Building Optimum Production Settings using De Novo Programming with Global Criterion Method

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ABSTRACT
This study consists of four main parts. In first part, a brief history of De Novo technique is introduced. In part two, mathematical definitions of Multicriteria De Novo Programming and Global Criterion Method are given with their respective principles. Part three shows a real firm application where the problem and solution parts are shown. Final part of the study concludes the study with explanations and future aims of the study group.

General Terms
De Novo Programming, Multicriteria Decision Making

Keywords
De Novo programming, Global Criterion Method, Optimal System Design.

1. INTRODUCTION
In mathematical techniques, aside objective functions, constraints affect the solution too. Where there are unused resources in a mathematical model, this causes objective functions occur in a lesser value. Thus, it is necessary to use the resources to their full aside the objective functions having the best values. Fully used constraints are named active constraints. In order to make a Linear Programming Problem an optimal one, all constraints have to be active constraints.Using De Novo assumption, it is possible to optimally use the constraints and the objective function takes the best possible value in accordance with directions of objective functions.

Multicriteria De Novo Programming was first studied by Zeleny [13]. According to De Novo assumption, it is possible to build an optimal model in accordance with a pre-defined budget and production means [15]. The most significant feature of the technique is that it creates an optimal system design, instead of optimizing the given system. System design is a process of creation, not selection, of alternatives [16]. The difference between an optimal system and optimizing a system is about using all the constraints to their full capacity [14]. De Novo formulation does not only deal with specifying the best mixture of output, but also with the best mixture of input too**[**11].

Having implied that Multicriteria De Novo Programming problems do not have a definitive solution technique, Zeleny [15] proposed a basic method to construct the optimal system design for solving a De Novo problem. Shi [9] defines six different types of optimum-path ratio. A new approach is also proposed for Multiobjective De Novo Programming [3]. Aside those solution types, fuzzy solutions are available too [7] bound to both positive and negative ideal solutions. Furthermore, Min-Max Goal Programming, bound to positive and negative ideal solutions, are used by Umarusman [12]. Babic and Pavic [1], Shi [10], Chen and Hsieh [5], Huang, *et al*. [6], Zhang *et al*. [17], and Chen and Tzeng [4] have contributed De Novo Programming literature with their studies.

2. GLOBAL CRITERION METHOD FOR DE NOVO TECHNIQUE
In this study, Global Criterion method is used for solving a Multicriteria De Novo Programming problem. Furthermore, traditional solution for De Novo assumption is also done and results are contrasted with the results obtained from Global Criterion method solution. In order to be able to compare the results under the same perception, "p" is taken "1" (p=1) to ensure the function is linear.

2.1 Multicriteria De Novo Programming
In this section, basic formulation of De Novo Programming, as defined by Zeleny [16], is shown. Formulation deals with both maximization and minimization type functions. Multicriteria De Novo Programming is mathematically composed as following:

$$Max\ Z_k = C^1 x$$
$$Min\ W_s = C^2 x$$
Subject to (1)
$$Ax - b \leq 0$$
$$pb \leq B$$
$$x \geq 0,$$

where $Z_k = C^1 x = \sum_{j=1}^{n} c_{kj} x_j$, *k=1,2,…,l*, are *l* objective functions Z_k to be maximized simultaneously. $W_s = C^2 x = \sum_{j=1}^{n} c_{sj} x_j$, s*=1,2,…r*, are *r* objective functions W_s to be minimized simultaneously. $C^1 \in \mathbb{R}^{lxn}$, $C^2 \in \mathbb{R}^{rxn}$ and $A \in \mathbb{R}^{mxn}$ are matrices of dimensions *lxn*, *rxn* and *mxn* respectively. $b \in \mathbb{R}^m$ is the *m*-dimensional unknown resource vector, $p \in \mathbb{R}^m$ is the vector of unit prices of *m* resources, and *B* is the given total budget. Solution of (1) is redefined by the best feasible solution of each objective function under budget restriction and re-defined resources. Each resource's unit price is used for restructuring the problem as a "knapsack" problem:

$$Max\ Z_k = C^1 x$$
$$Min\ W_s = C^2 x$$
Subject to (2)
$$Vx \leq B$$
$$x \geq 0,$$

where $Z_k = (Z_1, \dots, Z_l) \in \mathbb{R}^l$, $W_s = (W_1, \dots, W_r) \in \mathbb{R}^r$ and $V = (V_1, ,, ,. V_n) = pA \in \mathbb{R}^n$. Using the methodology of de novo single-criterion optimal, Problem (2) can be solved, for *x* and *b*, with respect to each to objective functions Z_k and W_s, respectively. Let vector $Z_k^* = (Z_1^*, \dots, Z_l^*)$ and vector $W_s^* = (W_1^*, \dots, W_r^*)$ show the obtained ideal solutions under budget restriction. Obviously, Z_k^* and W_s^* must be attainable for a given budget level *B*.

$$Min\ Vx$$

Subject to (3)

$$C^1 x \le Z_k^*$$
$$C^2 x \le W_s^*$$
$$x \ge 0,$$

Solving (3) identifies the minimum budget B^* at which the metaoptimum performance Z_k^* and W_s^* can be realized through x^* and b^*. Solving problem (3) B^* must exceed any given budget *B*. Optimum-path ration “r” can be used with a pre-defined budget “B”:

$$r = B/B^*. \quad (4)$$

Using “r”, final solution formulations can be defined as: $x = rx^*$, $b = rb^*$, $Z = rZ_k^*$ ve $W = rW_s^*$

2.2 The Global Criterion Method

The Global Criterion Method measures the distance by using Minkowski’s *Lp* metric. In this method, the aim is to minimize a function which defines a global criterion which is a measure of how close the decision maker can get to the ideal solution. Mathematical formulation is as follows [11].

$$F(x) = \sum_{k=1}^{l} \left[\frac{Z_k(x^*) - Z_k(x)}{Z_k(x^*)}\right]^p \quad (5)$$

Where $Z_k(x^*)$ is the value of objective function *l* at its individual optimumx^*, $Z_k(x)$ is the function itself, p $(1 \le p \le \infty)$ is integer valued exponent that serves to reflect the importance objectives. Boychuk and Ovchinnikov[2] suggest using *p*=1, whereas Salukvadze [8] suggests using *p*=2. Setting *p*=1 implies that equal importance is given to all deviations, while *p=2* implies that these deviations are weighted proportionately with the largest deviations having the largest weight. Setting p>2 means that more and more weight is given to the largest of deviations. In addition, where *p*=1 (1.1) function is linear, whereas *p*=2 makes it a non-linear function [11]. In order to keep the function linear, p value is taken 1. Global Criterion Method for minimization objectives can be composed as following:

$$\sum_{s=1}^{r} \left[\frac{W_s(x) - W_s(x^*)}{W_s(x^*)}\right]^p \quad (6)$$

Taking (5) and (6) under consideration together, maximization and minimization objectives can be written as following:

$$F(x) = \sum_{k=1}^{l} \left[\frac{Z_k(x^*) - Z_k(x)}{Z_k(x^*)}\right]^p + \sum_{s=1}^{r} \left[\frac{W_s(x) - W_s(x^*)}{W_s(x^*)}\right]^p \quad (7)$$

Using the constraints of (1) and using (7), the following proposed model can be used for solving the De Novo problem bound to Global Criterion Method:

$$Min\ G = \sum_{k=1}^{l} \left[\frac{Z_k(x^*) - Z_k(x)}{Z_k(x^*)}\right]^p + \sum_{s=1}^{r} \left[\frac{W_s(x) - W_s(x^*)}{W_s(x^*)}\right]^p$$

Subject to (8)

$$Ax - b \le 0$$
$$pb \le B$$
$$x \ge 0,$$

where, $Z_k = (Z_1, \dots, Z_l) \in \mathbb{R}^l$, $W_s = (W_1, \dots, W_r) \in \mathbb{R}^r$ and $V = (V_1, , , , . V_n) = pA \in \mathbb{R}^n, A \in \mathbb{R}^{mxn}$ is matrices of dimension mxn,$b \in \mathbb{R}^m$ is the *m*-dimensional unknown resource vector, $p \in \mathbb{R}^m$ is the vector of unit prices of *m* resources, and *B* is the given total budget.

3. ILLUSTRATIVE EXAMPLE

A production facility that produces four different types of plastic balls is taken for illustrating the problem. Use of raw materials and stock level are given in Table 1. The organization has a monthly budget of $8610 for given materials. Aside with raw material usage constraints, the firm management dictate some additional constraints to both protect the products’ visibility in the market and meet the forecasted market demand:

Table 1. Unists of Raw Material to be Used and Unit Prices

Raw Materials	PB1 (75gr)	PB2 (90gr)	PB3 (125gr)	PB4 (150gr)	Units in Stock	Unit Price ($/kg)
PVC (kg)	33	40	58	66	1000	5,3
DOP (kg)	32	38	56	64	1000	3,2
Paint (kg)	5	6	8	10	100	0,6
Wax (kg)	4	5	7	8	100	0,5

$$(PB_1 + PB_4) - (PB_2 + PB_3) \le 2050,$$
$$PB_1 \ge 3000, PB_2 \le 2000, PB_3 \le 3000,$$
$$PB_4 \ge 3500, PB_2 + PB_3 \ge 4000.$$

Aside with raw material and production constraints, firm management also define income, unit production and cost constraints. For income maximization, each plastic ball provide respectively (1.3;1.4;1.85;3.2), for cost minimization constraint, the respective values are (0.6;0.75;1.1;1.23), and for total unit production, values are (1;1;1;1). The Multi-Functional Linear Programming problem under those constraints occurs as following:

$$Max\ Z_1 = 1.3PB_1 + 1.42PB_2 + 1.85PB_3 + 3.2PB_4$$
$$Max\ Z_2 = PB_1 + PB_2 + PB_3 + PB_4$$
$$Min\ W_1 = 0.6PB_1 + 0.75PB_2 + 1.1PB_3 + 1.23PB_4$$

Subject to; (P1)

$$0.033PB_1 + 0.04PB_2 + 0.058PB_3 + 0.066PB_4 \le 1000$$
$$0.032PB_1 + 0.038PB_2 + 0.056PB_3 + 0.064PB_4 \le 1000$$
$$0.005PB_1 + 0.006PB_2 + 0.008PB_3 + 0.01PB_4 \le 100$$
$$0.004PB_1 + 0.005PB_2 + 0.007PB_3 + 0.008PB_4 \le 100$$
$$(PB_1 + PB_4) - (PB_2 + PB_3) \le 2050 PB_1 \ge 3000\ ;$$
$$PB_2 \le 2000;\ PB_3 \le 3500;\ PB_4 \ge 3500$$
$$PB_2 + PB_3 \ge 4000$$
$$PB_1,\ PB_2, PB_3, PB_4 \ge 0\ and\ integer.$$

In order to be able to solve (P1) with De Novo, the problem is re allocated with (1).

$Max\ Z_1 = 1.3PB_1 + .42PB_2 + 1.85PB_3 + .2PB_4$
$Max\ Z_2 = PB_1 + PB_2 + PB_3 + PB_4$
$Min\ W_1 = 0.6PB_1 + 0.75PB_2 + 1.1PB_3 + 1.23PB_4$

$Subject\ to;$ (P2)

$0.033PB_1 + 0.04PB_2 + 0.058PB_3 + 0.066PB_4 - b_1 \leq 0$
$0.032PB_1 + 0.038PB_2 + 0.056PB_3 + 0.064PB_4 - b_2 \leq 0$
$0.005PB_1 + 0.006PB_2 + 0.008PB_3 + 0.01PB_4 - b_3 \leq 0$
$0.004PB_1 + 0.005PB_2 + 0.007PB_3 + 0.008PB_4 - b_4 \leq 0$
$4.3b_1 + 3.2b_2 + 0.6b_3 + 0.5b_4 \leq 8610$
$(PB_1 + PB_4) - (PB_2 + PB_3) \leq 2050$

$PB_1 \geq 3000$

$PB_2 \leq 2000$

$PB_3 \leq 3500$

$PB_4 \geq 3500$

$PB_2 + PB_3 \geq 4000$
$PB_1, PB_2, PB_3, PB_4, b_1, b_2, b_3, b_4 \geq 0\ and\ integer.$

(P2) solution is conducted by using (1)-(4) for traditional De Novo. (1) or (2) can be used for determining each objective function. Ideal solutions for each objective occur as following:

$Z_1^* = 27674{,}03; PB_1 = 3000;\ PB_2 = 2000;$
$PB_3 = 3500;\ PB_4 = 4550$
$Z_2^* = 13050; PB_1 = 3000;\ PB_2 = 2000;$
$PB_3 = 3500;\ PB_4 = 4550$
$W_1^* = 10300; PB_1 = 3000;\ PB_2 = 2000;$
$PB_3 = 2450;\ PB_4 = 3500$

Clearly each objective is realized in different variable values. Table 2 shows the meta-optimums using (3).

Table 2. Meta-optimum solutions

Variables	Z_1	Z_2	W_1
PB_1	3000	3000	3000
PB_2	2000	2000	2000
PB_3	3500	3500	3500
PB_4	4550	4550	4550
b_1	659.2	659.2	659.2
b_2	682.3	682.3	682.3
b_3	100.5	100.5	100.5
b_4	82,9	82.9	82.9
Objective Function Values	27775	13050	12746.5

Variables that are determined by using De Novo in Table 2 is to be taken under with a budget of \$5119,670. Since $\$5119.670 \leq \$\,8610$, there is no need to determine an optimum path ratio.

Global Criterion method puts objective functions of (P2) by using (7) as following:

$$Min\ G = 1 - [0.0000652PB_1 + 0.0000549\ PB_2 + 0.0000364PB_3 + 0.0000724PB_4]$$

Based on this allocation, with use of (8), De Novo Problem solution Global model is as follows:

$$Max\ G = 0.0000652PB_1 + 0.0000549\ PB_2 + 0.0000364PB_3 + 0.0000724PB_4$$

Subject to (P3)

$0{,}033PB_1 + 0{,}04PB_2 + 0{,}058PB_3 + 0{,}066PB_4 - b_1 \leq 0$

$0{,}032PB_1 + 0{,}038PB_2 + 0{,}056PB_3 + 0{,}064PB_4 - b_2 \leq 0$

$0{,}005PB_1 + 0{,}006PB_2 + 0{,}008PB_3 + 0{,}01PB_4 - b_3 \leq 0$

$0{,}004PB_1 + 0{,}005PB_2 + 0{,}007PB_3 + 0{,}008PB_4 - b_4 \leq 0$

$4{,}3b_1 + 3{,}2b_2 + 0{,}6b_3 + 0{,}5b_4 \leq 8610$

$(PB_1 + PB_4) - (PB_2 + PB_3) \leq 2050$

$PB_1 \geq 3000$

$PB_2 \leq 2000$

$PB_3 \leq 3500$

$PB_4 \geq 3500$
$PB_2 + PB_3 \geq 4000$
$PB_1, PB_2, PB_3, PB_4, b_1, b_2, b_3, b_4 \geq 0\ and\ integer.$

Determined objective function values with (P3) solution occur as: $x^* = (3000; 2000; 3500; 4550)$. The calculated budget determined by given variables is \$5119,670. All calculations are given in Table 3 for illustrating a general summary.

Table 3. Summary

Variables	De Novo Solution			Global Criterion Solution		
	Z_1	Z_2	W_1	Z_1	Z_2	W_1
PB_1	3000			3000		
PB_2	2000			2000		
PB_3	3500			3500		
PB_4	4550			4550		
	Z_1	27775		Z_1	27775	
	Z_2	13050		Z_2	13050	
	W_1	12746,5		W_1	12746,5	

De Novo solution suggestion and Global solution suggestion are seen on the same variable values in Table 3. In Table 4, intial resource values, model suggested values and required budgets are given.

Table 4. Use of Capacity Values

Resources	Initial	De Novo Suggestion	Global Criterion Suggestion
b_1	1000	659.2	659.2
b_2	1000	682.3	682.3
b_3	100	100.5	100.5
b_4	100	82.9	82.9
Budget (\$)	8610	5119.670	5119.670

Inspecting Table 3 and Table 4, it is clear that suggested values of budgets by solutions are lesser from the initial budget. Taking budget in consideration only, there is a \$3470,33 reduction in subject value where all three objective functions are optimally satisfied. In addition, comparing (P2) objective function ideal solutions with meta-optimum and global solutions, ideal solution of all objective functions are satisfied on the Z_1 and Z_2, with same values whereas an increase is observed in W_1.This is a well calculated and expected situation. Goals of meta-optimum and global solutions together reach an optimal solution where all resources are fully used. Because of the characteristics of De Novo assumption, for (P2), ideal solutions for Z_1 and Z_2 occur on same values whereas W_1 occurs on different values. Meta-optimum solution enables all 3 objectives to be solved on the same value. Consequently, an increase in W_1 is unavoidable. Global method's result with p=1 shows that the solution is to occur on either of Z_1, Z_2 , W_1points. Because Global solution is conducted with $Min\, G = 1 - [0.0000652PB_1 + 0.0000549\, PB_2 + 0.0000364PB_3 + 0.0000724PB_4]$ equation, (P3) solution value is 0,56662 with given constraints. As a result, De Novo solution and Global solution both suggest that problem result should occur on the same value.

4. CONCLUSION

As explained in previous sections, budget given for the same level of production is significantly reduced by an improvement in problem constraints. It can be seen that both Global Criterion Method and simple De Novo solutions give the same values. However, eventhough it is no coincidence, this situation is merely a result of the problem design. Optimum solution occurs in first two objective functions for both methods, which results to return of the same value in final calculations. Our study group is aiming to continue studying multicriteria De Novo Programming with and without under Global Criterion Method in future.

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