Testing Hypothesis for New Class of Life Distribution NBUFR-t0

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ABSTRACT
A new concept of ageing distribution, namely new better (worse) than used in failure rate at specific time t_0 (NBUFR-t_0 (NWUFR-t_0)) are introduced. The problem is investigated how to prove that after a specified time t_0 of operation the failure rate of an item is greater than the corresponding failure rate of a new item. This problem occurs in various areas like for instance in industry, when designing a maintenance policy. A test statistics that based on the goodness of fit method are derived for testing exponentially versus the NBUFR-t_0 alternatives. The percentiles and powers of this test statistic are tabulated. The asymptotic efficiencies for some alternatives are derived. A medical data is taken as a practical application.

Keywords
NBUFR-t_0 ; NWUFR-t_0 ; Mont Carlo method; Hypotheses testing; NBUFR; U-statistic; Life testing; Exponential distribution; Goodness of fit testing; Efficiency; Power of test.

1. INTRODUCTION
The notion of aging for engineering systems has been characterized by several classes of life distributions in reliability. Various classes of life distributions have been introduced to describe several types of deterioration (improvement) that accompany aging. Unfortunately these types of classes are restricted to one type of applications since they present positive aging or negative aging for their dual classes throughout their life span of the underlying components or their systems.

[16] have introduced the new better than used at specific age at t_0 (NBU-t_0) and its dual (NWU-t_0). They have listed some types of situation where (NBU-t_0) aging or its dual might arise. [12] introduced the new better than used in convex ordering at age t_0 (NBUC-t_0), and its dual.

There are many situation in real life where the components of the system gradually deteriorate up time t_0 which is warranty guarantee time provided by most manufacturers, then maintenance through repairs or spare parts replacement take place after time t_0. Here maintenance is expected to improve the performance of the system but can not bring it back to a better situation than it was at age t_0. For some interesting examples, see [16].

One of the oldest problems in aging distributions is testing exponentiality against the known classes of aging distributions such as IFR, IFRA, NBU, NBARFR and others, for example see [23], [15], [2], [3] and [20] among others. With respect to the NBUFR class, see [6].

[16] introduced the concept of the class of life distributions called new better than used of specified age t_0. [11], [22] and [4] are investigated the testing of new better than used of specified age t_0 (NBU-t_0) alternatives and others as [19] deal with the problem with respect to increasing failure rate (IFR) and new better than used (NBU) after specified time t_0. [21] introduce the new better (worse) than used renewal failure rate at specied time t_0 NBURFR-t_0 (NWURFR-t_0).

In practice, one might be interested in the new better than used failure rate behaviour at specified age t_0. Therefore a new concept of ageing distribution namely new better (worth) than used failure rate of specified age t_0 (NBUFR-t_0 (NWUFR-t_0)). This class is generalization of NBUFR ([1]). We investigate the testing exponentiality problem of it alternatives here according to goodness of fit approach. The distributional properties of the test statistic are studied. The performances of a proposed test are studied in terms of powers for some of alternative life distributions. The proposed test is shown to be consistent for NBUFR-t_0 alternatives.

This paper is organized as follows. In Section 2 we will recall some definitions and facts about characteristics of lifetime distributions and about relationship between some classes of life distributions. Section 3 introduced main results for our study represented in the test statistic proposed for testing exponentially versus the NBUFR-t_0 alternatives.

2. DEFINITIONS and RELATIONSHIPS
Let X be a non-negative random variable representing the lifetime of a device, with distribution function $F(x)=P[X\le x]$, survival function $\overline{F}(x)=1-F(x)$ and density function f(x) (if it exists), then the corresponding failure rate r(t), when the distribution is absolutely continuous, is defined by

$$r(t)=\lim_{\delta t\to 0}\frac{P[T\le t+\delta t|T\ge t]}{\delta t}.$$

[1] introduced the new better than used failure rate class of life distribution and its dual and discussed some of its properties as follows

Definition 2.1 An absolutely continuous probability distribution F on [0, ∞) for which F(x)/x has a limit as x →0 from above is said to be NBUFR(NWUFR) if there exists a version r of the failure function such that

$$\tilde{r}(0)\le(\ge)r(t)$$

Where

$$\tilde{r}(0) = \lim_{t \to 0^+} t^{-1} \int_0^t r(x)\, dx.$$

Authors state that: An absolutely continuous distribution F on [0, ∞) for which F(x)/x has a limit as x →0 from above has the NBUFR property iff

$$\overline{F}(x+t) \le e^{-\tilde{r}(0)t}\, \overline{F}(x), \text{ for all x, t} \ge 0$$

[7] introduced the new better than used (NBU) and its dual new worse than used (NWU) classes of life distributions as follows

Definition 2.2 The random variable X, or its distribution function F(x), is said to be (or to have) new better (worse) than used (NBU (NWU)) if

$$\overline{F}(x+t) \le (\ge) \overline{F}(x)\, \overline{F}(t), \text{ for all x, t} \ge 0$$

[9] introduced the class of new better than used in convex ordering (NBUC) distributions. [10] suggested the class of new better than used of second order(2) as follows.

Definition 2.3 The random variable X, or its distribution function F(x), is said to be

new better than used in convex order (NBUC) if

$$\int_x^\infty \overline{F}(t+s)\, ds \le \overline{F}(t) \int_x^\infty \overline{F}(s)\, ds, \text{ for all x, t} \ge 0$$

new better than used in concave order (NBU(2)) if

$$\int_0^x \overline{F}(t+s)\, ds \le \overline{F}(t) \int_0^x \overline{F}(s)\, ds, \text{ for all x, t} \ge 0$$

We have the following chain of implications of the above mentioned classes as follows:

$$\text{NBU(2)} \Leftarrow \text{NBU} \Leftarrow \text{NBUC}$$

and

$$\text{NBU} \Rightarrow \text{NUBE} \Rightarrow \text{NBUFR}$$

[16] introduced the larger class of life distributions called new better than used of age t_0 defined as follows:

Definition 2.4 A non-negative random variable X is said to be new better than used of age t_0 if

$$\overline{F}(t_0 + x) \le \overline{F}(t_0)\ \overline{F}(x), \text{ for all x} \ge 0$$

The NBU property states that a used item of any age has stochastically smaller residual life length than does a new item, whereas the NBU-t_0 property states that a used item of age t_0 has stochastically smaller residual life length than does a new item.

[12] introduced the concept of new better than used in convex ordering and new better than used of second order (2) classes of age t_0 as follows.

Definition 2.5 A non-negative random variable X is said to be

new better than used in convex ordering of age t_0 (NBUC-t_0)if

$$\int_x^\infty \overline{F}(t_0 + y)\, dy \le \overline{F}(t_0) \int_x^\infty \overline{F}(y)\, dy, \text{ for all x} \ge 0$$

new better than used of second order(2)of age t_0 (NBU(2)-t_0)if

$$\int_0^x \overline{F}(t_0 + y)\, dy \le \overline{F}(t_0) \int_0^x \overline{F}(y)\, dy, \text{ for all x} \ge 0$$

It is seen from the above definitions that that

$$NBUC - t_0 \Leftarrow NBU - t_0 \Rightarrow NBU(2) - t_0$$

and

$$\begin{array}{c} NBU - t_0 \\ \Downarrow \\ NBUC - t_0 \Rightarrow \quad NBUE - t_0 \Leftarrow \quad NBU(2) - t_0. \end{array}$$

By the same concept, we can introduce new class of life distributions at specific age t_0. Namely new better than used in failure rate at specific age t_0 (NBUFR-t_0 (NWUFR- t_0)) as follows.

Dfinitione 2.6 An absolutely continuous distribution F on [t_0, ∞) is said to be new better (worse) than used in failure rate at specific time t_0 (NBUFR- t_0 (NWUFR- t_0)) if there exist a version r of the failure rate functions such that for specified t_0

$$r(t_0) \le (\ge)\ r(t_0 + t), \text{ for all t} \ge 0$$

Note that the border class where $r(t_0) = r(t + t_0)$ include only the following members:

all exponential distribution.
all life distribution defined on [0, t_0].
all life distribution defined freely on [0, t_0] and defined as $\overline{F}^{\,j}(t_0)\overline{F}(x - jt_0)$ for all $jt_0 \le x \le (j+1)t_0,\ j \ge 1$.

3. The U-STATISTIC TEST PROCEDURE

In this section we consider the problem of testing $H_0 : \overline{F}$ belongs to border distributions against $H_1 :$ $\overline{F}$ is strictly NBUFR-t_0 by using a goodness of fit approach. For more detail see [3]. The following is the form of our new class that called new better than used failure rate of specified age t_0

$r_F(t_0) \le r_F(t + t_0)$, $t \ge 0$ for fixed t_0 ,

that is equivalent to

$$\frac{f(t_0)}{\overline{F}(t_0)} \le \frac{f(t + t_0)}{\overline{F}(t + t_0)},\ t \ge 0 \text{ and fixed } t_0 .$$

In contrast to goodness of fit problems, where the test statistic is based on a measure of departure from H_0 and H_1 , we refer to the departure between the two sides of inequality by

$$\Delta_F = \int_0^\infty \left(\overline{F}(t_0) f(x + t_0) - f(t_0)\overline{F}(x + t_0)\right) dF_0(x)$$

A nonparametric test statistic for this class is introduced. This test is designed to test the hypothesis whether a

NBUFR-t_0 elements are strictly or not. The following lemma is essential for the development of our test statistic.

Lemma Let X be a random variable with NBUFR-t_0 distribution function F then

$\Delta_F =$

$$\bar{F}(t_0)E\left[e^{-(X-t_0)}I(X>t_0)\right]-f(t_0)E\left[1-e^{-(X-t_0)}I(X>t_0)\right]$$

Proof Since

$$\int_0^\infty \bar{F}(t+t_0)dF_0(t)=\int_0^\infty I(X>x+t_0)e^{-x}dx$$

$$=E\int_{t_0}^\infty I(X>u)e^{-(u-t_0)}du=\bar{F}(t_0)E\left[I(X>t_0)\left(1-e^{-(X-t_0)}\right)\right]$$

,

and

$$E\int_0^\infty f(x+t_0)e^{-x}dx=E\left[e^{-(x-t_0)}I(x\geq t_0)\right]$$

Then the result is simple to obtain.

$\Delta_F=0$ under H_0 and $\Delta_F>0$ under H_1. To estimate Δ_F let $X_1,X_2,X_3,\ldots X_n$ be a random sample from F, then F(t) will be empirically estimated by $\hat{F}_n(t)$ So the empirical form of Δ_F is as follows

$$\hat{\Delta}_F=\frac{1}{n^2}\sum_{i,j=1}^{n}\begin{pmatrix} I(X_i>t_0)I(X_j>t_0)e^{-(X_j-t_0)} \\ -\frac{1}{a}K\left(\frac{t_0-X_i}{a}\right)I(X_j>t_0)(1-e^{-(X_j-t_0)}) \end{pmatrix}$$

here $\hat{f}_n(t_0)$ is as follows

$$\hat{f}_n(t_0)=\frac{1}{na_n}\sum_{j=1}^{n}K\left(\frac{t_0-X_j}{a_n}\right)$$

For more details about K and the sequence $\{a_n\}$ See [13].

If we choose

$$\phi(X_1,X_2)=I(X_1>t_0)I(X_2>t_0)e^{-(X_2-t_0)}$$
$$-\frac{1}{a_n}K\left(\frac{X_1-t_0}{a_n}\right)I(X_2>t_0)(1-e^{-X_2-t_0}),$$

then $\hat{\Delta}(F)$ equivalents the following U-statistic cf [18].

$$U_n=\frac{1}{\binom{n}{2}}\sum_R \psi(X_i,X_j).$$

Where, the summation over all arrangements of X_{i1},X_{i2}, i: 1, 2, 3,..,n. The proof of the following theorem follows from the standard theory of U-statistic, which obtained the asymptotic distribution of $\hat{\Delta}_F$.

Theorem As $n\to\infty$, the random variable $\sqrt{n}\left(\hat{\Delta}_F-\Delta_F\right)$ is asymptotically normal with mean 0 and variance σ^2 where σ^2 is given in (3.1). Under H_0, σ_0^2 is given by (3.2).

Proof

Using standard U-statistics theory, cf- [18], we need only the asymptotic variance which is equal to

$$\sigma^2=V(E[\phi(X_1,X_2)\mid X_1]+E[\phi(X_1,X_2)\mid X_2])$$

Now since

$$E[\phi(X_1,X_2)\mid X_1]=[I(X_1\geq t_0)+f(t_0)]e^{t_0}E(I(X_2\geq t_0)e^{-X_2})$$
$$-f(t_0)\bar{F}(t_0)$$

and

$$[\phi(X_1,X_2)\mid X_2]=\bar{F}(t_0)I(X_2\geq t_0)e^{-(X_2-t_0)}$$
$$\mathrm{E}-(1-e^{-(X_2-t_0)})I(X_2\geq t_0)f(t_0)$$

Hence,

$$\sigma^2=V\begin{Bmatrix}(I(X>t_0)+f(t_0))E\left(I(X>t_0)e^{-(X-t_0)}\right)+ \\ \bar{F}_0(t_0)(I(X>t_0))e^{-(X-t_0)}- \\ f(t_0)I(X>t_0)(1-e^{-(X-t_0)})-f(t_0)\bar{F}_0(t_0))\end{Bmatrix}.$$

Under H_0,

$$\sigma_0^2=\frac{1}{4}\bar{F}^4(t_0)-\frac{1}{12}\bar{F}^4(t_0)(-6+7\bar{F}^{-1}(t_0)).$$

This completes the proof.

It is easy to explain that our proposed test is consistent and unbiased. To perform above test, calculate $\sqrt{\frac{n}{\sigma_0^2}}\hat{\Delta}_F$ and reject H_0 if this value exceeds $-Z_\alpha$, the lower α -percentile of the standard normal variate.

The asymptotic Pitman efficiacy of a statistic T_n is defined by

$$eff(T_n)=\frac{\left|\frac{\partial}{\partial\theta}E(T_n)\right|_{\theta\to\theta_0}}{\sigma_0}.$$

The following families of alternatives are used for efficiacy comparisons :

the Linear failure rate distribution: $\exp\left(-t-\theta t^2/2\right)$, $t\geq 0,\theta\geq 0$

the Makeham distribution: $e^{-t-\theta(t+e^{-t}-1)}$, $t>0,\theta\geq 0$

the c^* distribution

$$\bar{F}(t)=\begin{cases} e^{-t+\theta t^2(2t_0)^{-1}}, & 0\leq t<t_0, 0\leq\theta\leq 1 \\ e^{-t+\frac{\theta t^2}{2}t_0}, & t>t_0, 0\leq\theta\leq 1\end{cases}$$

the null exponential is attained at $\theta=0$.

Carrying out efficacy calculations for the above alternatives , we get

$$\frac{\sqrt{3}e^{-2t_0}\left(1-2t_0-e^{t_0}\left(1-2t_0+2t_0^2\right)\right)}{4\sqrt{e^{-4t_0}\left(-3+7e^{t_0}\right)}},$$

$$\frac{e^{-3t_0}}{\sqrt{3}\sqrt{e^{-4t_0}\left(-3+7e^{t_0}\right)}},$$

and

$$\begin{pmatrix} -2+t_0+12t_0^2-6t_0^3-8t_0^2e^{t_0(2+t_0)}-2(-2+t_0)t_0^2e^{\frac{1}{2}t_0(2+t_0)}+ \\ (2-5t_0+10t_0^2-4t_0^3)e^{2t_0^2}+(2+3t_0-6t_0^2+2t_0^3)e^{3t_0}+ \\ (-2-7t_0-12t_0^2+8t_0^3)e^{t_0} \end{pmatrix}\times$$

$$\left(\frac{4t_0(-2+t_0)\sqrt{(-3+7e^{t_0})e^{-4t_0}}}{\sqrt{3}\ e^{-4t_0}}\right)^{-1}$$

respectively.

For sample sizes 5, 6, 7…50 and using 5000 replications we tabulate the lower and upper percentiles for $\hat{\Delta}_F$ for t_0 =0.25 in table (Appendix). Its powers for sample sizes 10, 20 and 30 for the linear failure rate, weibull and gamma families are given in the following two tables respectively.

Power estimates of $\hat{\Delta}_F$ (t_0 = 0.25)

Distribution	θ	n			
		5	10	15	20
LFR	2	1.000	1.000	1.000	1.000
	3	1.000	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000
Weibull	2	1.000	1.000	1.000	1.000
	3	1.000	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000
Gamma	2	1.000	1.000	1.000	1.000
	3	1.000	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000

Power estimates of $\hat{\Delta}_F$ (t_0 = 0.5)

Distribution	θ	n			
		5	10	15	20
LFR	2	1.000	1.000	1.000	1.000
	3	1.000	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000
WEIBULL	2	1.000	1.000	1.000	1.000
	3	1.000	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000
Gamma	2	1.000	1.000	1.000	1.000
	3	1.000	1.000	1.000	1.000
	4	1.000	1.000	1.000	1.000

Thus we have shown that, based on Monte Carlo methods, our test statistic has a full powers for all alternatives.

Example 3.3 The data of this example consist of the survival times of 40 patients suffering from blood cancer (Leukemia) from one of the hospitals in Saudi Arabia, [14]. The orderd life times (in days) are:

115.0 181.0 255.0 418.0 441.0 461.0 516.0 739.0 743.0 789.0 807.0 865.0 924.0 983.0 1024.0 1062.0 1063.0 1165.0 1191.0 1222.0 1122.0 1251.0 1277.0 1290.0 1357.0 1369.0 1408.0 1455.0 1478.0 1549.0 1578.0 1578.0 1599.0 1603.0 1605.0 1696.0 1735.0 1799.0 1815.0 1852.0

To apply the test we choose t_0 =1.137 ($\approx$ 3 years). We obtain the value of our test for the blood cancer data with a corresponding one side and 0.05 level of significant hence H_0 is not rejected, agreeing with the conclusion of [14] and [3].

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Appendix

Table Percentiles for $\hat{\Delta}_F$ (t_0 = 0.25)

n	.01	.05	.10	.90	.95	.98	.99
5	-.0727	-.0299	.0071	.4149	.4808	.5461	.5998
6	-.0670	-.0179	.0150	.3821	.4446	.5237	.5685
7	-.0559	-.0095	.0231	.3753	.4283	.5031	.5509
8	-.0536	-.0046	.0242	.3503	.4026	.4667	.5163
9	-.0455	-.0054	.0280	.3400	.3884	.4538	.4902
10	-.0433	.0008	.0322	.3292	.3782	.4327	.4690
11	-.0418	.0072	.0355	.3183	.3618	.4154	.4588
12	-.0298	.0141	.0424	.3114	.3559	.4119	.4479
13	-.0300	.0133	.0434	.2981	.3407	.3868	.4177
14	-.0268	.0161	.0448	.2920	.3291	.3752	.4044
15	-.0244	.0227	.0469	.2899	.3305	.3723	.4032
16	-.0190	.0248	.0477	.2827	.3203	.3646	.3878
17	-.0177	.0231	.0471	.2713	.3106	.3485	.3724
18	-.0154	.0270	.0522	.2663	.3002	.3380	.3627
19	-.0144	.0243	.0500	.2656	.3014	.3416	.3660
20	-.0051	.0324	.0550	.2604	.2944	.3295	.3502
21	-.0086	.0290	.0513	.2543	.2885	.3244	.3490
22	-.0066	.0321	.0561	.2498	.2840	.3226	.3460
23	-.0028	.0339	.0567	.2490	.2783	.3136	.3366
24	-.0086	.0347	.0585	.2467	.2753	.3085	.3273
25	-.0029	.0350	.0572	.2476	.2730	.3116	.3345
26	-.0026	.0335	.0567	.2400	.2671	.2975	.3258
27	.0029	.0390	.0609	.2377	.2658	.2974	.3195
28	.0018	.0383	.0599	.2389	.2655	.2962	.3177
29	.0082	.0429	.0624	.2338	.2590	.2902	.3095
30	-.0003	.0366	.0601	.2295	.2556	.2887	.3090
31	.0028	.0442	.0631	.2290	.2546	.2854	.3081
32	.0068	.0421	.0615	.2243	.2526	.2809	.2998
33	.0099	.0446	.0621	.2210	.2479	.2789	.3007
34	.0130	.0450	.0626	.2232	.2481	.2764	.2950
35	.0058	.0419	.0631	.2217	.2443	.2710	.2876
36	.0110	.0423	.0624	.2193	.2437	.2728	.2923
37	.0108	.0477	.0657	.2157	.2411	.2683	.2863
38	.0131	.0467	.0642	.2155	.2392	.2673	.2834
39	.0108	.0449	.0662	.2132	.2381	.2661	.2875
40	.0162	.0466	.0653	.2147	.2337	.2591	.2798
41	.0129	.0474	.0651	.2078	.2297	.2539	.2711
42	.0165	.0458	.0655	.2114	.2317	.2590	.2739
43	.0119	.0473	.0646	.2081	.2298	.2538	.2736
44	.0155	.0489	.0669	.2056	.2272	.2535	.2702
45	.0218	.0501	.0666	.2056	.2266	.2538	.2720
46	.0189	.0495	.0659	.2047	.2255	.2528	.2676
47	.0195	.0506	.0666	.2018	.2221	.2459	.2670
48	.0221	.0492	.0667	.2028	.2237	.2446	.2619
49	.0217	.0505	.0671	.2016	.2233	.2472	.2609
50	.0202	.0506	.0675	.2006	.2226	.2423	.2565

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