A New Method of Knowledge Representation based on Simplex

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ABSTRACT
Based on the syllogism of formal logic and triangulation theory of algebraic topology, a new method of knowledge representation with 2-simplex called simplex representation is given in this paper. To show differences, the method is compared with others. The simplex representation has special characteristics in representation abilities, expression ability, and representation modality.

General Terms
Artificial Intelligence, Knowledge Representation

Keywords
Knowledge Representation, Simplex Representation, Assessing.

1. INTRODUCTION
Knowledge representation can be defined as the representation of knowledge in a structured manner. It means the most to artificial intelligence and knowledge management. Many researchers have focused their interests on the knowledge representation and have proposed some approaches which have been utilized widely, such as predicate logic, production rule, frame, script, semantic network and etc. [1, 2, 3]. This paper presents a new method of knowledge presentation by using 2-simplex.

2. THE CONCEPT OF VALUATION SIMPLEX
When people discussed the abstract problem, usually they can discover that, in the concrete space contacted with, most are the sub-spaces of Euclidean space. This is not an accidental phenomenon, because the Euclidean space is the expansion of mankind's three-dimensional space, it has the most actual meaning. In addition, the Euclidean spaces has more abundant geometry structure than the general topology space even the metric space, it can inspire people to study the structure problem concerning space with combination skill.

2.1 The preparative definitions of complex
There are some preparative definitions before the concept of valuation simplex is given [4].

Definition 1 (planoid) Suppose V^q is the q dimensional vector subspace of n dimensional Euclidean space E^n, if a_0 is a given fixed-point of E^n, then $a_0 + V^q = \{a_0 + v \mid v \in V^q\}$ is the subspace of E^n, a q-dimensional planoid of E^n.

Obviously, if $q = 0,1,2$, q-dimensional planoid respectively is a point, a line segment and a plane of E^n, and when $q = n$, n dimensional planoid is E^n.

Definition 2 (geometric noncorrelation) Given q+1 points $A = \{a_0, a_1, \cdots, a_q\}$, if vector { $a_1 - a_0$, $a_2 - a_0$,…, $a_q - a_0$ } is normal noncorrelation, then a set of points A is geometric noncorrelation.

Definition 3 Let { $a_0, a_1, \cdots, a_q$ } be a set of points on the Euclidean space R^n and geometric noncorrelation. Then the set of nonnegative track coordinate of points

$$\left\{ x = \sum_{i=0}^{q} \lambda_i a_i \in P(A) \middle| \sum_{i=0}^{q} \lambda_i = 1,\ \lambda_i \geq 0,\ i = 0,1,\cdots,q \right\}$$

is called q-simplex spanned by $\{a_0, a_1, \cdots, a_q\}$, and denoted by $[a_0, a_1, \cdots, a_q]$. If vertexes are not emphasized, it can also be short for $[s]$. q is called the dimension of the simplex, denoted by $\dim[s] = q$ .The ordered vector $\{\lambda_0, \lambda_1, \cdots, \lambda_q\}$ is still called the track coordinate of

$$x = \sum_{i=0}^{q} \lambda_i a_i$$

barycenter of the point , denoted by $x = (\lambda_0, \lambda_1, \cdots, \lambda_q)$.

That is, a simplex is uniquely spanned by its vertexes. For example, 0-simplex is a point, 1-simplex is a line segment, 2-simplex is a triangle, 3-simplex is a tetrahedroid, and so on.

According to the explanations of informatics[5], information should not be comprehended as the self of things and processes, but the apparent characteristics of things, it's represented by numbers, symbols, figures, and other abstract signs, These signs are called the token field of information, denoted by F, and the elements in F called field value. Based on the simplex in algebraic topology [6], further we have

Definition 4 Suppose $A=\{a_0,a_1,\cdots,a_q\}$ be a set of vertexes of simplex[S], and full mapping $\tau: A\to F$, F∈F, then $\tau(A)$ is called a valuation simplex, denoted by $[\tau(A)]=(\tau(a_0),\tau(a_1),\cdots,\tau(a_q))$.

Obviously, according to the above definitions, assessing is a q-dimensional simplex constitued by objects and rules in intellectualized space, while $q\le 3$,assessing can be denoted to eyeable form by 3-dimensional spaces and arrays or even smaller than 3. Actually, 1-simplex and 2-simplex are enough for the general consequence in real life. This conclusion is given by syllogism in form logics and triangulable theory in algebraic topology.

2.2 The Base of Logic and Mathematics on Simplex Representation

When people solve problems, faced object is always described or abstracted to a system in a certain way first, followed by relevant analysis and study. Now considering thus a problem in contrast: Given a system, can it be resolved to simple components combination in a certain way? The answer should be affirmative, and the triangulable theory in algebraic topology is one of the resolving methods.

(1) Triangulable Theory

Systems have structure, so any system can be abstracted to a geometric pattern with topology structure, denoted by G might as well. To resolve G, based on the knowledge of simplex, generally there is the following definition:

Definition 5 Suppose G is a topology space, if there is a geometric pattern K, its polyhedron |K| correspond with G by a certain homeomorphism,then G is called triangulable space, K is called the triangulation of G.

Sometimes K is also called pure dissection of G, and the homeomorphism of simplex in general is called curl simplex. Specially, the elements constitued pure dissction in 3-dimensional space are all 2-simplex, triangle, this is also the origin of the name “triangulation”. The dissection of the space is not alone generally.

For example, figure 1 represented three triangulations of flat ring

$$\{(x,y)\in E^2 \,\big|\, 1\le x^2+y^2\le 4\}$$

(a) is the triangulation constituted by the curl simplex of flat ring, it contains 6 vertexes, 12 edges and 6 triangles. When the curl part of (a) is straightened, it turns to another triangulation (b). When cut along the edge of (b) $[b_1,b_4]$ and straighten, it turns to triangulation (c), the two side edges $[b_1,b_4]$ is conglutinated in the arrowhead direction.

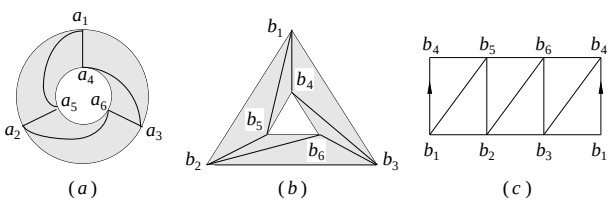

Fig 1: A sketch of triangulation of flat ring

Annulus is also triangulable. Annulus is the bent surface formed by the two round end of column, its triangulation is shown in figure 2.

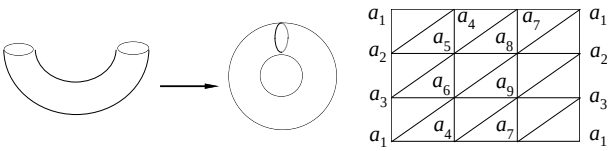

Fig 2: A sketch of triangulation of annulus

(2) Syllogism

The ancient philosopher Aristotle (A.D. 384-322) puts forward some main laws of form logics in his masterwork The Tool Theory, syllogism among them is still the basic basis of numerous reasoning methods.

Syllogism is indirect reasoning constituted by property judgment, among them, the property judgment means judgment that judge whether the thing have a certain property or not. Each property judgment implies a main item and a predicate item, main item means the concept to be judged, but predicate item means the concept that whether the judged object has or not. The indirect reasoning of syllogism is to make two property judgments as the premise to prove another property judgment as the conclusion.

For example:

All metals will expand after being heated

Copper is a metal (1)

So, the copper will expand after being heated

This is a syllogism proposition, the two property judgment premise can be denoted as A and B, the conclusion property judgment as C, now showed in common ways, denoted by

$$A * B \Rightarrow C \qquad (2)$$

"∗" means the connection between A and B, "⇒" means result conversion.

Let A, B and C be the vertexes of a 2-simplex (figure 3a), that is to say, there is a simplex [A, B, C] formally. Further, according to (2) A, B, C can be analyzed:

A: M (metal) be heated P (expand)

B: S (copper) belongs to M (metal) (3)

C: S (copper) be heated P (expand)

Obviously, (3) contains three different contents of M, S, P, if they are taken as three objects, and let φ1 =“be heated”, φ2=“belong to” denote two rules, the corresponding directions can be marked in the edges of assessing [M, S, P], showed as figure 3c.

Through above transformation, it can be found that, whether the expression form or its constitution can be described by a 2-simplex.

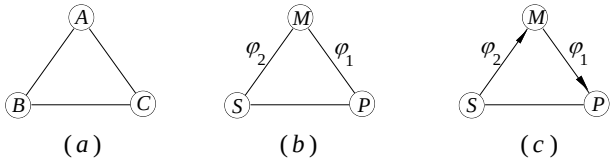

Fig 3: The 2-simplex expression of syllogism

3. THE SIMPLEX REPRESENTATION OF KNOWLEDGE

People ask for help from some abstract ideas (such as length, speed, color etc.) generally to express the basic characteristic of the thing while describe objective thing, some of these concepts are qualitative, some are quantitative, this text call them as indicative attributes generally.

Definition 6 All the constituted parts of described objects or problem solving are called objects.

The apparent index is "a tool" and it expressed a variety of characteristics of an object. Generally, the corresponding relation of objects and index can be divided into two categories: one is called fellow attributes if the same index corresponds to different objects, the other is called fellow objects if the same object corresponds to different indexes.

Let object correspond with points in intellectualized space, the relation between objects correspond with the boundary, when some rules are defined on the boundary, the combination of points and boundary has the general property of reasoning. Now let V and E be the family constituted by all the points and boundaries in intellectualized space, and let R denote the ordered family in the space, to express the relationship between points and boundaries, there is the definition:

Definition 7 Suppose V∈ V, E∈ E. if mapping φ: E→R, then the geometric pattern combined by V and E is called an assessing.

Assessing not only contains known qualities but also unknown qualities, it's a kind of information combination. With the help of rulers and knowledge, it can deduce unknown information of the assessing or renew an assessing by the known assessing or known information of assessing. The multiplying mechanism of the assessing shows its reason capability.

3.1 The method of Simplex Representation

There are three basic elements for simplex representation, namely, that the field value, rules and relationship are necessarily. The relations of the elements can be shown in figure 4.

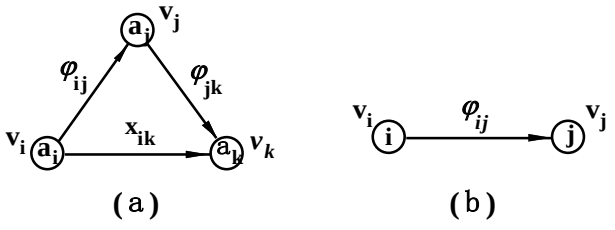

Fig 4: The basic form of simplex representation

Figure 3a shows a 2-simplex, marked [v_i,v_j,v_k], there v_i, v_j and vk is field value of event a_i, a_j and a_k respectively. φ_{ij} and φ_{jk} are the rules of among v_i, v_j and v_k is the transition between a_i and a_k. The line between two points is the direction line, which can be called an arc. Figure 3b shows a 1-simplex [v_i, v_j], it gives a cause and effect.

Under the rules' working, the net is developing with the simplex connecting one by one, and the net can be called the simplex net. In a general way, ○⟶○ is unilateral transition. ○⟷○ is bilateral transition, and it is shortly marked ○——○.

In the field of finance, prices of stocks will fall along with interest rate's rising, and it let stockholders be unhappy. Therefore, the event that "Rising of interest rate let most of stockholders be unhappy" can be given. Figure 5a shows a 2-simplex of the event R is interest rate, S is stock, and M is stockholders. The rule "+" means "Rising", "–" means "Falling".

When expressing the "Copper is a kind of metal", for instance, it can give another simplex in figure 5b, which is a 1-simplex. Co is copper, and Me is metal.

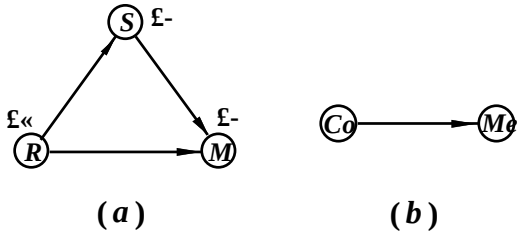

Fig 5: The examples of simplex representation

3.2 Characteristics of simplex representation

Simplex representation is a kind of knowledge representation method that has special features. Analyzed from the structure and methods of simplex representation, it has some characteristics quite obviously.

(1) Unity. The form of simplex is to be led a result by two premises, it has unitive 2-simplex representation method in knowledge representation and reasoning, this indicates that simplex representation implies reasoning.

(2) Concision. Simplex representation accords with the common expression method, "IF- THEN", substantially, this is the expression method that people often use most, intuitionistic, natural, easy to reason, and connected in the way of rules between simplexes, in this way knowledge can be expressed in concise network form of simplex.

(3) Continuity. Simplex representation always starts from a vertex, and then become a network gradually with the basic way that two vertexes make the third vertex, this process expresses the continuity of simplex representation.

4. CONCLUSION

In the artificial intelligence science, reasoning is an eternal topic, it expresses the level and ability of knowledge representation and knowledge application comprehensively. Through research, it can be discovered that, knowledge representation is the realistic problem that any reasoning method can't slide over. Usually their difference is the knowledge representation aspect compared to the already existed reasoning methods, i.e. characteristics of knowledge representation determines the characteristics of reasoning method, so determines the two characteristics of current intelligence reasoning research: one is continuous appearance of new methods and new viewpoints concerning reasoning, the other is that almost all the reasoning methods contain controversy to a certain degree.

In this kind of case, the simplex representation method of knowledge representation is a kind of trial. Although the research contents may be not overall, the thought proposed may be not deep, some results just is elementary, with developing, it is beneficial towards the study and investigation of qualitative problems, where ramification mechanism, unity of knowledge representation and reasoning process, search arithmetic based on potential function etc. are important questions of artificial intelligence, need to be studied further.

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