach in search interval selection of variables in making GA search more definitive towards reaching the optimal value. It also aims at the utility & effectiveness study of new step wise approach to Genetic Algorithm in solving non linear optimization problems with linear constraints.

3. NUMERICAL EXPERIMENTS
Three types of test objective functions have been considered for numerical experimentation & elite-GA© is used for obtaining GA solutions[31]. Table 1 gives the values of the GA parameter like pop size, crossover, mutation & number of iterations that are kept constant for the test & step run of elite-GA© for the numerical experiment. Each step consists of 5 run & each run has 40 iterations. A comparative studies of the optimal values obtained using GA with & without step wise approach with different techniques is carried out.

Table 1. Value of constant of GA parameters for all elite-GA© run

Population size Pop_size	Crossover Probability $P_{crossover}$ (%)	Mutation Probability $P_{mutation}$	Number of Iterations $N_{Iteration}$
20	20	0.001	40

3.1 Non-linear objective function with linear inequality constraints
3.1.1 Function:
Maximize $f(x_1,x_2) = 20\, x_1x_2 + 16x_2 - 2x_1^2 - x^2 - (x_1 + x_2)^2$
Subject to $x_1 + x_2 \le 5$
$0 \le x_1 \le 5$
$0 \le x_2 \le 5$

Conventional technique
The objective function 3.1.1 can be solved using Penalty function method that includes approximation of linear programming[32]. The details are reported in literature.

Genetic Algorithm technique
The present work optimizes the function 3.1.1 in stepwise manner using elite-GA©. The snapshot of the elite-GA© run mode for function 3.1.1 is shown in Fig. 1.

Table 2 gives the details of the best values of the function with the corresponding values of the search variables obtained during consecutive five test run. Based on these values of the objective function obtained the search interval range for the variables is set for carrying out the step 1 run. The values of the search variables set & the best values of the function obtained during the five consecutive run are listed in Table 3. This procedure is followed for the remaining steps.

Table 3 gives the details of search interval set for variables & best values obtained for consecutive five run for steps 1 to 5.

Table 4 gives the details of the best values of the objective function obtained for consecutive 25 run of GA with similar parameters without stepwise approach.

Table 2. Details of elite-GA© test run for function 3.1.1

Run	Limits of x_1		Limits of x_2		Best $f(x_1,x_2)$	x_1	x_2
	Lower	Upper	Lower	Upper		Best value	
1	1.0	5	1.0	5	42.340	3.150	1.273
2	1.0	5	1.0	5	44.516	1.538	3.345
3	1.0	5	1.0	5	44.825	1.785	2.545
4	1.0	5	1.0	5	44.496	2.812	1.734
5	1.0	5	1.0	5	45.837	2.032	2.648

Table 3. Search interval set for variables & best values obtained for consecutive five run for step 1 to 5

\	Steps	Run	Limits of x_1		Limits of x_2		Best $f(x_1,x_2)$	x_1	x_2
			Lower	Upper	Lower	Upper		Best value	
1	**Step 1**	1	1.0	3.5	1.0	3.5	42.697	3.066	1.328
2		2	1.0	3.5	1.0	3.5	46.003	2.571	2.312
3		3	1.0	3.5	1.0	3.5	44.503	2.768	1.733
4		4	1.0	3.5	1.0	3.5	46.283	2.248	2.704
5		5	1.0	3.5	1.0	3.5	45.125	1.960	2.391
6	**Step 2**	1	2.1	2.8	2.1	2.8	46.262	2.232	2.697
7		2	2.1	2.8	2.1	2.8	46.178	2.136	2.776
8		3	2.1	2.8	2.1	2.8	46.236	2.237	2.660
9		4	2.1	2.8	2.1	2.8	46.165	2.189	2.657
10		5	2.1	2.8	2.1	2.8	46.234	2.485	2.485
11	**Step 3**	1	2.2	2.7	2.2	2.7	46.287	2.282	2.661
12		2	2.2	2.7	2.2	2.7	46.214	2.470	2.470
13		3	2.2	2.7	2.2	2.7	46.306	2.299	2.665
14		4	2.2	2.7	2.2	2.7	45.930	2.414	2.308
15		5	2.2	2.7	2.2	2.7	46.193	2.421	2.469
16	**Step 4**	1	2.2	2.3	2.5	2.7	46.294	2.297	2.651
17		2	2.2	2.3	2.5	2.7	46.271	2.286	2.637
18		3	2.2	2.3	2.5	2.7	46.232	2.650	2.621
19		4	2.2	2.3	2.5	2.7	46.274	2.280	2.641
20		5	2.2	2.3	2.5	2.7	46.277	2.285	2.644
21	**Step 5**	1	2.28	2.3	2.65	2.68	46.297	2.297	2.656
22		2	2.28	2.3	2.65	2.68	**46.309**	2.296	2.673
23		3	2.28	2.3	2.65	2.68	46.306	2.298	2.666
24		4	2.28	2.3	2.65	2.68	46.308	2.296	2.671
25		5	2.28	2.3	2.65	2.68	46.296	2.281	2.674

Table 4. Details of best values of objective function obtained for 25 consecutive run

Sr. No.	Run	$f(x_1,x_2)$	Sr. No.	Run	$f(x_1,x_2)$
1	1	42.697	14	14	45.789
2	2	44.516	15	15	45.298
3	3	44.825	16	16	32.576
4	4	44.496	17	17	44.474
5	5	45.837	18	18	42.476
6	6	45.136	19	19	36.140
7	7	37.980	20	20	36.815
8	8	45.425	21	21	45.671
9	9	44.465	22	22	43.574
10	10	**45.938**	23	23	41.696
11	11	34.411	24	24	45.424
12	12	33.854	25	25	42.686
13	13	44.800			

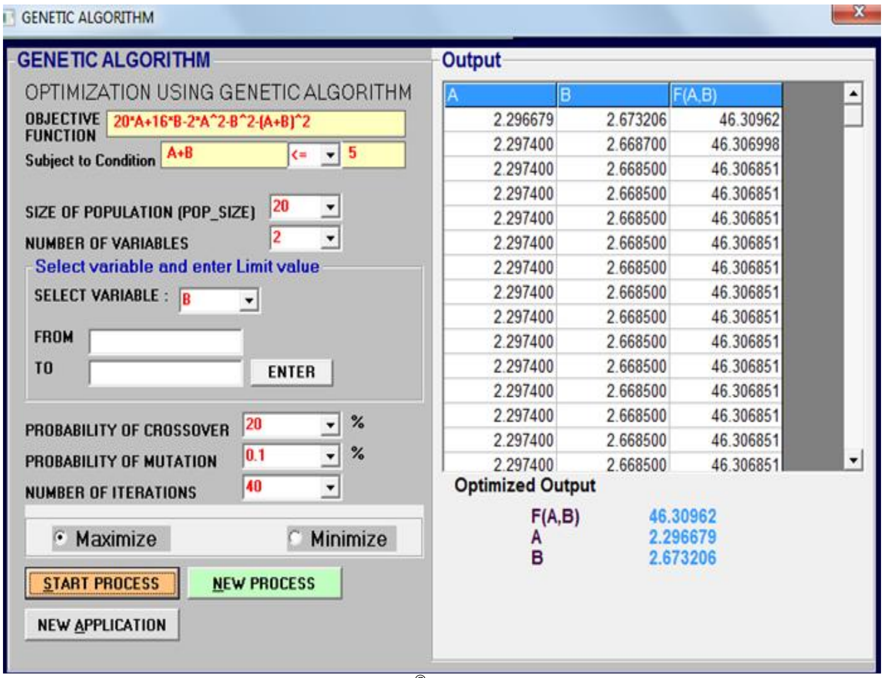

Fig 1: Snapshot of elite-GA© in run mode for function 3.1.1

Figs. 2 & 3 show graphs plotted between the best value of function obtained for 25 run of elite-GA© with & without stepwise approach respectively.

As can be seen from the nature of these graphs the stepwise approach in search interval selection results in directing the search towards optimal value more definitely.

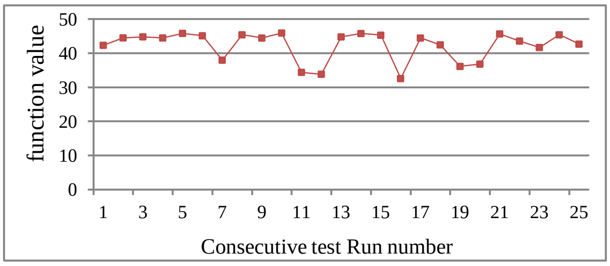

Fig 2: Values of objective function for 25 consecutive run without step wise approach

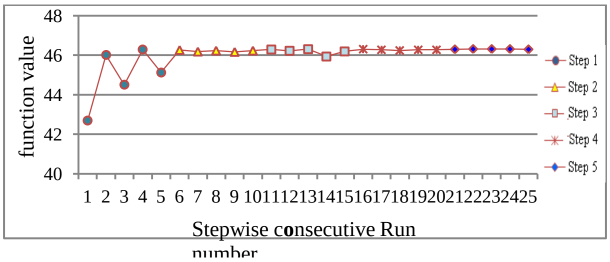

Fig 3: Values of objective function for five consecutive run for each step with stepwise approach

3.1.1.1 Results and Discussion

The objective function discussed in this part of the present work has been solved by Penalty function Constraint Optimization method & is reported in the literature. The comparison between the global values of the objective function obtained using Penalty function method & GA with and without stepwise approach is given in Table 5 & Fig 4 depicts the same. The % deviation in the optimal values obtained using GA technique with stepwise approach is of the order of 0.045% compared with the global value obtained using conventional technique. Thus these optimal solution obtained using GA technique with stepwise approach is acceptable.

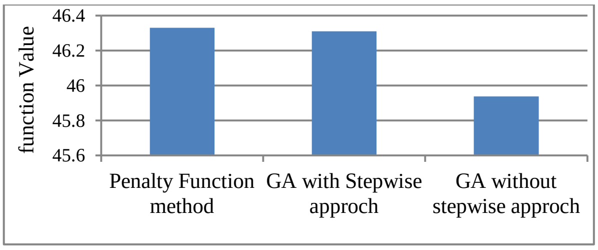

Fig 4: Comparison between global values of function 3.1.1 for conventional and GA with & without stepwise approach

Table 5. Comparison of optimum results obtained by using penalty function method and GA

Maximum value of function	Penalty function Method	GA With stepwise approach	GA without stepwise approach
$f(x_1, x_2)$	46.333	46.309	45.938
Search variables:			
x_1	2.333	2.296	2.514
x_2	2.666	2.673	2.279

3.1.2 Function:

Maximize $f(x_1, x_2) = 5x_1 - x_2^2 + 8x_2 - 2x_2^2$

Subject to $3x_1 + 2x_2 \leq 6$

$0 \leq x_1 \leq 2$

$0 \leq x_2 \leq 3$

Conventional technique

The objective function has been solved by Frank-Wolfe algorithm. The details are reported in the literature[2].

Genetic Algorithm technique
The present work optimizes the function in stepwise manner and elite-GA© is used for this purpose. The snapshot of GA in run mode for function 3.1.2 is shown in Fig. 5.

Table 6 gives the details of the best values of the function with the corresponding values of the search variables obtained during consecutive five test run. Based on the values of the objective function obtained the search interval range for the variables is set for carrying out the step 1 run. The values of the search variables set & the best values of the function obtained during the five consecutive run of steps 1-8 are listed in Table 7.

Table 6. Details of elite-GA© test run for function 3.1.2

Run	Limits of x_1		Limits of x_2		Best $f(x_1,x_2)$	x_1	x_2
	Lower	Upper	Lower	Upper		Best value	
1	0	2	0	3	8.421	0.0859	1.9870
2	0	2	0	3	8.769	0.1614	2.0752
3	0	2	0	3	6.734	1.3276	0.2476
4	0	2	0	3	11.105	1.0888	1.2407
5	0	2	0	3	8.554	1.0909	0.6380

Table 7. Search interval set for variables & best values obtained for consecutive five run for step 1 to 8

Sr. No.	Steps	Run	Limits of x_1		Limits of x_2		Best $f(x_1,x_2)$	x_1	x_2
			Lower	Upper	Lower	Upper		Best value	
1	**Step 1**	1	0	1	1	2	11.144	0.7751	1.745
2		2	0	1	1	2	9.662	0.5797	1.329
3		3	0	1	1	2	10.518	0.5936	1.779
4		4	0	1	1	2	11.077	0.7746	1.686
5		5	0	1	1	2	10.288	0.5961	1.589
6	**Step 2**	1	0.5	1	1.5	2	11.223	0.7819	1.807
7		2	0.5	1	1.5	2	11.156	0.8693	1.534
8		3	0.5	1	1.5	2	11.466	0.9857	1.504
9		4	0.5	1	1.5	2	11.077	0.7887	1.650
10		5	0.5	1	1.5	2	11.421	0.8944	1.645
11	**Step 3**	1	0.7	1	1.5	1.8	11.386	0.9515	1.517
12		2	0.7	1	1.5	1.8	11.254	0.8730	1.582
13		3	0.7	1	1.5	1.8	11.135	0.7879	1.687
14		4	0.7	1	1.5	1.8	11.135	0.8161	1.753
15		5	0.7	1	1.5	1.8	11.124	0.8254	1.598
16	**Step 4**	1	0.8	1	1.5	1.8	11.450	0.9166	1.617
17		2	0.8	1	1.5	1.8	11.369	0.9303	1.543
18		3	0.8	1	1.5	1.8	11.276	0.8434	1.661
19		4	0.8	1	1.5	1.8	11.225	0.8338	1.647
20		5	0.8	1	1.5	1.8	11.361	0.8874	1.620
21	**Step 5**	1	0.9	1	1.4	1.6	11.490	0.9760	1.532
22		2	0.9	1	1.4	1.6	11.443	0.9829	1.497
23		3	0.9	1	1.4	1.6	11.346	0.9815	1.453
24		4	0.9	1	1.4	1.6	11.431	0.9412	1.564
25		5	0.9	1	1.4	1.6	11.447	0.9459	1.560
26	**Step 6**	1	0.95	1	1.5	1.6	11.470	0.9814	1.513
27		2	0.95	1	1.5	1.6	11.462	0.9650	1.535
28		3	0.95	1	1.5	1.6	11.452	0.9698	1.522
29		4	0.95	1	1.5	1.6	11.452	0.9413	1.553
30		5	0.95	1	1.5	1.6	11.455	0.9666	1.529
31	**Step 7**	1	0.95	0.98	1.52	1.54	11.479	0.9798	1.520
32		2	0.95	0.98	1.52	1.54	11.477	0.9789	1.521
33		3	0.95	0.98	1.52	1.54	11.493	0.9718	1.541
34		4	0.95	0.98	1.52	1.54	11.483	0.9722	1.535
35		5	0.95	0.98	1.52	1.54	11.479	0.9746	1.529
36	**Step 8**	1	0.96	0.98	1.53	1.54	**11.494**	0.9782	1.531
37		2	0.96	0.98	1.53	1.54	11.490	0.9740	1.535
38		3	0.96	0.98	1.53	1.54	11.477	0.9725	1.531
39		4	0.96	0.98	1.53	1.54	11.488	0.9767	1.530
40		5	0.96	0.98	1.53	1.54	11.486	0.9754	1.531

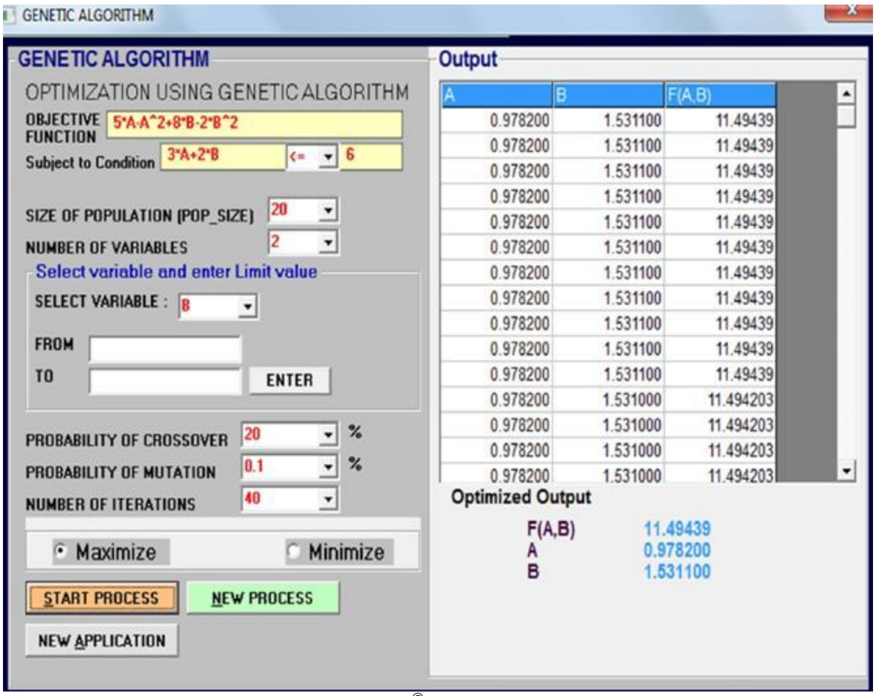

Fig 5: Snapshot of elite-GA© in run mode for function 3.1.2

Table 8 gives the details of the best values of the objective function obtained for consecutive 40 run of GA with similar parameters without stepwise approach.

Table 8. Details of best values of objective function obtained for 40 consecutive run without stepwise approach

Sr. No	Run	$f(x_1,x_2)$	Sr. No.	Run	$f(x_1,x_2)$
1	1	8.421	21	21	10.103
2	2	8.769	22	22	8.190
3	3	6.734	23	23	8.882
4	4	11.105	24	24	11.037
5	5	8.554	25	25	10.571
6	6	9.991	26	26	7.818
7	7	8.529	27	27	9.964
8	8	10.288	28	28	10.673
9	9	6.237	29	29	11.333
10	10	9.688	30	30	9.552
11	11	9.735	31	31	11.224
12	12	5.948	32	32	5.434
13	13	9.886	33	33	9.857
14	14	9.837	34	34	8.598
15	15	9.691	35	35	10.350
16	16	10.936	36	36	9.835
17	17	9.561	37	37	8.488
18	18	10.382	38	38	8.211
19	19	10.156	39	39	10.341
20	20	8.860	40	40	**11.447**

Figs. 6 & 7 show the graphs plotted between the best values of the objective function for 40 run of elite-GA© with & without stepwise approach respectively.

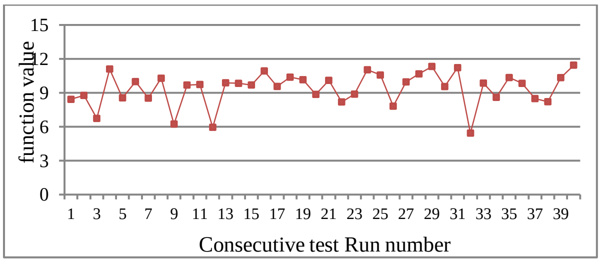

Fig 6: Values of objective function for 40 consecutive run without step wise approach

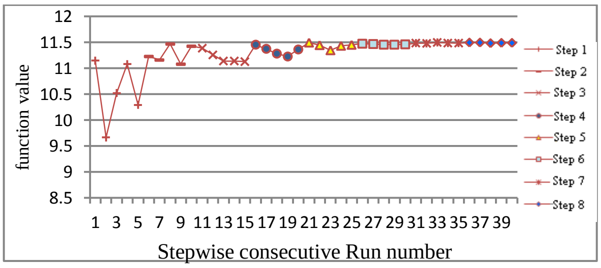

Fig 7: Values of objective function for five consecutive run for each step with stepwise approach

As can be seen from these graphs the stepwise approach in search interval selection results in directing the search towards optimal value more deterministically in this case also.

3.1.2.1 Results and Discussion

Frank-Wolfe algorithm is used as a conventional method for optimizing the given nonlinear objective function 3.1.2. This method is very extensive and time consuming. A large number of iterations are required to reach the optimum solution, & if done manually, would prove to be tedious method.

The comparison of the maximum values of the objective function as obtained by Frank-Wolfe algorithm & GA is given in table 9 & is depicted in Fig. 8. As can be seen from the bar graphics that, GA global optimal value is better than obtained by other methods including Frank-Wolfe algorithm & also with GA without stepwise approach. The % improvement over the conventional technique is 13.99 % hence the GA solution with stepwise approach is acceptable.

Table 9. Comparison of Frank-Wolfe algorithm with GA

Maximum value of function	Frank-Wolfe Algorithm	GA With stepwise approach	GA without stepwise approach
$f(x_1,x_2)$	10.083	11.494	11.447
Search variables:			
x_1	0.8333	0.978	1.025
x_2	1.1666	1.531	1.439

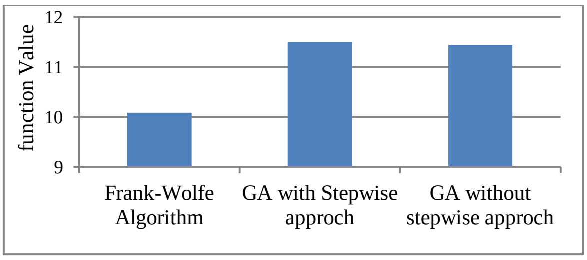

Fig 8; Comparison between values of function 3.1.2 for conventional and GA method

3.2 Non-linear objective function with non-linear inequality constraints

3.2.1 Function:

Maximize $f(x_1, x_2) = x_1x_2$
Subject to $x_1^2 + x_2 \le 3$
$0 \le x_1 \le 1$
\ $0 \le x_2 \le 2$

Conventional technique
The objective function has been solved by Sequential Unconstrained Minimization Technique (SUMT). The details are reported in the literature[2].

Genetic Algorithm technique
The present work optimizes the function 3.2.1 in stepwise manner and using elite-GA©. The snapshot of elite-GA© in run mode for function 3.2.1 is shown in Fig. 9.

Table 10 gives the details of the best values of the function

with the corresponding values of the search variables obtained during consecutive five test run. Based on the values of the objective function obtained the search interval range for the variables is set for carrying out the step 1 run. The values of the search variables set & the best values of the function obtained during the five consecutive run of step 1-6 are listed in table 11.

Table 10. Details of elite-GA© Test run for function 3.2.1

	Limits of x_1		Limits of x_2		Best $f(x_1,x_2)$	x_1	x_2
Run	Lower	Upper	Lower	Upper		Best value	
1	0	1	0	2	0.757	0.574	1.317
2	0	1	0	2	0.537	0.496	1.083
3	0	1	0	2	1.825	0.924	1.974
4	0	1	0	2	1.054	0.554	1.904
5	0	1	0	2	0.434	0.343	1.265

Table 11. Search interval set for variables & best values obtained for consecutive five run for step 1 to 6

Sr. No.	Steps	Run	Limits of x_1		Limits of x_2		Best	x_1	x_2
			Lower	Upper	Lower	Upper	$f(x_1,x_2)$	Best value	
1	**Step 1**	1	0.5	1	1.5	2	1.600	0.843	1.897
2		2	0.5	1	1.5	2	1.590	0.964	1.648
3		3	0.5	1	1.5	2	1.298	0.793	1.636
4		4	0.5	1	1.5	2	1.677	0.900	1.863
5		5	0.5	1	1.5	2	1.884	0.996	1.891
6	**Step 2**	1	0.8	1	1.85	2	1.950	0.988	1.973
7		2	0.8	1	1.85	2	1.860	0.940	1.978
8		3	0.8	1	1.85	2	1.929	0.988	1.951
9		4	0.8	1	1.85	2	1.841	0.926	1.988
10		5	0.8	1	1.85	2	1.963	0.989	1.985
11	**Step 3**	1	0.9	1	1.9	2	1.937	0.976	1.983
12		2	0.9	1	1.9	2	1.973	0.994	1.983
13		3	0.9	1	1.9	2	1.965	0.986	1.991
14		4	0.9	1	1.9	2	1.943	0.989	1.963
15		5	0.9	1	1.9	2	1.862	0.975	1.910
16	**Step 4**	1	0.95	1	1.95	2	1.964	0.991	1.981
17		2	0.95	1	1.95	2	1.964	0.987	1.989
18		3	0.95	1	1.95	2	1.939	0.988	1.961
19		4	0.95	1	1.95	2	1.974	0.998	1.977
20		5	0.95	1	1.95	2	1.979	0.998	1.982

21	**Step 5**	1	0.98	1	1.97	2	1.989	0.999	1.990
22		2	0.98	1	1.97	2	1.986	0.996	1.993
23		3	0.98	1	1.97	2	1.996	0.998	2.000
24		4	0.98	1	1.97	2	1.982	0.995	1.990
25		5	0.98	1	1.97	2	1.982	0.995	1.992
26	**Step 6**	1	0.99	1	1.99	2	1.998	0.999	1.998
27		2	0.99	1	1.99	2	**1.999**	0.999	1.999
28		3	0.99	1	1.99	2	1.985	0.996	1.992
29		4	0.99	1	1.99	2	1.992	0.998	1.996
30		5	0.99	1	1.99	2	1.995	0.999	1.996

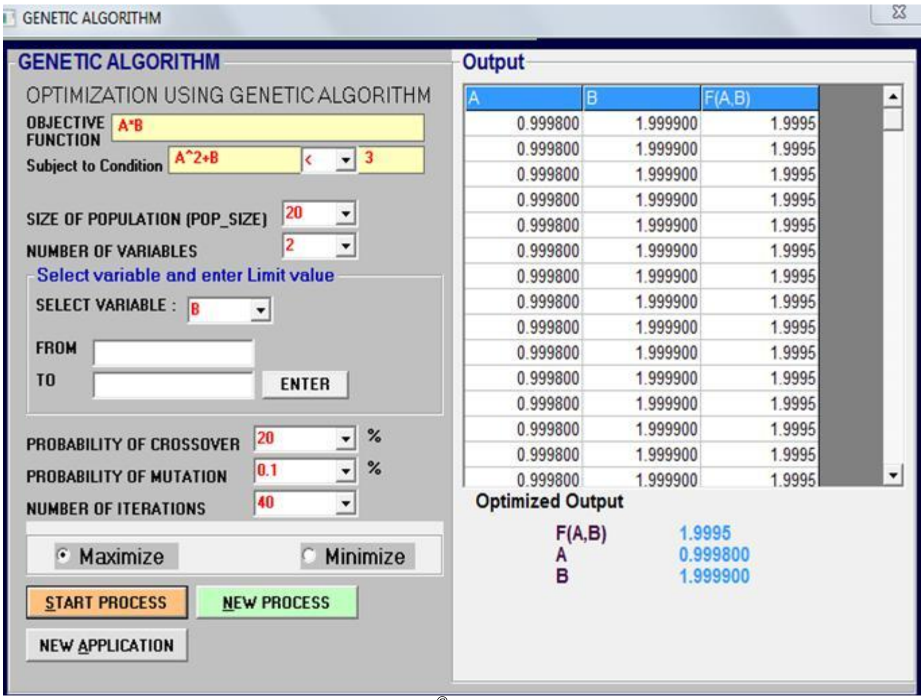

Fig 9: Snapshot of elite-GA© in run mode for function 3.2.1

Table 12 gives the details of the best values of the objective function 3.2.1 obtained for consecutive 30 run of GA with similar parameters without stepwise approach.

Table 12. Details of best values of objective function obtained for 25 consecutive run without step wise approach

Sr. No.	Run	$f(x_1,x_2)$	Sr. No.	Run	$f(x_1,x_2)$
1	1	0.757	16	16	1.193
2	2	0.537	17	17	1.019
3	3	1.825	18	18	0.305
4	4	1.054	19	19	0.984
5	5	0.434	20	20	1.262
6	6	0.548	21	21	0.986
7	7	0.328	22	22	0.196
8	8	1.722	23	23	0.830
9	9	1.732	24	24	0.451
10	10	**1.892**	25	25	1.655
11	11	1.179	26	26	0.380
12	12	1.361	27	27	0.885
13	13	0.814	28	28	0.907
14	14	1.167	29	29	1.725
15	15	0.815	30	30	0.198

Figs. 10 & 11 show the graphs plotted between the best value of function 3.2.1 obtained for 30 run of elite-GA© with & without stepwise approach respectively.

As can be seen from the nature of these graphs the stepwise approach in search interval selection results in directing the search towards optimal value more deterministically.

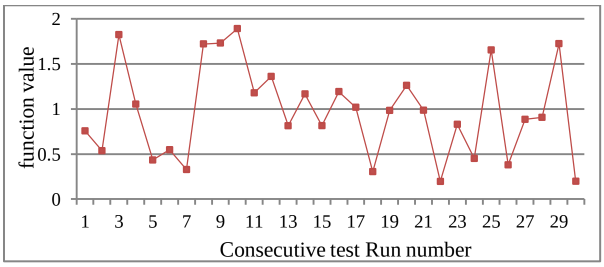

Fig 10: Values of objective function for 30 consecutive run without step wise approach

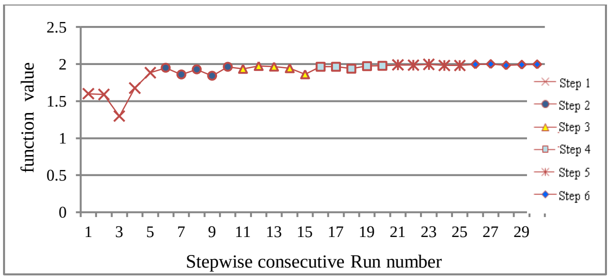

Fig 11: Values of objective function for five consecutive run for each step with stepwise approach

3.2.1.1 Results and Discussion

The objective function 3.2.1 discussed in this part of the present work has been solved by SUMT. Table 13 gives the comparison between the maximum values of the objective function as obtained using SUMT & GA with and without stepwise approach & the Fig. 12 depicts same graphically. It can be seen the table that the best global maximum value obtained & the percentage improvement is of the order of 0.45% over the next global value obtained using SUMT. It can be said that the GA with stepwise approach solution is acceptable.

Table 13. Comparison of optimum results obtained by using Sequential Unconstrained Minimization Technique and GAs

Maximum value of function	SUMT	GA With stepwise approach	GA without stepwise approach
$f(x_1, x_2)$	1.990	1.999	1.892
Search variables:			
x_1	0.998	0.999	0.9662
x_2	1.994	1.999	1.9583

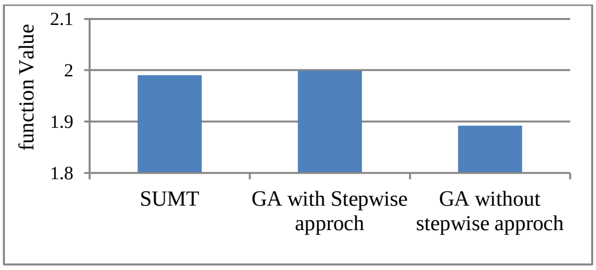

Fig 12: Comparison between values of function 3.2.1 for conventional and GA method

4. CONCLUSION

Genetic algorithm is a random search method with universality of approach in providing optimization solutions. One of the factors that limit the applicability of GA is in its uncertainty to reach optimal solution. Although several conventional optimization techniques have been reported in the literature, most of these techniques are problem specific & selective in nature. The present work has addressed to these limitation of conventional and GA technique & suggested novel stepwise approach in search interval selection of variables. Among the three non-linear optimization problems that are selected for numerical experimentation, two involved linear inequality constraint whereas the remaining has the non-linear inequality constraint. These are reported in literature & solved using Penalty function, Frank-Wolfe algorithm & Sequential unconstrained minimization technique respectively. The limitations of these methods are overcome by using GA with stepwise search interval selection approach. Based on the comparison between the best optimal values using GA with and without step wise approach & the conventional techniques, it can be concluded that the present work has successfully highlighted the utility of the novel approach to GA in providing better optimal solution to test optimization problems.

The present work is demonstrative & it is felt necessary to substantiate the claim by extending this stepwise search interval approach of GA in selection of variables to other problems of optimization.

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