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Coupled Fixed Point Theorem for Generalized Contraction in Complex-Valued Metric Spaces

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ABSTRACT
The aim of this paper is to prove a coupled fixed point theorem for a pair of mappings in complex valued metric space, which generalized the results of Marwan Amin Kutbi et al. [4]

General Terms:
47H10, 54H25

Keywords
Coupled fixed point theorem, Contractive type mapping, Complex valued metric space.

1. INTRODUCTION
In 2011, Azam et al [1] introduced the notion of complex valued metric space and proved a common fixed point theorem for a pair of contractive type mappings involving rational expressions which is a generalization of the classical Banach fixed point theorem. Subsequently, many authors have studied the existence and uniqueness of the fixed point and common fixed point of self-mappings in view of contrasting contractive conditions. Some of these observations are described in [2-6].

In 2006, Bhaskar and Lakshmikantham [9] introduced the concept of coupled fixed point for a given partially ordered set X. Recently, Marwan Amin Kutbi et al. [7] proved common coupled fixed point theorems for generalized contraction in complex valued metric space. In this paper, we proved a coupled fixed point theorem for a pair of mappings in complex valued metric space, which generalized the results of Marwan Amin Kutbi et al. [7].

2. PRELIMINARIES
Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$z_1 \precsim z_2$ if and only if $\mathrm{Re}(z_1) \le \mathrm{Re}(z_2)$, $\mathrm{Im}(z_1) \le \mathrm{Im}(z_2)$.

If follows that $z_1 \precsim z_2$ if one of the following conditions is satisfied

(i) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$, $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$,

(ii) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2)$, $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$,

(iii) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2)$, $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$,

(iv) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$, $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$.

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (i), (ii), and (iii) is satisfied and we will write $z_1 \prec z_2$ if and only if (iii) is satisfied.

REMARK 2.1. The following statements hold:

(i) If $a, b \in \mathbb{R}$ with $a \le b$, then $az \prec bz$, for all $z \in \mathbb{C}$

(ii) If $0 \precsim z_1 \precnsim z_2$, then $|z_1| < |z_2|$.

(iii) If $z_1 \precsim z_2$, $z_2 \prec z_3$, then $z_1 \prec z_3$.

DEFINITION 2.2 ([1]). Let X be a non empty set. Suppose that the mapping $d : X \times X \to \mathbb{C}$, satisfies:

(i) $0 \precsim d(x,y)$, for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$;

(ii) $d(x,y) = d(y,x)$, for all $x, y \in X$;

(iii) $d(x,y) \precsim d(x,z) + d(z,y)$, for all $x, y, z \in X$.

Then d is called a complex valued metric on X, and (X,d) is called a complex valued metric space.

EXAMPLE 2.3. Let $X = \mathbb{C}$. Define the mapping $d : X \times X \to \mathbb{C}$ by $d(z_1, z_2) = i|z_1 - z_2|$ for all $z_1, z_2 \in \mathbb{C}$. Then (X,d) is a complex valued metric space.

DEFINITION 2.4. Let (X,d) be a complex valued metric space and $\{x_n\}$ be a sequence in X.

(i) If for every $c \in \mathbb{C}$ with $0 \prec c$, there is $n_0 \in \mathbb{N}$ such that, for all $n > n_0$, $d(x_n, x) \prec c$, then $\{x_n\}$ is said to be convergent to $x \in X$. We denote this by $x_n \to x$, as $n \to +\infty$ or $\lim_{n\to\infty} x_n = x$.

(ii) If for every $c \in \mathbb{C}$ with $0 \prec c$ there is $n_0 \in \mathbb{N}$ such that, for all $n > n_0$, $d(x_n, x_{n+m}) \prec c$, then $\{x_n\}$ is called a cauchy sequence in (X,d).

(iii) If every cauchy sequence in X is convergent, then (X,d) is said to be a complete complex valued metric space.

LEMMA 2.5 ([1]). Let (X,d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.

LEMMA 2.6 ([1]). Let (X,d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$.

DEFINITION 2.7 ([9]). Let (X,d) be a complex valued metric space. Then an element $(x,y) \in X \times X$ is said to be a common coupled fixed point of $f, g : X \times X \to X$ if

$$x = f(x,y) = g(x,y),$$
$$y = f(y,x) = g(y,x),$$

EXAMPLE 2.8. Let $X = \mathbb{C}$ and define $d : X \times X \to \mathbb{C}$ by $d(z_1, z_2) = i|z_1 - z_2|$ with $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$. Then (X,d) is a complex valued metric space. Consider the mappings $f, g : X \times X \to X$ with $f(z_1, z_2) = \frac{x_1 x_2}{2} i$, $g(z_1, z_2) = \frac{x_1 y_2}{4} i$.

Here $(0,0)$ is the common coupled fixed point of f and g.

3. MAIN RESULTS

THEOREM 3.1. Let (X, d) be a complete complex-valued metric space, and let the mappings $f, g : X \times X \to X$ satisfying the condition

$$\begin{aligned} d(f(x,y), g(u,v)) \P\ & A\frac{d(x,u)+d(y,v)}{2} \\ & + B\frac{d(x, f(x,y))d(u, g(u,v))}{1+d(x,u)+d(y,v)} \\ & + C\frac{d(u, f(x,y))d(x, g(u,v))}{1+d(x,u)+d(y,v)} \\ & + D\frac{d(x, f(x,y))d(x, g(u,v))}{1+d(x,u)+d(y,v)} \\ & + E\frac{d(u, f(x,y))d(u, g(u,v))}{1+d(x,u)+d(y,v)} \qquad \text{(i)} \end{aligned}$$

for all $x, y, u, v \in X$ and A, B, C, D and E are nonnegative reals with $A + B + C + 2D + 2E < 1$. Then f and g have a unique common coupled fixed point.

PROOF. Let x_0 and y_0 be arbitrary points in X.

Define $x_{2k+1} = f(x_{2k}, y_{2k})$, $y_{2k+1} = f(y_{2k}, x_{2k})$ and

$x_{2k+2} = g(x_{2k+1}, y_{2k+1})$, $y_{2k+2} = g(y_{2k+1}, x_{2k+1})$,

for all $k = 0,1,2,3,\ldots$.

Then,

$$\begin{aligned} & d(x_{2k+1}, x_{2k+2}) \\ & = d(f(x_{2k}, y_{2k}), g(x_{2k+1}, y_{2k+1})) \\ & \P\ A\frac{d(x_{2k}, x_{2k+1}) + d(y_{2k}, y_{2k+1})}{2} \\ & + B\frac{d(x_{2k}, f(x_{2k}, y_{2k}))d(x_{2k+1}, g(x_{2k+1}, y_{2k+1}))}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \\ & + C\frac{d(x_{2k+1}, f(x_{2k}, y_{2k}))d(x_{2k}, g(x_{2k+1}, y_{2k+1}))}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \\ & + D\frac{d(x_{2k}, f(x_{2k}, y_{2k}))d(x_{2k}, g(x_{2k+1}, y_{2k+1}))}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \\ & + E\frac{d(x_{2k+1}, f(x_{2k}, y_{2k}))d(x_{2k+1}, g(x_{2k+1}, y_{2k+1}))}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \\ & \P\ A\frac{d(x_{2k}, x_{2k+1}) + d(y_{2k}, y_{2k+1})}{2} \\ & + B\frac{d(x_{2k}, x_{2k+1})d(x_{2k+1}, x_{2k+2})}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \\ & + C\frac{d(x_{2k+1}, x_{2k+1})d(x_{2k}, x_{2k+2})}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \\ & + D\frac{d(x_{2k}, x_{2k+1})d(x_{2k}, x_{2k+2})}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \\ & + E\frac{d(x_{2k+1}, x_{2k+1})d(x_{2k+1}, x_{2k+2})}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \end{aligned}$$

$$\begin{aligned} & \P\ A\frac{d(x_{2k}, x_{2k+1}) + d(y_{2k}, y_{2k+1})}{2} \\ & + B\frac{d(x_{2k}, x_{2k+1})d(x_{2k+1}, x_{2k+2})}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \\ & + D\frac{d(x_{2k}, x_{2k+1})d(x_{2k}, x_{2k+2})}{1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})} \qquad \text{(ii)} \end{aligned}$$

This implies that

$$\begin{aligned} |d(x_{2k+1}, x_{2k+2})| \le & A\frac{|d(x_{2k}, x_{2k+1}) + d(y_{2k}, y_{2k+1})|}{2} \\ & + B\frac{|d(x_{2k}, x_{2k+1})d(x_{2k+1}, x_{2k+2})|}{|1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})|} \\ & + D\frac{|d(x_{2k}, x_{2k+1})d(x_{2k}, x_{2k+2})|}{|1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})|} \qquad \text{(iii)} \end{aligned}$$

Since $|1+d(x_{2k}, x_{2k+1})+d(y_{2k}, y_{2k+1})| > |d(x_{2k}, x_{2k+1})|$, so we get

$$\begin{aligned} |d(x_{2k+1}, x_{2k+2})| \le & \frac{A|d(x_{2k}, x_{2k+1})| + A|d(y_{2k}, y_{2k+1})|}{2} \\ & + B|d(x_{2k+1}, x_{2k+2})| + D|d(x_{2k}, x_{2k+2})| \end{aligned}$$

$$\begin{aligned} |d(x_{2k+1}, x_{2k+2})| \le & \frac{A|d(x_{2k}, x_{2k+1})| + A|d(y_{2k}, y_{2k+1})|}{2} \\ & + B|d(x_{2k+1}, x_{2k+2})| + D|d(x_{2k}, x_{2k+1})| \\ & + D|d(x_{2k+1}, x_{2k+2})| \end{aligned}$$

$$\begin{aligned} (1-B-D)|d(x_{2k+1}, x_{2k+2})| \le & \frac{A+2D}{2}|d(x_{2k}, x_{2k+1})| \\ & + \frac{A}{2}|d(y_{2k}, y_{2k+1})| \end{aligned}$$

$$\begin{aligned} |d(x_{2k+1}, x_{2k+2})| \le & \frac{A+2D}{2(1-B-D)}|d(x_{2k}, x_{2k+1})| \\ & + \frac{A}{2(1-B-D)}|d(y_{2k}, y_{2k+1})| \qquad \text{(iv)} \end{aligned}$$

Similarly,

$$\begin{aligned} |d(y_{2k+1}, y_{2k+2})| \le & \frac{A+2D}{2(1-B-D)}|d(y_{2k}, y_{2k+1})| \\ & + \frac{A}{2(1-B-D)}|d(x_{2k}, x_{2k+1})| \qquad \text{(v)} \end{aligned}$$

Also,

$$\begin{aligned} & d(x_{2k+2}, x_{2k+3}) \\ & = d(g(x_{2k+1}, y_{2k+1}), f(x_{2k+2}, y_{2k+2})) \\ & = d(f(x_{2k+2}, y_{2k+2}), g(x_{2k+1}, y_{2k+1})) \\ & \P\ A\frac{d(x_{2k+2}, x_{2k+1}) + d(y_{2k+2}, y_{2k+1})}{2} \\ & + B\frac{d(x_{2k+2}, f(x_{2k+2}, y_{2k+2}))d(x_{2k+1}, g(x_{2k+1}, y_{2k+1}))}{1+(x_{2k+2}, x_{2k+1})+d(y_{2k+2}, y_{2k+1})} \\ & + C\frac{d(x_{2k+1}, f(x_{2k+2}, y_{2k+2}))d(x_{2k+2}, g(x_{2k+1}, y_{2k+1}))}{1+(x_{2k+2}, x_{2k+1})+d(y_{2k+2}, y_{2k+1})} \\ & + D\frac{d(x_{2k+2}, f(x_{2k+2}, y_{2k+2}))d(x_{2k+2}, g(x_{2k+1}, y_{2k+1}))}{1+(x_{2k+2}, x_{2k+1})+d(y_{2k+2}, y_{2k+1})} \\ & + E\frac{d(x_{2k+1}, f(x_{2k+2}, y_{2k+2}))d(x_{2k+1}, g(x_{2k+1}, y_{2k+1}))}{1+(x_{2k+2}, x_{2k+1})+d(y_{2k+2}, y_{2k+1})} \end{aligned}$$

$$\precsim A\frac{d(x_{2k+2},x_{2k+1})+d(y_{2k+2},y_{2k+1})}{2}$$
$$+B\frac{d(x_{2k+2},x_{2k+3})d(x_{2k+1},x_{2k+2})}{1+(x_{2k+2},x_{2k+1})+d(y_{2k+2},y_{2k+1})}$$
$$+C\frac{d(x_{2k+1},x_{2k+3})d(x_{2k+2},x_{2k+2})}{1+(x_{2k+2},x_{2k+1})+d(y_{2k+2},y_{2k+1})}$$
$$+D\frac{d(x_{2k+2},x_{2k+3})d(x_{2k+2},x_{2k+2})}{1+(x_{2k+2},x_{2k+1})+d(y_{2k+2},y_{2k+1})}$$
$$+E\frac{d(x_{2k+1},x_{2k+3})d(x_{2k+1},x_{2k+2})}{1+(x_{2k+2},x_{2k+1})+d(y_{2k+2},y_{2k+1})} \quad \text{(vi)}$$

This implies that

$$|d(x_{2k+2},x_{2k+3})|$$
$$\le \frac{A}{2}|d(x_{2k+2},x_{2k+1})|+\frac{A}{2}|d(y_{2k+2},y_{2k+1})|$$
$$+B\frac{|d(x_{2k+2},x_{2k+3})|\cdot|d(x_{2k+1},x_{2k+2})|}{|1+d(x_{2k+2},x_{2k+1})+d(y_{2k+2},y_{2k+1})|}$$
$$+E\frac{|d(x_{2k+1},x_{2k+3})\cdot d(x_{2k+1},x_{2k+2})|}{|1+d(x_{2k+2},x_{2k+1})+d(y_{2k+2},y_{2k+1})|} \quad \text{(vii)}$$

Since $|1+d(x_{2k+2},x_{2k+1})+d(y_{2k+2},y_{2k+1})|>|d(x_{2k+1},x_{2k+2})|$, we get

$$|d(x_{2k+2},x_{2k+3})|\le \frac{A}{2}|d(x_{2k+2},x_{2k+1})|$$
$$+\frac{A}{2}|d(y_{2k+2},y_{2k+1})|$$
$$+B|d(x_{2k+2},x_{2k+3})|$$
$$+E|d(x_{2k+1},x_{2k+2})|$$
$$+E|d(x_{2k+2},x_{2k+3})|$$

$$(1-B-E)|d(x_{2k+2},x_{2k+3})|\le \frac{A}{2}|d(x_{2k+2},x_{2k+1})|$$
$$+\frac{A}{2}|d(y_{2k+2},y_{2k+1})|$$
$$+E|d(x_{2k+1},x_{2k+2})|$$

$$|d(x_{2k+2},x_{2k+3})|\le \frac{A+2E}{2(1-B-E)}|d(x_{2k+1},x_{2k+2})|$$
$$+\frac{A}{2(1-B-E)}|d(y_{2k+1},y_{2k+2})| \quad \text{(viii)}$$

Similarly,

$$|d(y_{2k+2},y_{2k+3})|\le \frac{A+2E}{2(1-B-E)}|d(y_{2k+1},y_{2k+2})|$$
$$+\frac{A}{2(1-B-E)}|d(x_{2k+1},x_{2k+2})| \quad \text{(ix)}$$

Adding (iv)-(ix), we get

$$|d(x_{2k+1},x_{2k+2})|+|d(y_{2k+1},y_{2k+2})|$$
$$\le \frac{A+D}{1-B-D}|d(x_{2k},x_{2k+1})|+\frac{A+D}{1-B-D}|d(y_{2k},y_{2k+1})|$$

$$|d(x_{2k+2},x_{2k+3})|+|d(y_{2k+2},y_{2k+3})|$$
$$\le \frac{A+E}{1-B-E}|d(x_{2k+1},x_{2k+2})|$$
$$+\frac{A+E}{1-B-E}|d(y_{2k+1},y_{2k+2})| \quad \text{(x)}$$

On substituting $k=\max\left\{\frac{A+D}{1-B-D},\frac{A+E}{1-B-E}\right\}$, we obtain that

$$|d(x_n,x_{n+1})|+|d(y_n,y_{n+1})|$$
$$\le k(|d(x_{n-1},x_n)|+|d(y_{n-1},y_n)|)$$
$$\le k^2(|d(x_{n-2},x_{n-1})|+|d(y_{n-2},y_{n-1})|)$$
$$\vdots$$
$$\le k^n(|d(x_0,x_1)|+|d(y_0,y_1)|)$$

Without loss of generality, we take $m>n$, since $0<k<1$, so we get

$$|d(x_n,x_m)|+|d(y_n,y_m)|$$
$$\le |d(x_n,x_{n+1})|+|d(y_n,y_{n+1})+\ldots$$
$$+|d(x_{m-1},x_m)|+|d(y_{m-1},y_m)|$$
$$\le k^n(|d(x_0,x_1)+d(y_0,y_1)|)$$
$$+k^{n+1}(|d(x_0,x_1)|+|d(y_0,y_1)|)+\ldots$$
$$+k^{m-1}(|d(x_0,x_1)|+|d(y_0,y_1)|)$$
$$\le (k^n+k^{n+1}+\ldots+k^{m-1})(|d(x_0,x_1)|+|d(y_0,y_1)|)$$
$$\le \frac{k^n}{1-k}(|d(x_0,x_1)|+|d(y_0,y_1)|)\to 0 \text{ as } n\to\infty$$

This implies that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequence in X.

Since X is complete, there exists $x,y\in X$ such that $x_n\to x$ and $y_n\to y$ as $n\to\infty$. Next, we show that $x=f(x,y)$ and $y=f(y,x)$.

Suppose on the contrary that $x\ne f(x,y)$ and $y\ne f(y,x)$ so that $0\prec d(x,f(x,y))=l_1$ and $0\prec d(y,f(y,x))=l_2$,

$$l_1=d(x,f(x,y))$$
$$\precsim d(x,x_{2k+2})+d(x_{2k+2},f(x,y))$$
$$\precsim d(x,x_{2k+2})+d(g(x_{2k+1},y_{2k+1}),f(x,y))$$

$$\precsim d(x,x_{2k+2})+A\frac{d(x_{2k+1},x)+d(y_{2k+1},y)}{2}$$
$$+B\frac{d(x_{2k+1},g(x_{2k+1},y_{2k+1}))d(x,f(x,y))}{1+d(x_{2k+1},x)+d(y_{2k+1},y)}$$
$$+C\frac{d(x,g(x_{2k+1},y_{2k+1}))d(x_{2k+1},f(x,y))}{1+d(x_{2k+1},x)+d(y_{2k+1},y)}$$
$$+D\frac{d(x_{2k+1},g(x_{2k+1},y_{2k+1}))d(x_{2k+1},f(x,y))}{1+d(x_{2k+1},x)+d(y_{2k+1},y)}$$
$$+E\frac{d(x,g(x_{2k+1},y_{2k+1}))d(x,f(x,y))}{1+d(x_{2k+1},x)+d(y_{2k+1},y)}$$

$$\precsim d(x,x_{2k+2})+A\frac{d(x_{2k+1},x)+d(y_{2k+1},y)}{2}$$
$$+B\frac{d(x_{2k+1},x_{2k+2})d(x,f(x,y))}{1+d(x_{2k+1},x)+d(y_{2k+1},y)}$$
$$+C\frac{d(x,x_{2k+2})d(x_{2k+1},f(x,y))}{1+d(x_{2k+1},x)+d(y_{2k+1},y)}$$
$$+D\frac{d(x_{2k+1},x_{2k+2})d(x_{2k+1},f(x,y))}{1+d(x_{2k+1},x)+d(y_{2k+1},y)}$$
$$+E\frac{d(x,x_{2k+2})d(x,f(x,y))}{1+d(x_{2k+1},x)+d(y_{2k+1},y)}$$

$$|l_1| \P ||d(x,x_{2k+2})| + \frac{A}{2}|d(x_{2k+1},x)| + \frac{A}{2}|d(y_{2k+1},y)|$$
$$+ \frac{B|d(x_{2k+1},x_{2k+2})|\cdot|d(x,f(x,y)|}{|1+d(x_{2k+1},x)+d(y_{2k+1},y)|}$$
$$+ \frac{C|d(x,x_{2k+2})|\cdot|d(x_{2k+1},f(x,y)|}{|1+d(x_{2k+1},x)+d(y_{2k+1},y)|}$$
$$+ \frac{D|d(x_{2k+1},x_{2k+2})|\cdot|d(x_{2k+1},f(x,y)|}{|1+d(x_{2k+1},x)+d(y_{2k+1},y)|}$$
$$+ \frac{E|d(x,x_{2k+2})|\cdot|d(x,f(x,y)|}{|1+d(x_{2k+1},x)+d(y_{2k+1},y)|}$$

By taking $k \to \infty$, we get $|d(x,f(x,y))| = 0$, which is a contradiction so that $x = f(x,y)$. Similarly, $y = f(y,x)$. Similarly, it follows that $x = g(x,y)$ and $y = g(y,x)$. This implies that (x,y) is a common coupled fixed point of f and g. Next, we show that f and g have a unique common coupled fixed point. Assume that $(x^*,y^*) \in X$ be another common coupled fixed point of f and g. Then

$$d(x,x^*) = d(f(x,y),g(x^*,y^*))$$
$$\P\ A\frac{d(x,x^*)+d(y,y^*)}{2}$$
$$+ B\frac{d(x,f(x,y))d(x^*,g(x^*,y^*))}{1+d(x,x^*)+d(y,y^*)}$$
$$+ C\frac{d(x^*,f(x,y))d(x,g(x^*,y^*))}{1+d(x,x^*)+d(y,y^*)}$$
$$+ D\frac{d(x,f(x,y))d(x,g(x^*,y^*))}{1+d(x,x^*)+d(y,y^*)}$$
$$+ E\frac{d(x^*,f(x,y))d(x^*,g(x^*,y^*))}{1+d(x,x^*)+d(y,y^*)}$$

$$|d(x,x^*)| \le A\frac{|d(x,x^*)+d(y,y^*)|}{2}$$
$$+ B\frac{|d(x,f(x,y))|\cdot|d(x^*,g(x^*,y^*))|}{|1+d(x,x^*)+d(y,y^*)|}$$
$$+ C\frac{|d(x^*,f(x,y))|\cdot|d(x,g(x^*,y^*))|}{|1+d(x,x^*)+d(y,y^*)|}$$
$$+ D\frac{|d(x,f(x,y))d(x,g(x^*,y^*))|}{|1+d(x,x^*)+d(y,y^*)|}$$
$$+ E\frac{|d(x^*,f(x,y))|\cdot|d(x^*,g(x^*,y^*))|}{|1+d(x,x^*)+d(y,y^*)|}$$

Since $|1+d(x,x^*)+d(y,y^*)| > |d(x,x^*)|$, we get

$$|d(x,x^*)| \le A\frac{|d(x,x^*)+d(y,y^*)|}{2} + C|d(x,x^*)|$$
$$\le \left(\frac{A}{2-A-2C}\right)|d(y,y^*)|$$

Similarly,

$$|d(y,y^*)| \le \left(\frac{A}{2-A-2C}\right)|d(x,x^*)|$$

$$|d(x,x^*)| + |d(y,y^*)|$$
$$\le \left(\frac{A}{2-A-2C}\right)(|d(x,x^*)| + |d(y,y^*)|)$$

which is a contradiction because $A+B+C+2D+2E<1$. Thus we get $x = x^*$, $y = y^*$, which proves the uniqueness of common coupled fixed point of f and g.

COROLLARY 3.2. If f is a self-mapping defined on a complete complex-valued metric space (X,d) satisfying the condition

$$d(f(x,y),f(u,v))$$
$$\P\ A\frac{d(x,u)+d(y,v)}{2}$$
$$+ B\frac{d(x,f(x,y))d(u,f(u,v))}{1+d(x,u)+d(y,v)}$$
$$+ C\frac{d(u,f(x,y))d(x,f(u,v))}{1+d(x,u)+d(y,v)}$$
$$+ D\frac{d(x,f(x,y))d(x,f(u,v))}{1+d(x,u)+d(y,v)}$$
$$+ E\frac{d(u,f(x,y))d(u,f(u,v))}{1+d(x,u)+d(y,v)} \quad \text{for all } x,y,u,v \in X$$

where A,B,C,D and E are nonnegative reals with $A+B+C+2D+2E<1$. Then f has a unique coupled fixed point.

REMARK 3.1.

(i) Theorem 3.1 generalized Theorem 10 of [7] after subsituting $D = E = 0$.
(ii) Theorem 3.1 generalized Corollary 11 of [7] after subsituting $D = E = 0$ and $f = g$.
(iii) Theorem 3.1 generalized Corollary 2.2 of [8] after subsituting $B = C = D = E = 0$ and $g = f$.

4. ACKNOWLEDGEMENT

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