

Numerical Analysis of Reliability and Availability of the Web based Software System

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ABSTRACT

Today is the world of computers' tasks where least human interventions are required and the software behave like the brain for computers. A small error in the software sub-system can cause a failure in the complete system that leads to disastrous failures which differ in their impact depending on the operations of an organization. Therefore, the analysis of software systems for their reliability and availability is of great significance. Reliability & Availability are the two most important measures for evaluating the quality of the software system and represents user-oriented view of software quality. Now a days Web based software system are the most famous one with the dawn in internet technology. Presently almost every organization is using this software system. This paper describes the numerical analysis of reliability & availability of web based software system based on architecture.

Keywords

Reliability and availability analysis, Software architecture, Software component, Markov process, software quality

1. INTRODUCTION

Software systems are increasingly entering consumers' everyday life. These software systems are a part of almost all the computerized products developed and used by organizations as well as the consumers. Due to the uncertainties associated with parameters like software failure and repair rates, which either cannot be accurately measured in limited time frames through testing, or may vary on different customer sites. Reliability and availability analysis must be able to accommodate the uncertainties and produce meaningful results. Reliability is defined here as the probability of the failure-free operation of a software system for a specified period of time in a specified environment [1]. Availability is used to indicate the probability of a system or equipment being in operating condition at any time t , given that it was in operating condition at $t = 0$. Reliability and availability are often defined as attributes of dependability, which is the ability to deliver service that can justifiably be trusted [2]. With the dawn of internet technology, today web services become the most powerful tool for information sharing & transactions. Here we are analyzing the architecture-based reliability and availability of the Web based software system by using Markov chain process. Section 2 describes the web architecture and its component. As well as we explain in brief the system description, notations and certain assumptions of the present work. In section 3 the mathematical model for the web based software system is derived on the basis of Markov model. After that formulation of Chapman-Kolmogorov differential equation is done for determining the reliability and availability of the web-based software system. The behavior analysis of the system is carried out in section 4 for various combinations of repair and failure rates of the sub systems. The conclusion based on the numerical analysis is finally presented in section 5.

2. OVERVIEW OF WEB BASED SOFTWARE SYSTEM

A web-based software system is an application that uses the internet infrastructure and web technologies to deliver their functionality and can be accessed through the web browser [3]. The software and database reside on a central server rather than being installed on the desktop system and is accessed over a network. Two-tier and three-tier architecture are two commonly used approaches for web based software systems. We are taking three-tier architecture of the web system for analysis.

2.1 Description of Three-Tier Architecture

Most applications deployed on the web, implement a three-tier architecture comprising of a database tier at the bottom, the application tier in the middle and the client tier on top. Three-tier architecture is the most common approach used for web based software systems. Three-tier architecture consists of the following three layers (tiers)

(i) Client-tier: This tier is responsible for the presentation of data, receiving user events and controlling the user interface. The actual business logic is hidden from client tier.

(ii) Application-server-tier: This tier protects the data from direct access by the clients and is not present in two-tier architecture. An application server is a software framework dedicated to the efficient execution of procedures for supporting the construction of applications. It processes the inputs it receives from the clients and interacts with the database. This tier consists of the web server, web scripting language and the scripting language engine.

(iii) Data-server-tier: This tier consists of data base management system which manages the storage, retrieval of data as well as allows simultaneous access, provides security, data integrity and support to the applications. It consists of several components like applications interface or the libraries to communicate with the database management system, structured query language interpreter to check the syntax of the statements, query evaluator for evaluating a query and data access modules that manage access to the data stored on disk which includes a transactions manager, a recovery manager, the main memory buffer manager, data security manager and the file and access method manager.

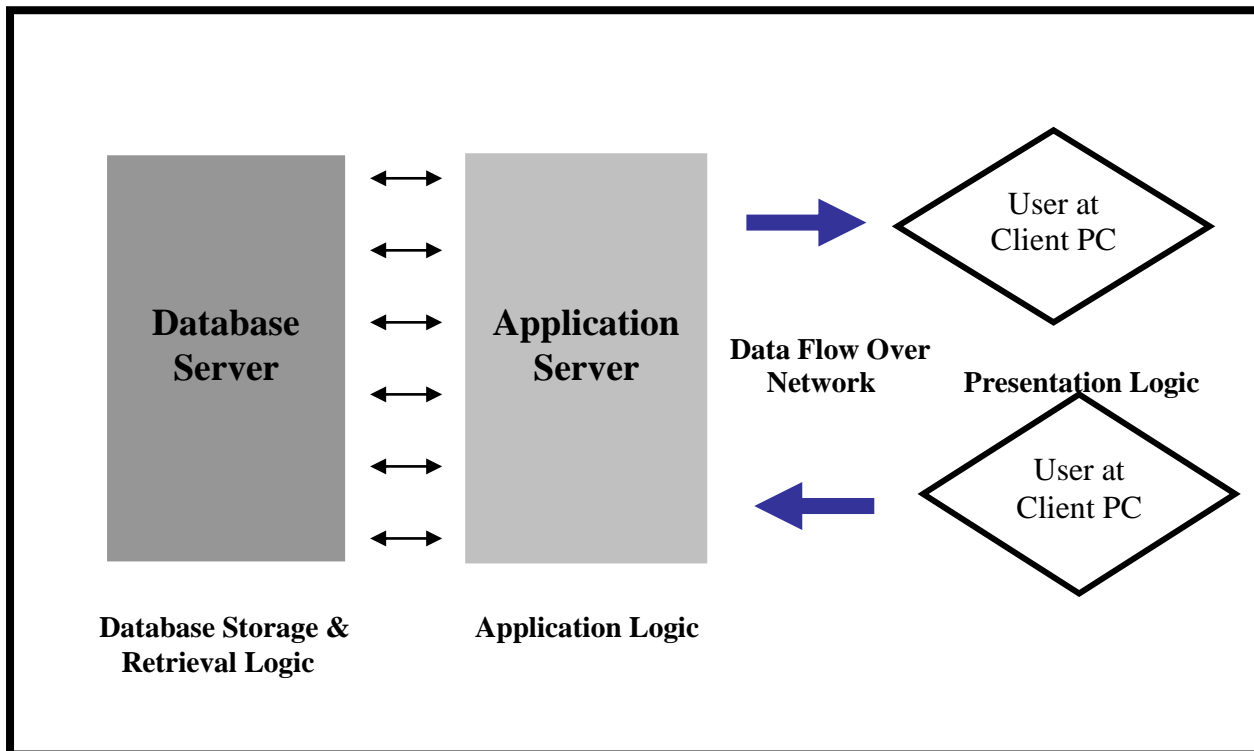


Figure 2.1

2.2 Components of the Web-Based Software System

The web-based software system consists of four main components namely

(i) Application servers (AP): Four application servers namely A1, A2, A3 and A4 are considered here, which are present in the system. The system works in full capacity if all the APs are in working state. As soon as failure occurs in one of the APs the system goes to the reduced state. The user should get the results in some desired amount of time after which the failure in the system is assumed. If the failure occurs in the three or more APs, we assume that the system fails as the user gets the web pages after a long time of wait. So, at least three APs must be running to keep the system working. The system fails if three out of four APs fail.

(ii) Database servers (DB): The two database servers, namely, D1 and D2 are considered here, used for processing various database queries of the system. Both the DBs should be working to get the result of the query in minimum possible time. If one of the DBs fails the performance of the system goes down and the system is said to be working in reduced state. If both DBs fail the system goes to the failed state.

(iii) Routers (RT): The two routers, namely, R1 and R2 are considered here, used for transferring data packets to the destinations based on their addresses. If there is a failure in one of the RTs the availability of the system goes

down and it works in the reduced state. If both the RTs fail the system fails.

(iv) Backbone Networks: we are considering that there is no failure in the backbone networks.

3. MATHEMATICAL FORMULATION OF WEB-BASED SOFTWARE SYSTEM

In this section we are developing the state based Markov model for web based software system on the basis of components of the web system described in section 2.2

3.1 Description of States

The description of states shown in fig. 3.1 is presented in the quadruple form:

(State, number of application servers working, number of database servers working, number of routers working).

We are considering that the system remains in either of three states i.e. Good state, Reduced state or Failed state, depending on the number of database server, application server and routers working. We start with the state 0 when all the **APs, DBs & RTs** are working. The state is represented as **(0,4,2,2)**. If we take a general combination of **DBs, APs & RTs** then total number of possible states would be 45 for Markov model but all of these states are not valid. Out of 45 only 28 states are valid since failure state after failed state is not considered for the model. The state descriptions of other states are shown below

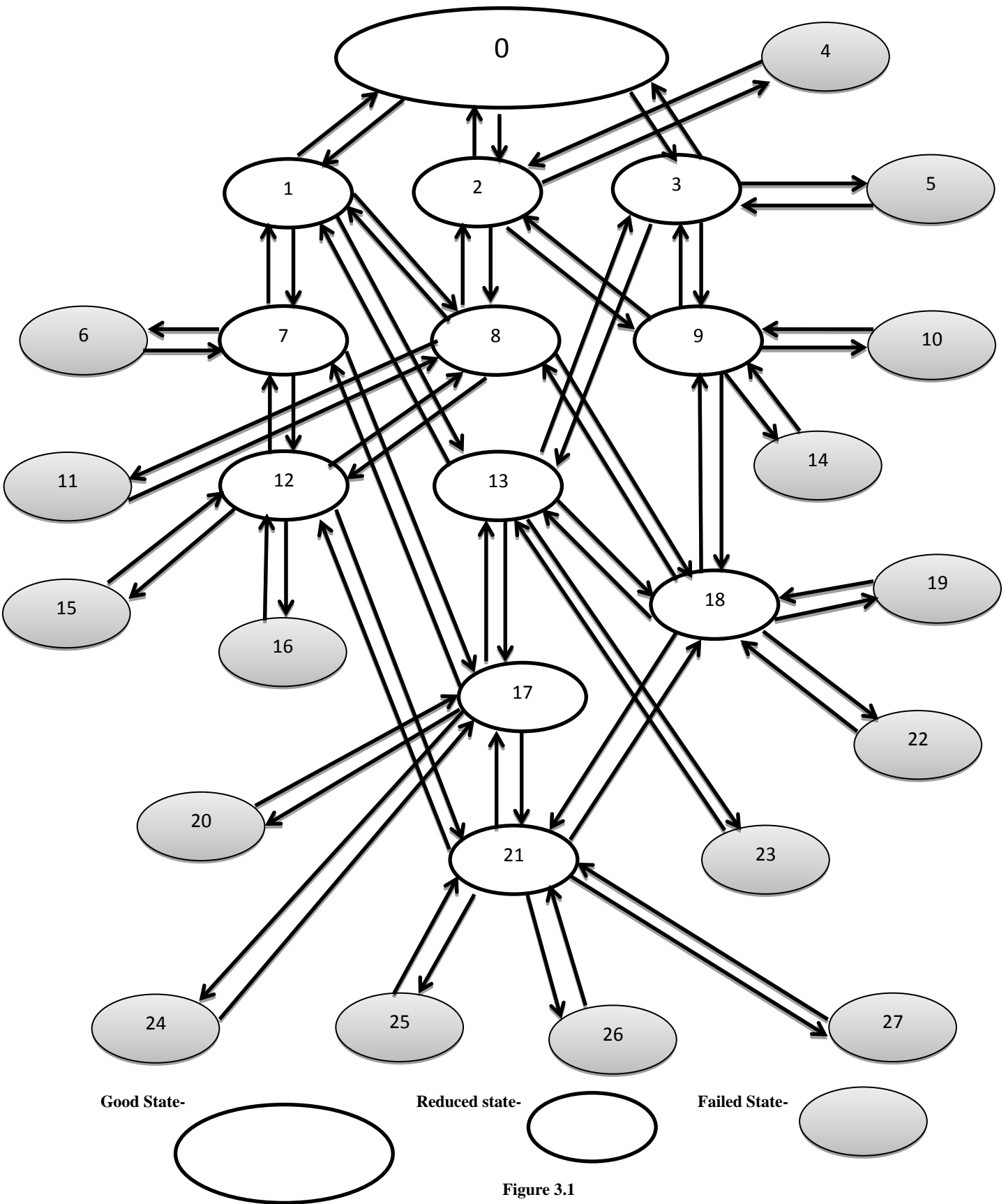


Figure 3.1

(1,3,2,2), (2,4,1,2), (3,4,2,1), (4,4,0,2), (5,4,2,0), (6,1,2,2), (7,2,2,2), (8,3,1,2), (9,4,1,1), (10,4,0,1), (11,3,0,2), (12,2,1,2), (13,3,2,1), (14,4,1,0), (15,2,0,2), (16,1,1,2), (17,2,2,1), (18,3,1,1), (19,3,0,1), (20,2,2,0), (21,2,1,1), (22,3,1,0), (23,3,2,0), (24,1,2,1), (25,1,1,1), (26,2,1,0) and (27,2,0,1).

3.2 Description of Edges

The description about all the edges shown in fig. 3.1 is presented in the following triplet form:

(source node, destination node, weight of the edge)

The weight of the edge is either repair rate or failure rate of a subsystem. Thus the triplet **(0,1,4X₁)** represents that **state 0** is transferred to **state 1** with failure rate **4X₁**. On the basis of the transition diagram, descriptions of other connectors are presented in the following triplet:

(0,2,2X₂), (0,3,2X₃), (1,0,Y₁), (1,7,3X₁), (1,8,2X₂), (1,13,2X₃), (2,0,Y₂), (2,4,X₂), (2,8,4X₁), (2,9,2X₃), (3,0,Y₃), (3,5,X₃), (3,9,2X₂), (3,13,4X₁), (4,2,Y₂), (5,3,Y₃), (6,7,Y₁), (7,1,Y₁), (7,6,2X₁), (7,12,2X₂), (7,17,2X₃), (8,1,Y₂), (8,2,Y₁), (8,11,X₂), (8,12,3X₁), (8,18,2X₃), (9,2,Y₃), (9, 3,Y₂), (9,10,X₂), (9,14,X₃), (9,18,4X₁), (10,9,Y₂), (11,8,Y₂), (12,7,Y₂), (12,8,Y₁), (12,15,X₂), (12,16,2X₁), (12,21,2X₃), (13,1,Y₃), (13,3,Y₁), (13,17,X₁), (13,18,2X₂), (13,23,X₃), (14,9,Y₃), (15,12,Y₂), (16,12,Y₁), (17,7,Y₃), (17,13,Y₁), (17,20,X₃), (17,21,2X₂), (17,24,2X₁), (18,8,Y₃), (18,9,Y₁), (18,13,Y₂), (18,19,X₂), (18,21,3X₁), (18,22,X₃), (19,18,Y₂), (20,17,Y₃), (21,12,Y₃), (21,17,Y₂), (21,18,Y₁), (21,25,2X₁), (21,26,X₃), (21,27,X₂), (22,18,Y₃), (23,13,Y₃), (24,17,Y₁), (25,21,Y₁), (26,21,Y₃) and (27,21,Y₂).

3.3 Notations

P_i(t): Probability that the system is in state i at time t (i = 0, 1, ..., 27).

P'_i(t): Derivative of P_i(t) with respect to time t (i = 0, 1, ..., 27).

X₁: Failure rate of application server caused by fault in software.

Y₁: Software repair rate in application server

X₂: Failure rate of database server caused by fault in software.

Y₂: Software repair rate in database server.

X₃: Failure rate of routers caused by fault in router software.

Y₃: Software repair rate in router.

3.4 Transient State for Reliability Analysis

With probability considerations of various states, in transition diagram shown in fig. 3.1, the following system of linear differential equations is obtained at time t using mnemonic rule. The differential equation of **state 0** is written as
 $P_0(t)(t+\Delta t) = [1-(4X_1+2X_2+2X_3)]P_0(t)\Delta t - Y_1P_1(t)\Delta t + Y_2P_2(t)\Delta t + Y_3P_3(t)\Delta t$

Dividing both sides by Δt and taking limit as $\Delta t \rightarrow 0$, we get

$$P_0'(t) + (4X_1 + 2X_2 + 2X_3)P_0(t) = Y_1P_1(t) + Y_2P_2(t) + Y_3P_3(t) \dots \dots \dots (3.1)$$

Similarly, differential equations for the other states can be written as

$$P_1'(t) + (3X_1 + Y_1 + 2X_2 + 2X_3)P_1(t) = 4X_1P_0(t) + Y_1P_7(t) + Y_2P_8(t) + Y_3P_{13}(t) \dots \dots \dots (3.2)$$

$$P_2'(t) + (Y_2 + 4X_1 + X_2 + 2X_3)P_2(t) = 2X_2P_0(t) + Y_1P_8(t) + Y_3P_9(t) + Y_2P_4(t) \dots \dots \dots (3.3)$$

$$P_3'(t) + (Y_3 + 4X_1 + 2X_2 + X_3)P_3(t) = 2X_3P_0(t) + Y_3P_5(t) + Y_1P_{13}(t) + Y_2P_9(t) \dots \dots \dots (3.4)$$

$$P_4'(t) + Y_2P_4(t) = X_2P_2(t) \dots \dots \dots (3.5)$$

$$P_5'(t) + Y_3P_5(t) = X_3P_3(t) \dots \dots \dots (3.6)$$

$$P_6'(t) + Y_1P_6(t) = 2X_1P_7(t) \dots \dots \dots (3.7)$$

$$P_7'(t) + (2X_1 + 2X_2 + Y_1)P_7(t) = 3X_1P_1(t) + Y_1P_6(t) + Y_2P_{12}(t) + Y_3P_{17}(t) \dots \dots \dots (3.8)$$

$$P_8'(t) + (Y_1 + Y_2 + 3X_1 + X_2 + 2X_3)P_8(t) = 2X_2P_1(t) + 4X_1P_2(t) + Y_1P_{12}(t) + Y_2P_{11}(t) + Y_3P_{18}(t) \dots \dots \dots (3.9)$$

$$P_9'(t) + (Y_3 + Y_2 + 4X_1 + X_2 + X_3)P_9(t) = 2X_3P_2(t) + 2X_2P_3(t) + Y_2P_{10}(t) + Y_3P_{14}(t) + Y_1P_{18}(t) \dots \dots \dots (3.10)$$

$$P_{10}'(t) + Y_2P_{10}(t) = X_2P_9(t) \dots \dots \dots (3.11)$$

$$P_{11}'(t) + Y_2P_{11}(t) = X_2P_8(t) \dots \dots \dots (3.12)$$

$$P_{12}'(t) + (Y_1 + Y_2 + X_2 + 2X_1 + 2X_3)P_{12}(t) = 3X_1P_8(t) + 2X_2P_7(t) + Y_2P_{15}(t) + Y_1P_{16}(t) + Y_3P_{21}(t) \dots \dots \dots (3.13)$$

$$P_{13}'(t) + (Y_3 + Y_1 + 2X_2 + 3X_1 + X_3)P_{13}(t) = 2X_3P_1(t) + 4X_1P_3(t) + Y_2P_{18}(t) + Y_1P_{17}(t) + Y_3P_{23}(t) \dots \dots \dots (3.14)$$

$$P_{14}'(t) + Y_3P_{14}(t) = X_3P_9(t) \dots \dots \dots (3.15)$$

$$P_{15}'(t) + Y_2P_{15}(t) = X_2P_{12}(t) \dots \dots \dots (3.16)$$

$$P_{16}'(t) + Y_1P_{16}(t) = 2X_1P_{12}(t) \dots \dots \dots (3.17)$$

$$P_{17}'(t) + (Y_1 + Y_3 + X_3 + 2X_1 + 2X_2)P_{17}(t) = 2X_3P_7(t) + 3X_1P_{13}(t) + Y_2P_{21}(t) + Y_3P_{20}(t) + Y_1P_{24}(t) \dots \dots \dots (3.18)$$

$$P_{18}'(t) + (Y_1 + Y_2 + Y_3 + 3X_1 + X_2 + X_3)P_{18}(t) = 2X_2P_{13}(t) + 2X_3P_8(t) + 4X_1P_9(t) + Y_2P_{19}(t) + Y_1P_{21}(t) + Y_3P_{22}(t) \dots \dots \dots (3.19)$$

$$P_{19}'(t) + Y_2P_{19}(t) = X_2P_{18}(t) \dots \dots \dots (3.20)$$

$$P_{20}'(t) + Y_3P_{20}(t) = X_3P_{17}(t) \dots \dots \dots (3.21)$$

$$P_{21}'(t) + (Y_1 + Y_2 + Y_3 + 2X_1 + X_2 + X_3)P_{21}(t) = 3X_1P_{18}(t) + 2X_2P_{17}(t) + 2X_3P_{12}(t) + Y_1P_{25}(t) + Y_2P_{27}(t) + Y_3P_{26}(t) \dots \dots \dots (3.22)$$

$$P_{22}'(t) + Y_3 P_{22}(t) = X_3 P_{18}(t) \dots \dots \dots (3.23)$$

$$P_{23}'(t) + Y_3 P_{23}(t) = X_3 P_{13}(t) \dots \dots \dots (3.24)$$

$$P_{24}'(t) + Y_1 P_{24}(t) = 2X_1 P_{17}(t) \dots \dots \dots (3.25)$$

$$P_{25}'(t) + Y_1 P_{25}(t) = 2X_1 P_{21}(t) \dots \dots \dots (3.26)$$

$$P_{26}'(t) + Y_3 P_{26}(t) = X_3 P_{21}(t) \dots \dots \dots (3.27)$$

$$P_{27}'(t) + Y_2 P_{27}(t) = X_2 P_{21}(t) \dots \dots \dots (3.28)$$

With initial condition $P_0(0) = 1$ and $P_j(0) = 0$ for $(j = 1, 2, \dots, 27) \dots \dots \dots (3.29)$

The initial condition is based on the assumption that all the components are in the working state in the beginning. The system of linear differential equations (3.1-3.28) is called Chapman-Kolmogorov differential equation. Once the system of differential equations (3.1-3.28) together with initial condition (3.29) has been solved, the reliability $R(t)$ of the system can be calculated using the following relation

$$R(t) = P_0(t) + P_1(t) + P_2(t) + P_3(t) + P_7(t) + P_8(t) + P_9(t) + P_{12}(t) + P_{13}(t) + P_{17}(t) + P_{18}(t) + P_{21}(t) \dots \dots \dots (3.30)$$

Mean time between failures (MTBF) has been calculated using

$$MTBF = \int_0^t R(t) dt \dots \dots \dots (3.31)$$

3.5 Steady State for Availability Analysis

System analysts are always interested in the long run availability. For this, we need to find the steady state probability of the system which can be obtained by imposing the condition as

$$\frac{d}{dt} \rightarrow 0, \text{ as } t \rightarrow \infty.$$

Thus, the system of linear differential equations (3.1-3.28) now reduces to the following system of linear equations:

$$(4X_1 + 2X_2 + 2X_3)P_0(t) = Y_1 P_1(t) + Y_2 P_2(t) + Y_3 P(t) \dots \dots \dots (3.32)$$

$$(3X_1 + Y_1 + 2X_2 + 2X_3)P_1(t) = 4X_1 P_0(t) + Y_1 P_7(t) + Y_2 P_8(t) + Y_3 P_{13}(t) \dots \dots \dots (3.33)$$

$$(Y_2 + 4X_1 + X_2 + 2X_3)P_2(t) = 2X_2 P_0(t) + Y_1 P_8(t) + Y_3 P_9(t) + Y_2 P_4(t) \dots \dots \dots (3.34)$$

$$(Y_3 + 4X_1 + 2X_2 + X_3)P_3(t) = 2X_3 P_0(t) + Y_3 P_5(t) + Y_1 P_{13}(t) + Y_2 P_9(t) \dots \dots \dots (3.35)$$

$$Y_2 P_4(t) = X_2 P_2(t) \dots \dots \dots (3.36)$$

$$Y_3 P_5(t) = X_3 P_3(t) \dots \dots \dots (3.37)$$

$$Y_1 P_6(t) = 2X_1 P_7(t) \dots \dots \dots (3.38)$$

$$(2X_1 + 2X_2 + Y_1)P_7(t) = 3X_1 P_1(t) + Y_1 P_6(t) + Y_2 P_{12}(t) + Y_3 P_{17}(t) \dots \dots \dots (3.39)$$

$$(Y_1 + Y_2 + 3X_1 + X_2 + 2X_3)P_8(t) = 2X_2 P_1(t) + 4X_1 P_2(t) + Y_1 P_{12}(t) + Y_2 P_{11}(t) + Y_3 P_{18}(t) \dots \dots \dots (3.40)$$

$$(Y_3 + Y_2 + 4X_1 + X_2 + X_3)P_9(t) = 2X_3 P_2(t) + 2X_2 P_3(t) + Y_2 P_{10}(t) + Y_3 P_{14}(t) + Y_1 P_{18}(t) \dots \dots \dots (3.41)$$

$$Y_2 P_{10}(t) = X_2 P_9(t) \dots \dots \dots (3.42)$$

$$Y_2 P_{11}(t) = X_2 P_8(t) \dots \dots \dots (3.43)$$

$$(Y_1 + Y_2 + X_2 + 2X_1 + 2X_3)P_{12}(t) = 3X_1 P_8(t) + 2X_2 P_7(t) + Y_2 P_{15}(t) + Y_1 P_{16}(t) + Y_3 P_{21}(t) \dots \dots \dots (3.44)$$

$$(Y_3 + Y_1 + 2X_2 + 3X_1 + X_3)P_{13}(t) = 2X_3 P_1(t) + 4X_1 P_3(t) + Y_2 P_{18}(t) + Y_1 P_{17}(t) + Y_3 P_{23}(t) \dots \dots \dots (3.45)$$

$$Y_3 P_{14}(t) = X_3 P_9(t) \dots \dots \dots (3.46)$$

$$Y_2 P_{15}(t) = X_2 P_{12}(t) \dots \dots \dots (3.47)$$

$$Y_1 P_{16}(t) = 2X_1 P_{12}(t) \dots \dots \dots (3.48)$$

$$(Y_1 + Y_3 + X_3 + 2X_1 + 2X_2)P_{17}(t) = 2X_3 P_7(t) + 3X_1 P_{13}(t) + Y_2 P_{21}(t) + Y_3 P_{20}(t) + Y_1 P_{24}(t) \dots \dots \dots (3.49)$$

$$(Y_1 + Y_2 + Y_3 + 3X_1 + X_2 + X_3)P_{18}(t) = 2X_2 P_{13}(t) + 2X_3 P_8(t) + 4X_1 P_9(t) + Y_2 P_{19}(t) + Y_1 P_{21}(t) + Y_3 P_{22}(t) \dots \dots \dots (3.50)$$

$$Y_2 P_{19}(t) = X_2 P_{18}(t) \dots \dots \dots (3.51)$$

$$Y_3 P_{20}(t) = X_3 P_{17}(t) \dots \dots \dots (3.52)$$

$$(Y_1 + Y_2 + Y_3 + 2X_1 + X_2 + X_3)P_{21}(t) = 3X_1 P_{18}(t) + 2X_2 P_{17}(t) + 2X_3 P_{12}(t) + Y_1 P_{25}(t) + Y_2 P_{27}(t) + Y_3 P_{26}(t) \dots \dots \dots (3.53)$$

$$Y_3 P_{22}(t) = X_3 P_{18}(t) \dots \dots \dots (3.54)$$

$$Y_3 P_{23}(t) = X_3 P_{13}(t) \dots \dots \dots (3.55)$$

$$Y_1 P_{24}(t) = 2X_1 P_{17}(t) \dots \dots \dots (3.56)$$

$$Y_1 P_{25}(t) = 2X_1 P_{21}(t) \dots \dots \dots (3.57)$$

$$Y_3 P_{26}(t) = X_3 P_{21}(t) \dots \dots \dots (3.58)$$

$$Y_2 P_{27}(t) = X_2 P_{21}(t) \dots \dots \dots (3.59)$$

The system of linear equations (3.32-3.59) together with the normalizing condition

$$\sum_{i=0}^{27} P_i = 1$$

can be solved to find the unknown $P_i(t)$ ($i = 1, \dots, 27$). Once these unknowns are known, the system availability $A(\infty)$ can be calculated using the following relation

$$A(\infty) = P_0 + P_1 + P_2 + P_3 + P_7 + P_8 + P_9 + P_{12} + P_{13} + P_{17} + P_{18} + P_{21} \dots \dots \dots (3.60)$$

4. BEHAVIOUR ANALYSIS OF WEB-BASED SYSTEM

4.1 Transient State

In this section we have studied the effect of software failure and repair rates of application server, database server and routers on the reliability of the system following the approach of Gupta et al. [13]. The system of differential equation (3.1-3.28) together with initial condition (3.29) has been solved numerically using Runge-Kutta fourth order method, assuming step size $h=0.005$ as one hour and finally computed reliability of the system using relation (3.30) for various combination of failure and repair rates of the subsystems. The data for failure and repair rates of the various subsystems is the actual data taken in the units of per hour. The MTBF is finally computed from the equation (3.31) using Simpson rule.

4.1.1 Variation in the reliability of the system with the change in software failure rates of Application server

The reliability of the system has been calculated for various values of software failure rates ($X_1=0.02, 0.03, 0.04, 0.05, 0.06$) of the Application server and keeping other parameters: $X_2=0.03, X_3=0.01, Y_1=1, Y_2=3, Y_3=2$ fixed. We have also computed MTBF of the system and the results are presented in Table 4.1.

Table 4.1

Time (hrs.)	$X_1=0.02$	$X_1=0.03$	$X_1=0.04$	$X_1=0.05$	$X_1=0.06$
50	0.99971	0.99953	0.99931	0.99891	0.99835
100	0.99961	0.99931	0.99873	0.99793	0.99661
150	0.99959	0.99921	0.99852	0.99749	0.99605
200	0.99958	0.99919	0.99848	0.99735	0.99582
250	0.99958	0.99918	0.99845	0.99733	0.99571
300	0.99958	0.99918	0.99845	0.99732	0.99573
350	0.99958	0.99918	0.99845	0.99733	0.99572
400	0.99958	0.99918	0.99846	0.99732	0.99574
450	0.99958	0.99918	0.99846	0.99733	0.99574
500	0.99958	0.99918	0.99846	0.99733	0.99574
MTBF	433.155	432.996	432.711	432.250	431.613

4.1.2 Variation in the reliability of the system with the change in software failure rates of database server

The reliability of the system has been calculated for various values of software failure rates ($X_2=0.01, 0.02, 0.03, 0.04, 0.05$) of the database server and keeping other parameters: $X_1=0.04, X_3=0.01, Y_1=1, Y_2=3, Y_3=2$ fixed. We have also computed MTBF of the system and the results are presented in Table 4.2.

Table 4.2

Time (hrs.)	$X_2=0.01$	$X_2=0.02$	$X_2=0.03$	$X_2=0.04$	$X_2=0.05$
50	0.99947	0.99944	0.99932	0.99916	0.99896
100	0.99893	0.99885	0.99876	0.99862	0.99843
150	0.99871	0.99866	0.99855	0.99838	0.99821
200	0.99866	0.99858	0.99847	0.99832	0.99813
250	0.99863	0.99858	0.99846	0.99830	0.99812
300	0.99863	0.99856	0.99845	0.99830	0.99811
350	0.99863	0.99856	0.99845	0.99831	0.99811
400	0.99863	0.99856	0.99846	0.99831	0.99811
450	0.99863	0.99856	0.99846	0.99831	0.99811
500	0.99863	0.99856	0.99846	0.99831	0.99811
MTBF	432.782	432.752	432.704	432.640	432.558

4.1.3 Variation in the reliability of the system with the change in software failure rates of routers

The reliability of the system has been calculated for various values of software failure rates ($X_3=0.01, 0.02, 0.03, 0.04, 0.05$) of the routers and keeping other parameters: $X_1=0.04, X_2=0.03, Y_1=1, Y_2=3, Y_3=2$ fixed. We have also computed MTBF of the system and the results are presented in Table 4.3.

Table 4.3

Time (hrs.)	$X_3=0.01$	$X_3=0.02$	$X_3=0.03$	$X_3=0.04$	$X_3=0.05$
50	0.99932	0.99918	0.99896	0.99865	0.99829
100	0.99876	0.99863	0.99837	0.99805	0.99764
150	0.99855	0.99840	0.99816	0.99781	0.99741
200	0.99847	0.99835	0.99809	0.99776	0.99735
250	0.99846	0.99833	0.99807	0.99774	0.99732
300	0.99845	0.99834	0.99807	0.99775	0.99731
350	0.99845	0.99833	0.99807	0.99774	0.99733
400	0.99846	0.99833	0.99807	0.99774	0.99733
450	0.99846	0.99833	0.99807	0.99774	0.99733
500	0.99846	0.99833	0.99807	0.99774	0.99733
MTBF	432.704	432.644	432.539	432.395	432.215

4.1.4 Variation in the reliability of the system with the change in software repair rates of Application server

The reliability of the system has been calculated for various values of software repair rates ($Y_1=1, 1.1, 1.2, 1.3, 1.4$) of the Application server keeping other parameters: $X_1=0.04, X_2=0.03, X_3=0.01, Y_2=3, Y_3=2$ fixed. We have also computed MTBF of the system and the results are presented in Table 4.4.

Table 4.4

Time (hrs.)	$Y_1 = 1.0$	$Y_1 = 1.1$	$Y_1 = 1.2$	$Y_1 = 1.3$	$Y_1 = 1.4$
50	0.99932	0.99937	0.99941	0.99945	0.99949
100	0.99876	0.99896	0.99909	0.99922	0.99931
150	0.99855	0.99881	0.99901	0.99915	0.99926
200	0.99847	0.99876	0.99897	0.99914	0.99925
250	0.99846	0.99878	0.99898	0.99913	0.99926
300	0.99845	0.99876	0.99898	0.99914	0.99926
350	0.99845	0.99876	0.99898	0.99914	0.99926
400	0.99846	0.99877	0.99898	0.99914	0.99926
450	0.99846	0.99877	0.99898	0.99914	0.99926
500	0.99846	0.99877	0.99898	0.99914	0.99926
MTBF	432.704	432.822	432.908	432.972	433.021

4.1.5 Variation in the reliability of the system with the change in software repair rates of database server

The reliability of the system has been calculated for various values of software repair rates ($Y_2=3, 3.1, 3.2, 3.3, 3.4$) of the database server keeping the other parameters fixed: $X_1=0.04, X_2=0.03, X_3=0.01, Y_1=1, Y_3=2$. We have also computed MTBF of the system and the results are presented in Table 4.5.

Table 4.5

Time (hrs.)	$Y_2 = 3.0$	$Y_2 = 3.1$	$Y_2 = 3.2$	$Y_2 = 3.3$	$Y_2 = 3.4$
50	0.99934	0.99934	0.99934	0.99935	0.99936
100	0.99876	0.99877	0.99879	0.99881	0.99881
150	0.99855	0.99855	0.99856	0.99858	0.99858
200	0.99848	0.99848	0.99851	0.99851	0.99853
250	0.99846	0.99847	0.99848	0.99849	0.99851
300	0.99845	0.99846	0.99847	0.99849	0.99852
350	0.99846	0.99847	0.99848	0.99849	0.99851
400	0.99846	0.99847	0.99848	0.99849	0.99851

450	0.99846	0.99847	0.99848	0.99849	0.99851
500	0.99846	0.99847	0.99848	0.99849	0.99851
MTBF	432.705	432.711	432.715	432.719	432.723

4.1.6 Variation in the reliability of the system with the change in software repair rates of routers

The reliability of the system has been calculated for various values of software repair rates ($Y_3=2, 2.1, 2.2, 2.3$) of the router keeping the other parameters: $X_1=0.04, X_2=0.03, X_3=0.01, Y_1=1, Y_2=3$ fixed. We have also computed MTBF of the system and the results are presented in Table 4.6.

Table 4.6

Time (hrs.)	$Y_3 = 2.0$	$Y_3 = 2.1$	$Y_3 = 2.2$	$Y_3 = 2.3$
50	0.999323	0.999326	0.999329	0.999332
100	0.998767	0.998772	0.998776	0.998779
150	0.998546	0.998551	0.998555	0.998558
200	0.998479	0.998484	0.998488	0.998491
250	0.998461	0.998466	0.998470	0.998473
300	0.998457	0.998461	0.998466	0.998469
350	0.998467	0.998462	0.998466	0.998469
400	0.998460	0.998464	0.998469	0.998472
450	0.998460	0.998465	0.998469	0.998472
500	0.998460	0.998465	0.998469	0.998472
MTBF	432.7049	432.7066	432.7082	432.7089

4.2 Steady State

In this section we have studied the effect of software failure and repair rates of application server, database server and routers on the availability of the system following the approach of Gupta et al. [13]. The system of linear equation (3.32-3.59) together with the normalizing condition

$$\sum_{i=0}^{27} P_i = 1$$

have been solved using Gauss Jacobi method. Finally, the availability of the system has been calculated using relation (3.60) for various combinations of failure and repair rates of the subsystems. The data for failure and repair rates of the various subsystems is the actual data taken in the units of per hour.

4.2.1 Sensitivity analysis of availability of system to software failure rates of application server and database server

The availability of the system has been calculated for various values of software failure rates of application server and database server keeping the other parameters: $X_3= 0.01, Y_1=1, Y_2=3, Y_3=2$ fixed. The values of X_1 are taken as: 0.03, 0.04, 0.05, 0.06 and that of X_2 as: 0.01, 0.02, 0.03, and 0.04 and the results are presented in Table 4.7.

Table 4.7

Availability	$X_2=0.01$	$X_2=0.02$	$X_2=0.03$	$X_2=0.04$	% Variation
$X_1=0.03$	0.9994	0.9993	0.9992	0.9990	0.0324
$X_1=0.04$	0.9986	0.9986	0.9985	0.9983	0.0323
$X_1=0.05$	0.9975	0.9974	0.9973	0.9972	0.0323
$X_1=0.06$	0.9959	0.9958	0.9957	0.9956	0.0320
% Variation	0.3455	0.3452	0.3453	0.3451	

4.2.2 Sensitivity analysis of availability of system to software failure rates of application server and router software

The availability of the system has been calculated for various values of software failure rates of application server and router software keeping the other parameters: $X_2=0.03,$

$Y_1=1, Y_2=3, Y_3=2$ fixed. The values of X_1 are taken as: 0.03, 0.04, 0.05, 0.06 and that of X_3 as: 0.01, 0.02, 0.03, and 0.04 and the results are presented in Table 4.8.

Table 4.8

Availability	$X_3=0.01$	$X_3=0.02$	$X_3=0.03$	$X_3=0.04$	% Variation
$X_1=0.03$	0.9992	0.9990	0.9988	0.99846	0.0720
$X_1=0.04$	0.9985	0.9983	0.9981	0.99774	0.0717
$X_1=0.05$	0.9973	0.9972	0.9969	0.99661	0.0717
$X_1=0.06$	0.9957	0.9956	0.9954	0.99502	0.0716
% Variation	0.3453	0.3452	0.3449	0.34493	

5. CONCLUDING OBSERVATIONS

The variation in the reliability and availability of the system with the change in software failure and repair rates of application server, database server and routers is presented in the Tables 4.1-4.8. The results given in the Table 4.1 reveals that MTBF of the system decreases by approximately 0.35% and the reliability of the system decreases by approximately 0.13% with the increase in the software failure rate of application server from 0.02 to 0.06 and the reliability decreases by approximately 0.012% with increase in time from 50 to 500 hours. The results given in the Table 4.2 show that, MTBF of the system and the reliability of the system decrease by approximately 0.05% with the increase in the software failure rates of database server from 0.01 to 0.05 and the reliability decrease by approximately 0.086% with increase in time from 50 to 500 hours. Table 4.3 reveals that MTBF of the system decreases by approximately 0.11% and the reliability of the system decreases by approximately 0.1% with the increase in the software failure rate of router software from 0.01 to 0.05 and the reliability decreases by approximately 0.086% with increase in time from 50 to 500 hours. It can be noted from Table 4.4 that MTBF of the system increases by approximately 0.072% and the reliability of the system increases by approximately 0.017% with the increase in the software repair rate of application server from 1 to 1.4 and the reliability decreases by approximately 0.086% with increase in time from 50 to 500 hours. Table 4.5 reveals that MTBF of the system increases by approximately 0.0041% and the reliability of the system increases by approximately 0.0041% with the increase in the software repair rate of database server from 3 to 3.4 and the reliability decreases by approximately 0.086% with increase in time from 50 to 500 hours. The results given in the Table 4.6 shows that MTBF of the system increases by approximately 0.0012% and the reliability of the system increases by approximately 0.00097% with the increase in the software repair rate of router software from 2 to 2.3 and the reliability decreases by approximately 0.086% with increase in time from 50 to 500 hours. Sensitivity analysis presented in Table 4.7 and 4.8 shows that the variation in the software failure rates of application server affects the availability of the system to maximum extent as compare to database server and router software. The results reveal that the maximum effect on the MTBF, reliability and availability of the system is due to variation in software failure and repair rates of application server as compare to database server and router. The effect of software failure and repair rates of application server on the reliability of the system has also been presented in the graph. Thus in order to maximize the reliability and availability of the web-based software system, the failure rates of application servers should be minimized by and the repair rates should be maximized. To achieve this level of redundancy should be increased in case of application servers.

For software systems where reliability and availability are very critical parameters, the difference in achieved and required levels of reliability and availability, by analyzing results, can help in determining the testing intensity or required manpower for the projects. The results also describe the operational performance of software system. Moreover, by analyzing the effect of failure and repair rates of various components software system for it's the reliability and availability, we can also identify the most sensitive component of the software system. Here, it is Application server. The failure rates of this sensitive component should be minimized and the repair rates should be maximized in order to achieve the desired level of the reliability and availability of the system.

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