

Optimal Inventory Model for Time Dependent Decaying Items with Stock Dependent Demand Rate and Shortages

Rekha Rani Chaudhary
Government Engineering
College
Bharatpur, Rajasthan, India

Vikas Sharma
Banasthali Vidyapith
Rajasthan, India

Urvashi Chaudhary
D.N. College, Meerut,
U.P., India

ABSTRACT

This paper deals with in developing an inventory model for time dependent decaying items as bread, food stuff, green vegetables and medicine etc. The rate of deterioration is time dependent and considers a linear function of time. Demand rate is stock dependent in linear form. It is a common phenomenon observed in the super market that a large pile of goods attracts more customers. Shortages are allowed and shortages are completely backlogged. The numerical example is given to illustrate the model developed. The model is solved analytically by maximizing the total profit.

Keywords

Inventory, EOQ Model, deteriorating items, stock dependent demand rate

1. INTRODUCTION

It is usually observed that a large quantity of goods on self in a superstore will lead the customer to buy more goods and that situation create the greater demand of the goods. This situation motivates the retailer to increase their order quantity and the retailers earn the more profit to increase their revenue .In this situation there must arise the problem of space allocation for each item and investment re requirement resulting from the increased inventory levels. The situation become more complex when the inventory is deteriorates in nature. Deterioration of physical goods is one of the important factors in any inventory and production system. The deteriorating items with stock dependent demand rate and shortages have received much attention of several researches in the recent year because most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables and food stuffs from depletion by direct spoilage wile kept in store. Ghare and Schrader [1963] developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was proposed by Shah and Jaiswal [1976].Inventory models with a time dependent rate of deterioration were considered by Covert and Philip [1973]. Some of the significant recent work in this field have been done by Chung and Ting [1993],Fujiwara [1993], , Hariga and Benkherouf [1994], Wee [1995], Giri andChaudhuri[1997], Jalan and Chaudhuri[1999], Structural properties of an inventory system with deterioration and trended demand. Burwell [1997] developed economic lot size model for price-dependent demand under quantity and freight discounts. Inventory model for ameliorating items for price dependent demand rate was proposed by Mondal et.al [2003] and inventory model with price and time dependent demand was developed by You [2005]. In general holding cost is assumed to be known and constant. But in realistic

condition holding cost may not Goh [1994] considered various functions to describe holding cost. M. Mandal and M. maiti [1999] considered inventory of damageable items with variable replenishment rate, stock dependent demand and some units in hand. Inventory models for a single deteriorating item with stock dependent demand rate have been studied extensively in the last decade by Datta and pal, Giri and chaudhuri [1998], Padmanabhan and Vrat [1995] Presented inventory models for deteriorating items with stock dependent selling rate and derived the profit functions for both without backlogging and complete backlogging cases. They assumed the selling rate as a function of current inventory level and rate of deterioration as a constant. They assumed the total average cost, warehouse space, inventory cost, purchase and selling prices to be vague and imprecise.

In this paper we developed an economic order quantity (EOQ) in which deterioration rate is linear function of time and shortages are considered as completely backlogged. Demand rate is stock dependent in linear form as $D(t) = \alpha + \beta I(t)$. We solve the model to optimized the total profit which is maximum. Model is illustrated with numerical examples.

2. ASSUMPTIONS AND NOTATIONS

The fundamental Notations are used to develop the model.

- i. $I(t)$ be the inventory level at time t ($0 \leq t < T$).
- ii. T is the cycle length.
- iii. q is The ordering quantity is.
- iv. A_0 is the ordering cost.
- v. s is the selling price per unit item .
- vi. h is the inventory holding cost per unit item per unit time.
- vii. C_1 is the shortage cost per unit item per unit time.
- viii. C_2 is the deterioration cost per unit item per unit time.
- ix. $D(t)$ is the Demand rate.
- x. $\theta(t)$ is the time dependend deterioration rate
- xi. Q is the optimal inventory level.

The fundamental Assumptions are used to develop the model.

For the present Model, it is assumed that

- i. Demand $D(t)$ is dependent on the inventory level or stock dependent in linear form:

$$D(t) = \alpha + \beta I(t)$$

Where $\alpha > 0$, $0 \leq \beta \leq 1$, β is the stock dependent demand rate parameter

(5)

ii. The deterioration rate $\theta(t)$ is dependent on time in linear for

$$\theta(t) = bt$$

iii. The ordering cost A_0 is constant.

iv. The cycle length is assumed $0 < t < T$.

3. FORMULATION AND SOLUTION

The length of the cycle is T . Due to the demand and deterioration of the items the inventory is depleted during time $t-1$. At the time t_1 the inventory level becomes zero and shortages occurring in the period (t_1, T) which is completely backlogged.

Let $I(t)$ be the inventory level at time t ($0 \leq t < T$).

The differential equation Can be expressed when the instantaneous state over $(0, T)$ are given by

$$\frac{dI(t)}{dt} + bt.I(t) = -(\alpha + \beta I(t)) \quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -(\alpha + \beta I(t)) \quad t_1 \leq t \leq T \quad (2)$$

$$\text{With } I(t_1) = 0 \text{ at } t = t_1$$

From equation (1) we get

$$I(t) = \left[1 - \beta t - \frac{bt^2}{2} \right] \left[\alpha(t_1 - t) + \frac{\beta}{2}(t_1^2 - t^2) + \frac{b}{6}(t_1^3 - t^3) \right] \quad 0 \leq t \leq t_1 \quad (3)$$

From equation (2) we get

$$I(t) = [1 - \beta t] \left[\alpha(t_1 - t) + \frac{\beta}{2}(t_1^2 - t^2) \right] \quad t_1 \leq t \leq T \quad (4)$$

The holding cost during the time period 0 to t_1

$$H = H = \int_0^{t_1} I(t) dt$$

The total holding cost during the time period 0 to t_1

$$H = h. \int_0^{t_1} I(t) dt$$

$$H = h \int_0^{t_1} \left[1 - \beta t - \frac{bt^2}{2} \right] \left[\alpha(t_1 - t) + \frac{\beta}{2}(t_1^2 - t^2) + \frac{b}{6}(t_1^3 - t^3) \right] dt$$

Now total holding cost will be

$$H = h \left[\frac{\alpha}{2} t_1^2 + \frac{\beta}{3} t_1^3 \left(1 - \frac{\alpha}{2} \right) + \frac{t_1^4}{8} (b - \beta^2 - ab) - \frac{1}{12} b \beta t_1^5 - \frac{b^2}{72} t_1^6 \right]$$

The total deterioration cost during the time period 0 to t_1 is given by

$$D_T = C_2 \int_0^{t_1} \theta(t). I(t) dt$$

$$D_T = C_2 \int_0^{t_1} bt. \left[1 - \beta t - \frac{bt^2}{2} \right] \left[\alpha(t_1 - t) + \frac{\beta}{2}(t_1^2 - t^2) + \frac{b}{6}(t_1^3 - t^3) \right] dt$$

Now total deterioration cost will be

$$D_T = C_2 b \left[\frac{1}{6} \alpha t_1^3 + \beta t_1^4 \left(\frac{1}{8} - \frac{1}{12} \alpha \right) + t_1^5 \left(\frac{b}{20} - \frac{1}{15} \beta^2 - \frac{1}{40} ab \right) - \beta t_1^6 \left(\frac{1}{36} + \frac{1}{48} b \right) - \frac{1}{112} b^2 t_1^7 \right] \quad (6)$$

The total shortage cost during the time period t_1 to T is given by

$$Sh = C_1 \int_{t_1}^T I(t) dt$$

$$Sh = -C_1 \int_{t_1}^T [1 - \beta t] \left[\alpha(t_1 - t) + \frac{\beta}{2}(t_1^2 - t^2) \right] dt$$

$$Sh = C_1 \left[T^3 \left(\frac{\beta}{6} - \frac{\alpha\beta}{3} \right) + T^2 \left(\frac{\alpha}{2} + \frac{\alpha}{2} \beta t_1 + \frac{\beta^2}{4} t_1^2 \right) - T \left(\alpha t_1 + \frac{\beta}{2} t_1^2 \right) - T^4 \frac{\beta^2}{8} + \frac{\alpha}{2} t_1^2 + \frac{1}{3} \beta t_1^3 \left(1 - \frac{1}{2} \alpha \right) - \frac{1}{8} \beta^2 t_1^4 \right] \quad (7)$$

From equation (5), (6) & (7) the total profit per unit time can define

$$U(T, t_1) = s. (\alpha + \beta I(t)) - \frac{1}{T} [A_0 + H + D_T + S_h]$$

$$U(T, t_1) = s \left[\alpha + \beta \left\{ 1 - \beta t - \frac{bt^2}{2} \right\} \left\{ \alpha(t_1 - t) + \frac{\beta}{2}(t_1^2 - t^2) + \frac{b}{6}(t_1^3 - t^3) \right\} \right] - \frac{1}{T} \left[A_0 + h \left\{ \frac{\alpha}{2} t_1^2 + \frac{\beta}{3} t_1^3 \left(1 - \frac{\alpha}{2} \right) + \frac{t_1^4}{8} (b - \beta^2 - ab) - \frac{1}{12} b \beta t_1^5 - \frac{b^2}{72} t_1^6 \right\} + C_2 b \left\{ \frac{1}{6} \alpha t_1^3 + \beta t_1^4 \left(\frac{1}{8} - \frac{1}{12} \alpha \right) + t_1^5 \left(\frac{b}{20} - \frac{1}{15} \beta^2 - \frac{1}{40} ab \right) - \beta t_1^6 \left(\frac{1}{36} + \frac{1}{48} b \right) - \frac{1}{112} b^2 t_1^7 \right\} + C_1 \left\{ T^3 \left(\frac{\beta}{6} - \frac{\alpha\beta}{3} \right) + T^2 \left(\frac{\alpha}{2} + \frac{\alpha}{2} \beta t_1 + \frac{\beta^2}{4} t_1^2 \right) - T \left(\alpha t_1 + \frac{\beta}{2} t_1^2 \right) - T^4 \frac{\beta^2}{8} + \frac{\alpha}{2} t_1^2 + \frac{1}{3} \beta t_1^3 \left(1 - \frac{1}{2} \alpha \right) - \frac{1}{8} \beta^2 t_1^4 \right\} \right] \quad (8)$$

Our main objective to maximize the profit function $U(T, t_1)$, the necessary condition for maximize the profit are

$$\frac{\partial U(T, t_1)}{\partial T} = 0 \text{ and } \frac{\partial U(T, t_1)}{\partial t_1} = 0$$

$$\frac{\partial U(T, t_1)}{\partial T} = 0$$

$$+ \frac{1}{T^2} \left[A_0 + h \left\{ \frac{\alpha}{2} t_1^2 + \frac{\beta}{3} t_1^3 \left(1 - \frac{\alpha}{2} \right) + \frac{t_1^4}{8} (b - \beta^2 - \alpha b) - \frac{1}{12} b \beta t_1^5 - \frac{b^2}{72} t_1^6 \right\} + C_2 b \left\{ \frac{1}{6} \alpha T^3 + \beta t_1^4 \left(\frac{1}{8} - \frac{1}{12} \alpha \right) + t_1^5 \left(\frac{b}{20} - \frac{1}{15} \beta^2 - \frac{1}{40} \alpha b \right) - \beta t_1^6 \left(\frac{1}{36} + \frac{1}{48} b \right) - \frac{1}{112} b^2 T^7 \right\} \right] - \left[C_1 \left\{ \frac{2T}{3} \left(\frac{\beta}{2} - \frac{\alpha\beta}{1} \right) + \frac{\alpha}{2} + \frac{\alpha}{2} \beta t_1 + \frac{\beta^2}{4} t_1^2 - 3T^2 \frac{\beta^2}{8} - \frac{1}{T^2} \frac{\alpha}{2} t_1^2 - \frac{1}{3T^2} \beta t_1^3 \left(1 - \frac{1}{2} \alpha \right) + \frac{1}{8T^2} \beta^2 t_1^4 \right\} \right] = 0$$

(9)

And $\frac{\partial U(T, t_1)}{\partial t_1} = 0$

$$s\beta \left[\alpha + \beta t_1 + \frac{1}{2} b t_1^2 - \alpha \beta t_1 - \beta^2 t t_1 - \frac{1}{2} \beta b t t_1^2 - \frac{\alpha}{2} b t^2 - \frac{\beta}{2} b t_1 t^2 - \frac{b^2}{4} t^2 t_1^2 \right] - \frac{1}{T} \left[h \left\{ \alpha t_1 + \beta t_1^2 \left(1 - \frac{\alpha}{2} \right) + \frac{1}{2} t_1^3 (b - \beta^2 - \alpha b) - \frac{1}{12} b t_1^4 (5\beta + b t_1) \right\} + C_2 b \left\{ \frac{1}{2} \alpha t_1^2 + \beta t_1^3 \left(\frac{1}{2} - \frac{1}{3} \alpha \right) + t_1^4 \left(\frac{b}{4} - \frac{1}{3} \beta^2 - \frac{1}{8} \alpha b \right) - \frac{\beta t_1^5}{2} \left(\frac{1}{3} + \frac{1}{4} b \right) - \frac{1}{16} b^2 t_1^6 \right\} + C_1 \left\{ T \left(\frac{\alpha}{2} \beta + \frac{\beta^2}{2} t_1 \right) - (\alpha + \beta t_1) + \frac{1}{T} \alpha t_1 + \frac{1}{T} \beta t_1^2 \left(1 - \frac{1}{2} \alpha \right) - \frac{1}{2T} \beta^2 t_1^3 \right\} \right]$$

(10)

With the software mathematica-5.1, compute the optimal value of T* and t1* simultaneously by equation no. (9) and equation no. (10). The optimal value of profit function U* (T, t1) is determined by equation no. (8). The optimal value of T* and t1* satisfy the sufficient conditions for maximizing profit function U* (T,t1) are.

$$\frac{\partial^2 U(T, t_1)}{\partial T^2} < 0, \quad \frac{\partial^2 U(T, t_1)}{\partial t_1^2} < 0$$

(11)

And $\frac{\partial^2 U(T, t_1)}{\partial T^2} \cdot \frac{\partial^2 U(T, t_1)}{\partial t_1^2} - \left(\frac{\partial^2 U(T, t_1)}{\partial T \partial t_1} \right)^2 > 0$ And at T = T*
 optimal value t1 = t1*

(12)

4. NUMERICAL EXAMPLE

Example-1. Let us consider $A = 400, \alpha = 200, \beta = 0.3, b = 0.2, h = 1.2, C_1 = 1.1, C_2 = 0.02$

Based on above input data and Using the software mathematica-5.1, calculate the optimal value of U*(T,t1), T* and t1* simultaneously by equation no. (8), equation no. (9) And equation no. (10).

$$U^* (T, t_1) = 1379.62, T^* = 2.8031, t_1^* = 1.6482$$

Example -2 . Let us consider $A = 350, \alpha = 150, \beta = 0.21, b = 0.17, h = 1.1, C_1 = 1.4, C_2 = 0.01$

$$P (T, t_1) = 1281.50, T^* = 2.22141, t_1^* = 1.48592$$

5. SENSITIVITY ANALYSIS

We have study the effects of changes of the parameters on the optimal values of S(T, t1), T* & t1* derived by proposed method .The sensitivity analysis is performed in view of the numerical example given above. We have executed sensitivity analysis by changing the parameters a , b ,α , and β as +20% , +50% , -20% and -50%. All remaining parameters have original values with respect to these changes. The corresponding changes in S(T,t1) T* & t1* are showed in below table.1

parameters	% change	T*	t1*	S (T, t1)
a	-50	3.95571	2.58215	78.1315
	-20	2.97069	2.08240	771.315
	+20	3.00993	1.70750	2508.6869
	+50	3.08009	1.91768	4110.4955
α	-50	2.95645	2.02388	1589.5347
	-20	2.87390	2.12810	1570.8666
	+20	3.67173	1.93818	1565.3933
	+50	3.76248	1.94990	1545.7257
β	-50	3.39729	2.22046	1499.6804
	-20	3.30004	1.86548	1500.3134
	+20	3.20408	1.91124	1550.1047
	+50	3.10878	1.75736	1553.5795
b	-50	5.01628	1.99738	342.3532
	-20	4.87102	1.87911	967.1480
	+20	3.00856	1.75506	3998.8802
	+50	3.19306	1.63501	2599.5976

We study above table.1 brings out the following.

We observed that as parameter a and b increase, optimal value of T* and t1* decrease while the average total profit S (T, t1) of an inventory system increases, whereas parameter a and b decrease, optimal value of T* and t1* increase while the average total profit S (T, t1) of an inventory system decreases. It is interesting to observe that deterioration parameter α increase, optimal value of T* and t1* increase while the average total profit S (T, t1) of an inventory system decreases. If deterioration parameter α decrease, optimal value of T* and t1* decrease while the average total profit S (T, t1) of an inventory system increases. Second deterioration parameter β increase, optimal value of T* and t1* decrease while the average total profit S (T, t1) of an inventory system increases. If deterioration parameter β decrease, optimal value of T* and t1* increase while the average total profit S (T, t1) of an inventory system decreases.

6. GRAPHICAL REPRESENTATION OF CONCAVITY OF THE PROFIT FUNCTION

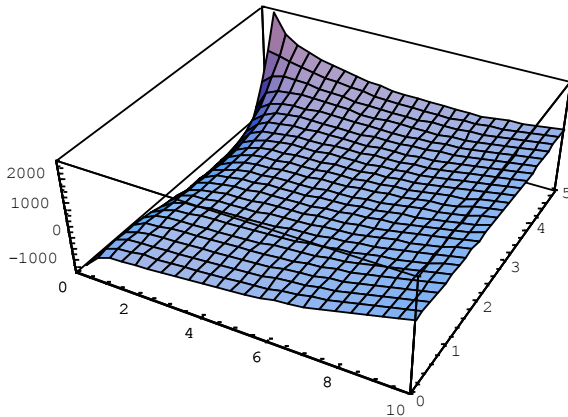


Fig. 1

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