Numerical Study of Liquid Metal MHD Duct Flow under Hydrodynamic "Slip" Condition

Dipjyoti Sarma
Department of Mathematics
Dibrugarh University
Dibrugarh-786004

P.N.Deka
Department of Mathematics
Dibrugarh University
Dibrugarh-786004

ABSTRACT
A numerical study for steady MHD flow of liquid metal through a square duct with slip walls has been carried out. An intense external magnetic field is acting normal to two walls of the square duct which are considered as slip walls. The numerical solutions for velocity and induced magnetic field have been obtained by using a 5 point stencil central difference scheme. Solutions for velocity and induced field for different values of Hartmann number and with consideration of a fixed value of slip length parameter are presented graphically.

Keywords
Liquid metal, Hartmann layer, Square duct, Slip length parameter

1. INTRODUCTION
The study of liquid metal MHD flow through duct of different geometrical cross section is gaining importance due to its practical application such as casting of steel, aluminum reduction etc. The study of liquid metal MHD flow also arises in the development of tritium breeder coolant blanket of a fusion reactor. In such a of MHD duct flow in a fusion environment certain issues arises which are not observed on ordinary MHD flow. Certain additional issues may arise if one considers a high Hartmann number MHD flow. The slip condition is one of such a issue which may arise, particularly where silicon carbide coated insulated walls are included as interior layer of the duct, in a very high Hartmann number liquid metal flow.

The issue of "slip and no-slip" condition is a matter of interest in certain cases of MHD flow problems. Slip condition for fluid at solid boundary was first proposed by Navier [1]. According to Navier's proposed slip boundary condition, velocity V_s at a solid surface is proportional to the shear stress at the surface. Mathematically $V_s = \alpha \frac{\partial V}{\partial y}$, where α is the slip length. If $\alpha = 0$, no slip boundary condition is obtained. In order to characterize fluid flow over patterned surfaces the slip based boundary conditions have been sought in variety of applications most notably in micro-fluids design. Thompson and Troian [2] obtained a generalized boundary condition to relate the degree of slip to the underlying static properties and dynamic interactions of the walls and the fluid. Boundary condition of the flow problem cannot be deduced from the continuum differential equation themselves as well as not easy to determine experimentally. For kinematic reason the normal component of velocity must vanish at the impermeable wall. On the other hand the parallel component of velocity when extrapolated toward the wall may match that of the wall only at some distance α away from it. This phenomenon is known as slip with α is the slip length [3]. For finite values of α, fluid slip occurs at the wall, but its effect depends upon the length scale of the flow [4].

Zhu and Granick [5] made experimental investigations to establish the limits of hydrodynamic no-slip boundary condition. Bazant and Vinogradova [6] made a tensorial generalization of the Navier slip boundary condition and used it for solving for flows around anisotropic textured surfaces. Pioneering work on MHD duct flow under hydro-dynamic "slip" condition using Fourier transformation was done by Smolentsev [7].

Kamrin et al. [8] derive a general formula for the effective slip describing equivalent fluid motion at the mean surface as depicted by the linear velocity profile that arises far from it. The solution of fluid velocity at the walls, solutions of MHD problems involving contact with solid wall requires knowledge not only of the governing differential equations but also the boundary condition imposed on the tangential component of fluid velocity at the walls [9]. Recently Sarma et al. [10] numerically studied the flow of liquid metal through a square duct under the action of strong transverse magnetic field.

In this paper a study on MHD liquid metal flow through a square duct in high Hartmann number environment has been presented. For such a high Hartmann number environment, in most cases, insulated coatings are employed to reduce the associated pressure drop in flow due to intense induced current in the fluid. In this problem Hartmann walls of the duct are considered to be associated with slip condition and the side walls with usual no slip condition.

2. FORMULATION OF THE PROBLEM
In this problem, a steady motion of liquid metal through a square duct has been considered. It is assumed that flow in the duct has been driven by pressure gradient and a strong transverse magnetic field is acting on the flow.

Here, the following assumptions are made:

(i) The flow is fully developed,

(ii) Hartmann walls are treated as slip walls and

(iii) side walls are assumed to be of no slip.

The fluid velocity and magnetic field distribution are $\overline{V} = \left[0, 0, V_z(x, y)\right]$ and $\overline{B} = \left[B_0, 0, B_z(x, y)\right]$ respectively.

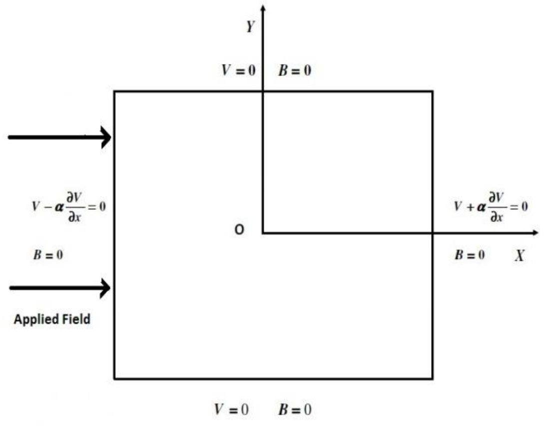

Fig. 1: Geometrical model of the flow problem

The equations describing the MHD duct flow problem are

$$\rho\left[\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}.\nabla\right)\vec{V}\right]+\nabla p=\vec{J}\times\vec{B}+\mu\nabla^2\vec{V} \qquad (1)$$

and $$\frac{\partial\vec{B}}{\partial t}=\nabla\times\left(\vec{V}\times\vec{B}\right)+\lambda\nabla^2\vec{B} \qquad (2)$$

The above systems of equations under given velocity and magnetic distribution become

$$0=-\frac{\partial p}{\partial z}+\frac{B_0}{\mu_e}\frac{\partial B_z}{\partial x}+\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)V_z \qquad (3)$$

$$0=B_0\frac{\partial V_z}{\partial x}+\frac{1}{\mu_e\sigma}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)B_z \qquad (4)$$

Normalizing V_z and B_z in terms of $\left(\partial p/\partial z\right),a,\sigma,\mu$ and μ_e Eq. (3) and Eq. (4) after removing asterisks become

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+M\frac{\partial B}{\partial x}=-1 \qquad (5)$$

and $$\frac{\partial^2 B}{\partial x^2}+\frac{\partial^2 B}{\partial y^2}+M\frac{\partial V}{\partial x}=0 \qquad (6)$$

where

$$V^*=V_z\mu\Big/\left(-\frac{\partial p}{\partial z}\right)a^2,\ x^*=x/a$$
$$B^*=B_z\mu^{1/2}\Big/\left(-\frac{\partial p}{\partial z}\right)a^2\sigma^{1/2}\mu_e,\ y^*=y/a,$$

$$M=B_0a\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}}$$

The boundary conditions at the Hartmann walls and the side walls are

$$\left.\begin{array}{l} x=\pm1:V\pm\alpha\dfrac{\partial V}{\partial x}=0\ ;B=0 \\ y=\pm1:V=0\ ;B=0 \end{array}\right\} \qquad (7)$$

2.1 Numerical Methods

Here, Eq. (5) and Eq. (6) are coupled non-linear equations which are to be solved using boundary condition given in Eq.(7). In view of complexities it is not convenient to obtain a closed form solution particularly in the case of high Hartmann number flow. Here attempt has been made to solve the governing equations by finite difference method. These equations Eq. (5) and Eq.6) with boundary condition given in Eq.(7)are expressed in finite difference equation by utilizing a 5-point stencil centered finite difference scheme with mesh size $h=1/N$ where N is a preassigned positive integer. The resulting finite difference representations are

$$\begin{aligned} V_{i,j}&=0.25\left(V_{i+1,j}+V_{i-1,j}+V_{i,j+1}+V_{i,j-1}\right)+ \\ &\quad C_1+C_2\left(B_{i,j+1}-B_{i,j-1}\right) \end{aligned} \qquad (8)$$

$$\begin{aligned} B_{i,j}&=0.25\left(B_{i+1,j}+B_{i-1,j}+B_{i,j+1}+B_{i,j-1}\right)+ \\ &\quad +C_2\left(V_{i,j+1}-V_{i,j-1}\right) \end{aligned} \qquad (9)$$

Where, $C_1=\frac{h^2}{4}$, $C_2=\frac{Mh}{8}$

The numerical boundary conditions for present duct flow are as follows

For the case of side walls

$$V(i,1)=0,\ \mathrm{V}(i,N+1)=0;$$
$$B(i,1)=0,\ \mathrm{B}(i,N+1)=0;$$

and

for Hartmann walls

$$V(N+1,j)=-\frac{2h}{\alpha}\mathrm{V}(N,j)+\mathrm{V}(N-1,j);$$
$$V(1,j)=-\frac{2h}{\alpha}\mathrm{V}(2,j)+\mathrm{V}(3,j);$$
$$B(1,j)=0,\ \mathrm{B}(N+1,j)=0$$

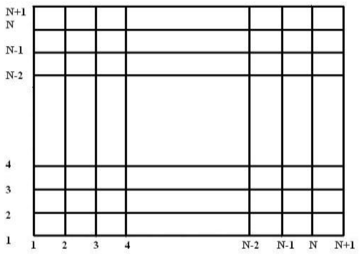

Fig. 2: Grid system

The convergence of each of the computed value of variable V, B at different grid points are checked by using root-mean-square residuals R_s for each flow variable. Convergence is considered to be achieved when $R_s < 10^{-7}$, where

$$R_B = \sqrt{\sum_{i=2}^{N}\sum_{j=2}^{N}\left(B_{i,j}^{n+1} - B_{i,j}^{n}\right)^2}$$

$$R_V = \sqrt{\sum_{i=1}^{N+1}\sum_{j=2}^{N}\left(V_{i,j}^{n+1} - V_{i,j}^{n}\right)^2}$$

3. RESULT AND DISCUSSION

The velocity profile of liquid metal flow inside the square duct has been obtained by numerical solution of Eq. (8) and Eq. (9) using finite difference scheme for different magnetic field strengths. The computations are carried out for Hartmann number $\boldsymbol{M}$ =100, 200 and 300. In the duct, along with increase of imposed field strength, pressure drop phenomena become intense due to increased $\overline{\boldsymbol{J}} \times \overline{\boldsymbol{B}}$ force. The computed results on velocities are plotted in Fig.6 to Fig.8. The pressure drop phenomenon is clearly visible in the 3d graphical presentation of flow field. It is observed that increased pressure drop effect is observed in Fig.6-Fig.8 with the gradually increased Hartmann number.

In this investigation slip effect at the Hartmann walls is included. Due to the slip velocity at walls a lift has been observed in the velocity profile in the duct. It is observed that the effect of slip condition of Hartmann walls is countered by pressure drop effect due to intense magnetic field.

In earlier studies on liquid metal flow through duct with consideration of high Hartmann number that generally arise in tritium breeder blanket design nuclear in fusion reactors, slip condition at the walls are not considered. Many observational reports [11], [12] and [13] suggest that slip may arise at walls in a liquid metal MHD duct flow. In a very high imposed magnetic field condition, associated pressure drop be considerably reduced if silicon carbide coating is applied for electrical insulation of the walls. For such insulated walls slip condition is most likely occurs.

The problem of MHD flow through duct with consideration of hydrodynamic slip condition for the Hartmann walls was carried out by Smolentsev [7] analytically by using Fourier Transformation. In this, investigation, the solution has been obtained for flow with Hartmann number upto $M = 100$.

The computed result of this investigation are in good agreement with analytic solution for 3d- flow obtained by Smolentsev [7], where value of Hartmann number is small. The numerical approach considered in this study is more effective over the analytical approach. It is possible to investigate duct flow with slip walls for much higher value of Hartmann number.

From the above investigation it is observed that if slip wall condition is included flow in duct may experience a lift. In the fusion reactor environment, for designing tritium breeder-coolant blanket it is essential to consider the liquid metal such as PbLi through a very high Hartmann number condition due to the presence of intense plasma confining magnetic field. In such duct flow to reduce the pressure drop phenomena insulated interior is required. In such insulated interior silicon carbide layer is introduced for which slip condition is most likely to involve. Numerical approach for dealing with liquid metal flow for much higher Hartmann number with slip effect at the wall is important.

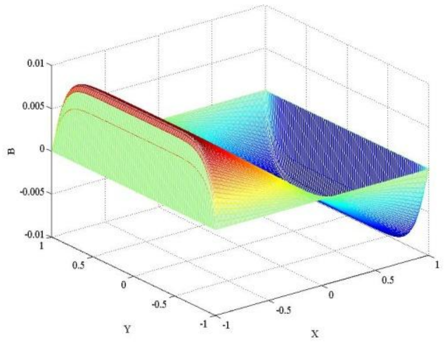

Fig. 3: 3D Variation of induced magnetic field for $M = 100$

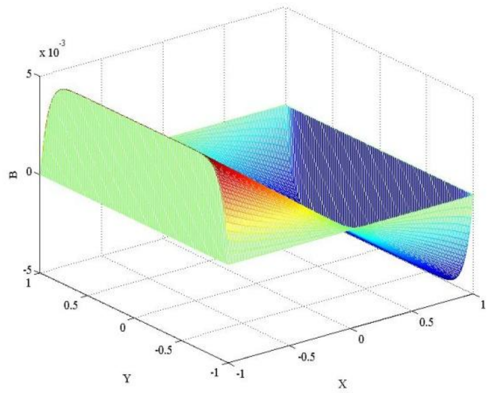

Fig. 4: 3D Variation of induced magnetic field for $M = 200$

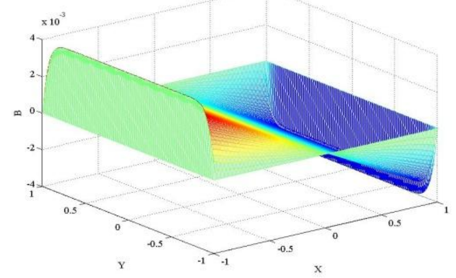

Fig. 5: 3D Variation of induced magnetic field for $M = 300$

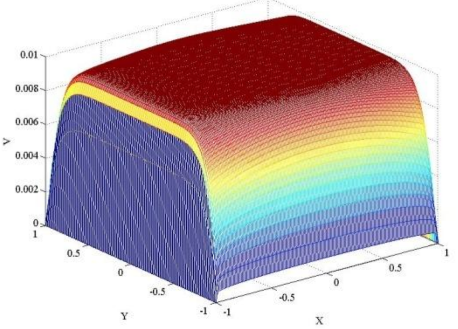

Fig. 6: 3D Variation of Velocity profile for $M = 100$

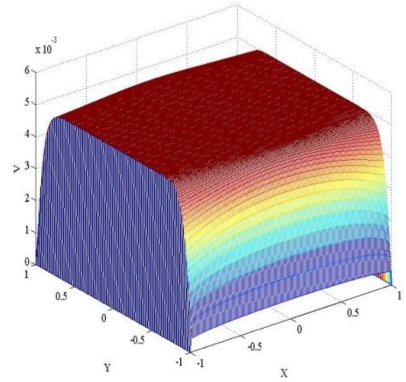

Fig. 7: 3D Variation of Velocity profile for $M = 200$

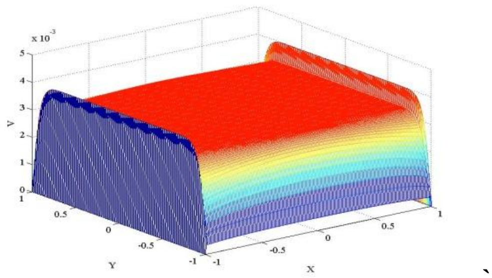

Fig. 8: 3D Variation of Velocity profile for $M = 300$

4. ACKNOWLEDGMENTS
This work was supported by University Grant Commission, India under grant number 39-44/2010(SR).

5. REFERENCES
[1] Navier, C.L.M.H. 1823. Sur les du mouvement des fluids. Mem. Acad. R. Sci. Inst. France. 6. 389-440.

[2] Thompson, P.A. and Troian, S.M. 1997. A general boundary condition for liquid flow at solid surfaces. Nature. 389(25 Sept). 360-362.

[3] Cieplak, M., Koplik, J. and Banavar, J.R. 2001. Boundary Conditions at a Fluid-Solid Interface. Physical Review Letter. 86(5). 803-806.

[4] Tretheway, D.C. and Meinhart, C.D. 2002. Apparent fluid slip at hydrophobic microchannel walls. Physics of Fluids. 14(3). 9-12.

[5] Zhu, Y. and Granick, S. 2002. Limits of Hydrodynamic No-Slip Boundary Condition. Physical Review Letters. 88(10). 106102(1-4).

[6] Bazant, M.Z. and Vinogradova, O.I. 2008 Tensorial hydrodynamic slip. Journal of Fluid Mechanics. 613. 125-134.

[7] Smolentsev, S. 2009. MHD duct flows under hydrodynamic "slip" condition. Theoretical Computational Fluid Dynamics. 23(6). 557-570.

[8] Kamrin, K., Bazant, M.Z. and Stone, H.A. 2010. Effective slip boundary conditions for arbitrary periodic surfaces: the surface mobility tensor. Journal of Fluid Mechanics. 658. 409-437.

[9] Brenner, H. 2011. Beyond the no-slip condition. Physics Review E. 84(4). 046309(1-8).

[10] Sarma, D., Hazarika, G.C. and Deka, P.N. 2013. Numerical study of liquid metal MHD flow through a Square duct under the action of strong transverse magnetic field. International Journal of Computer Application. 71(8). 29-32.

[11] Pint, B.A., More, K.L., Meyer, H.M. and Distefano, J.R. 2005. Recent progress addressing compatibility issues relevant to fusion environment. Fusion Science and Technology. 47(4). 851-855.

[12] Noborio, K., Kasada, R. ,Yamamato, Y.,Nam,G. and Konishi, S. 2009. Compatibility of materials for advanced blanket with liquid LiPb. SOFE 2009, 23rd IEEE/NPSS Symposium on. San Diego CA.

[13] Morley, N.B.,Medina, A. and Abdou, M.A. (2009). Measurements of specific electrical contact resistance between SiC and lead-lithium eutectic alloy. Fusion Science and Technology. 56(July). 195-200.