On Beta Combination Labeling Graphs

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ABSTRACT
Let G(V,E) be a graph with p vertices and q edges.A graph G(p,q) is said to be a Beta combination graph if there exist a bijection f: V(G) $\rightarrow$ {1,2 …., p } such that the induced function B_f: E(G)$\rightarrow$N, N is a natural number, given by B_f (uv)= $\frac{[f(u)+f(v)]!}{f(u)!f(v)!}$,every edges uv $\in$ G and are all distinct and the function f is called the Beta combination labeling of G [8].In this paper, we prove quadrilateral snake Q_n,double triangular snake , alternate triangular snake $A(T_n)$, alternate quadrilateral snake $A(Q_n)$, helm H_n ,the gear graph,Comb $P_n \Theta K_1$,the graph $C_n \Theta K_1$ and the diamond graph are the Beta combination graphs.

General Terms
Mathematical subject classification (2010) 05C78.

Keywords
Beta combination graph and Beta combination labeling.

1. INTRODUCTION
Graph labeling ,where the vertices and edges are assigned real values or subsets of a set are subject to certain conditions.Throughout this paper, by a graph we mean a finite, undirected, simple graph. The vertex set and the edge set of a graph G are denoted by V(G) and E(G) respectively. Let G(p,q) be a graph with p = | V(G) | vertices and q = | E(G)| edges. A detailed survey of graph labeling can be found in [6].Combinations play a major role in combinatorial problems.The concept of beta combination labeling of graphs was introduced in [8] which is a logical-mathematical attempt. We use the following definitions in the subsequent sections.

Definition 1.1. [8]
A graph G(p,q) is said to be a Beta combination graph if there exist a bijection f: V(G) $\rightarrow$ {1,2 …., p } such that the induced function Bf: E(G) $\rightarrow$ N, N is a natural number, given by Bf (uv) = $\frac{[f(u)+f(v)]!}{f(u)!f(v)!}$, every edges uv $\in$ G and are all distinct and the function f is called the Beta combination labeling.

Definition 1.2.[5]
A quadrilateral snake is obtained from a path u1,u2,…,un by joining ui,ui+1 to new vertices vi ,wi .That is ,every edge of the path is replaced by the cycle.

Definition 1.3.[5]
A triangular snake is obtained from a path v1,v2,….,vn by joining vi and vi+1 to a new vertex wi for i=1,2,….,n-1.

Definition 1.4.[5]
A double triangular snake consists of two triangular snakes that have a common path.That is , a double triangular snake is obtained from a path $u_1,u_2,\ldots,u_n$ by joining u_i and u_{i+1} to new vertex v_i for i=1,2,…,n-1 and to a new vertex w_i for i=1,2,…,n-1.

Definition 1.5.[7]
An alternate triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, \ldots,u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i .That is every alternative edge of a path is replaced by a cycle C_3.

Definition 1.6.[7]
An alternate quadrilateral snake $A(Q_n)$ is obtained from a path $u_1, u_2, \ldots,u_n$ by joining u_i and u_{i+1}(alternatively) to new vertex v_i ,w_i respectively and then joining v_i and w_i .That is every alternative edge of a path is replaced by a cycle C_4.

Definition 1.7.[5]
The wheel W_n ($n \geq 3$) is obtained by joining all nodes of cycle C_n to a further node called the center, and contains (n+1) nodes and 2n edges.

Definition 1.8 .[5]
The helm H_n is the graph obtained from a wheel by attachaing a pendent edge at each vertex of the n-cycle.

Definition 1.9 .[5]
A gear graph is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of the n-cycle.

Definition 1.10.[5]
The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1(which has p points)and p copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

In this paper, we prove quadrilateral snake Q_n,double triangular snake , alternate triangular snake $A(T_n)$, alternate quadrilateral snake $A(Q_n)$, helm H_n ,the gear graph,Comb $P_n \Theta K_1$,the graph $C_n \Theta K_1$ and the diamond graph are the Beta combination graphs.

2. MAIN RESULTS
Theorem: 2.1 The quadrilateral snake Q_n is a beta combination graph.

Proof: Let Qn be the quadrilateral snake with 3n-2 vertices u1, u2, …,un ,v1,v2,..,vn-1, w1,w2, …,wn-1.Let u1, u2, …,un be the vertices of the path Pn and every ui and ui+1 are joined to new vertices vi and wi respectively and every vi and wi are joined by an edge vi wi for $1 \leq i \leq$ n-1.

Then $E(Q_n)$={ u_iu_{i+1} ; u_iv_i ; $u_{i+1}w_i$; v_i w_i if $1 \leq i \leq n-1$ } and $|E(Q_n)|$ = 4n-4.Define a bijection f:$V(Q_n) \rightarrow$\{1,2,…,3n-2\} by $f(u_i)=3i-2$ if $1 \leq i \leq n$ and $f(v_i)=3i-1$ if $1\leq i\leq n-1$; $f(w_i)=3i$ if $1\leq i\leq n-1$.And f induces B_f: $E(Q_n) \rightarrow$N by

$B_f(uv) = \dfrac{[f(u)+f(v)]!}{f(u)!f(v)!}$, for every edges uv of Q_n and are all distinct.

Theorem: 2.2 Every double triangular snake is a beta combination graph.

Proof: Let G be a double triangular snake with 3n-2 vertices $\{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_{n-1}, w_1, w_2, \ldots, w_{n-1}\}$.

$E(G) = \{ u_iu_{i+1} \; ; \; u_iv_i \; ; u_iw_i \; ; u_{i+1}v_i \; ; \; u_{i+1} w_i \text{ if } 1 \le i \le n-1 \}$ and $|E(G)| = 5n-5$. Define $f: V(G) \to \{1,2,\ldots,3n-2\}$ by

$f(u_i) = 2n-2+i$ if $1 \le i \le n$ and $f(v_i) = 2i-1$ if $1 \le i \le n-1$;

$f(w_i) = 2i$ if $1 \le i \le n-1$. And f induces that $B_f: E(G) \to N$ by

$B_f(uv) = \dfrac{[f(u)+f(v)]!}{f(u)!f(v)!}$, for every edges uv of G and are all distinct.

Example:2.3 The beta combination labeling of a double triangular snake is shown in the Fig-1.

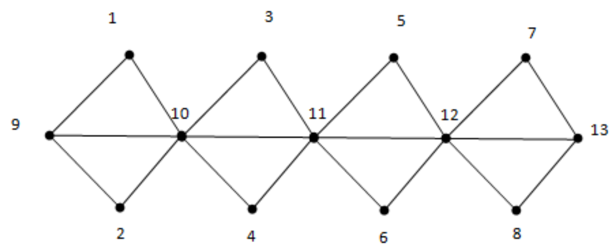

Fig-1.

Theorem: 2.4 Every alternate triangular snake $A(T_n)$ is a beta combination graph.

Proof:
Case (i) If the triangle starts from u_1.

In this case $A(T_n)$ has (3n/2) (n is even) vertices $\{u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_{n/2}\}$ such that $u_1,u_2,\ldots,u_n$ be the vertices of P_n and every u_{2i-1} and u_{2i} are adjacent to v_i for $1 \le i \le (n/2)$.

$E(A(T_n)) = \{u_iu_{i+1}$ if $1 \le i \le n-1$; $u_{2i-1}v_i$; v_iu_{2i} if $1 \le i \le (n/2)\}$

and $|E(A(T_n))| = 2n-1$. Define $f: V(A(T_n)) \to \{1,2,\ldots,(3n/2)\}$ by $f(u_{2i-1}) = 3i-2$ if $1 \le i \le (n/2)$; $f(u_{2i}) = 3i$ if $1 \le i \le (n/2)$ and

$f(v_i) = 3i-1$ if $1 \le i \le (n/2)$.

Case (ii) If the triangle starts from u_2.

In this case $A(T_n)$ has (3n-2)/2 (n is even) vertices $\{u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_{((n-2)/2)}\}$ such that $u_1,u_2,\ldots,u_n$ be the vertices of path P_n and every u_{2i} and u_{2i+1} are adjacent to v_i for $1 \le i \le (n-2)/2$.

$E(A(T_n)) = \{u_iu_{i+1}$ if $1 \le i \le n-1$; $u_{2i}v_i$; v_iu_{2i+1} if $1 \le i \le (n-2)/2\}$.

and $|E(A(T_n))| = 2n-3$. Define $f: V(A(T_n)) \to \{1,2,\ldots,(3n-2)/2\}$ by $f(u_{2i-1}) = 3i-2$ if $1 \le i \le (n/2)$; $f(u_{2i}) = 3i-1$ if $1 \le i \le (n/2)$ and $f(v_i) = 3i$ if $1 \le i \le (n-2)/2$.

In the above two cases, f induces $B_f: E(A(T_n)) \to N$ by

$B_f(uv) = \dfrac{[f(u)+f(v)]!}{f(u)!f(v)!}$, for every edges uv of $A(T_n)$ and are all distinct.

Theorem: 2.5 Every alternate quadrilateral snake $A(Q_n)$ is a beta combination graph.
Proof: Case (i) If the quadrilateral starts from u_1.
In this case $A(Q_n)$ has 2n vertices $\{u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_{n/2}, w_1,w_2,\ldots,w_{n/2}\}$ such that $u_1,u_2,\ldots,u_n$ be the vertices of P_n and u_{2i-1} is adjacent to v_i and u_{2i} is adjacent to w_i and v_i is adjacent to w_i for $1 \le i \le (n/2)$.

$$E(A(Q_n)) = \begin{cases} u_iu_{i+1} \text{ if } 1 \le i \le n-1 \\ u_{2i-1}v_i \; ; u_{2i}w_i \; ; v_iw_i \text{ if } 1 \le i \le \dfrac{n}{2} \end{cases}$$

and $|E(A(Q_n))| = (5n-2)/2$. Define $f: V(A(Q_n)) \to \{1,2,\ldots,2n\}$ by $f(u_{2i-1}) = 4i-3$ if $1 \le i \le (n/2)$; $f(u_{2i}) = 4i$ if $1 \le i \le (n/2)$;

$f(v_i) = 4i-2$ if $1 \le i \le (n/2)$ and $f(w_i) = 4i-1$ if $1 \le i \le (n/2)$.

Case (ii) If the quadrilateral starts from u_2. In this case $A(Q_n)$ has 2n-2 vertices $\{u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_{(n-2)/2}, w_1,w_2,\ldots,w_{(n-2)/2}\}$ such that $u_1,u_2,\ldots,u_n$ be the vertices of path P_n and every u_{2i} is adjacent to v_i and u_{2i+1} is adjacent to w_i and v_i is adjacent to w_i for $1 \le i \le (n-2)/2$.

$$E(A(Q_n)) = \begin{cases} u_iu_{i+1} \text{ if } 1 \le i \le n-1 \\ u_{2i}v_i \; ; u_{2i+1}w_i \; ; v_iw_i \text{ if } 1 \le i \le \left(\dfrac{n-2}{2}\right) \end{cases}$$

and $|E(A(Q_n))| = (5n-8)/2$. Define $f: V(A(Q_n)) \to \{1,2,\ldots,2n-2\}$ by $f(u_{2i-1}) = 4i-3$ if $1 \le i \le (n/2)$; $f(u_{2i}) = 4i-2$ if $1 \le i \le (n/2)$; $f(v_i) = 4i-1$ if $1 \le i \le (n-2)/2$ and $f(w_i) = 4i$ if $1 \le i \le (n-2)/2$.

In the above two cases, f induces $B_f: E(A(Q_n)) \to N$ by

$B_f(uv) = \dfrac{[f(u)+f(v)]!}{f(u)!f(v)!}$, for every edges uv of $A(Q_n)$ and are all distinct.

Example:2.6 The beta combination labeling of alternate quadrilateral snake $A(Q_4)$ is shown in the Fig-2.

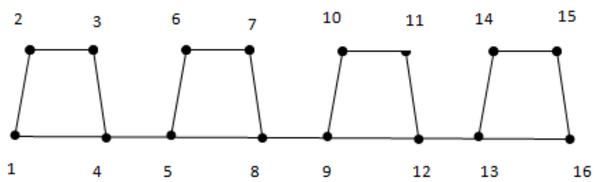

Fig-2.

Theorem: 2.7 Every helm H_n is a beta combination graph.
Proof: Let H_n be the helm graph with 2n+1 vertices $u_1, u_2, \ldots, u_n, u_{n+1}, v_1, v_2, \ldots, v_n$ such that $u_1, u_2, \ldots, u_n$ be the vertices of cycle C_n and u_{i+1} be the center vertex and every vertex v_i is adjacent to u_i through n pendent edges for $1 \le i \le n$.
Let $\{u_iu_{i+1}$ if $1 \le i \le n-1$; u_1u_n ; u_iu_{n+1} if $1 \le i \le n$; $u_i v_i\}$ be the 3n edges of H_n. Define f: $V(H_n) \rightarrow \{1,2, \ldots, 2n+1\}$ by

$f(u_i) = n+i$ if $1 \le i \le n+1$ and $f(v_i) = i$ if $1 \le i \le n$. And f induces that B_f: $E(H_n) \rightarrow N$ by $B_f(uv) = \frac{[f(u)+f(v)]!}{f(u)!f(v)!}$, for every edges uv of H_n and are all distinct.

Theorem:2.8 Every gear graph admits a beta combination labeling.
Proof: Let G be a gear graph with 2n+1 vertices $u_1, u_2, \ldots, u_n, u_{n+1}, v_1, v_2, \ldots, v_n$ such that $u_1, u_2, \ldots, u_n$ be the vertices of cycle C_n and every v_i between every pair of adjacent vertices of C_n for $1 \le i \le n$. $E(G) = \{u_iu_{n+1}$ if $1 \le i \le n$; u_iv_i if $1 \le i \le n$; v_iu_{i+1} if $1 \le i \le n-1$; $v_n u_1\}$ and $|E(G)| = 3n$. Define a bijection $f: V(G) \rightarrow \{1,2,\ldots,2n\}$ by $f(u_i)=2i-1$ if $1 \le i \le n$ and $f(u_i) = 2n+1$; $f(v_i)=2n-2(i-1)$ if $1 \le i \le n$. And f induces that B_f: $E(G) \rightarrow N$ by $B_f(uv) = \frac{[f(u)+f(v)]!}{f(u)!f(v)!}$, for every edges uv of G and are all distinct.
Example:2.9 The beta combination labeling of helm H_5 is shown in the Fig-3.

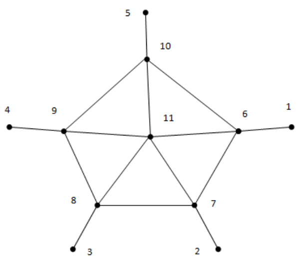

Fig-3.

Theorem:2.10 The Comb $P_n \Theta K_1$ admits a beta combination labeling.
Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of Path P_n and v_i be the vertex of i^{th} copy of the complete graph K_1 for $1 \le i \le n$. Therefore Comb $P_n \Theta K_1$ has 2n vertices.
Then $E(P_n \Theta K_1) = \{u_iu_{i+1}$ if $1 \le i \le n-1$; u_iv_i if $1 \le i \le n\}$ and $|E(P_n \Theta K_1)| = 2n-1$. Define $f: V(P_n \Theta K_1) \rightarrow \{1,2,\ldots,2n\}$ by $f(u_i)=2i-1$ and $f(v_i)=2i$ if $1 \le i \le n$. And f induces that B_f: $E(P_n \Theta K_1) \rightarrow N$ by $B_f(uv) = \frac{[f(u)+f(v)]!}{f(u)!f(v)!}$, for every edges uv of $P_n \Theta K_1$ and are all distinct.

Example: 2.11 The beta combination labeling of Comb $P_5 \Theta K_1$ is shown in the Fig-4 .

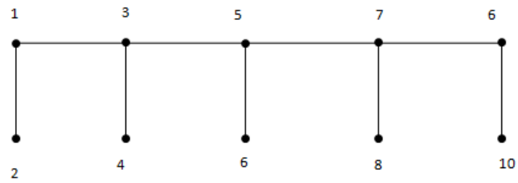

Fig-4

Theorem :2.1 The graph $C_n \Theta K_1$ is a beta combination graph.
Proof: Let $u_1, u_2, \ldots, u_n$ be the n vertices of C_n and v_i be the vertex of i^{th} copy of the complete graph K_1 for $1 \le i \le n$. Therefore $C_n \Theta K_1$ has 2n vertices. $E(C_n \Theta K_1) = \{u_iu_{i+1}$ if $1 \le i \le n-1$; u_1u_n ; u_iv_i if $1 \le i \le n\}$ and $|E(C_n \Theta K_1)| = 2n$.
Define a bijection $f: V(C_n \Theta K_1) \rightarrow \{1,2,\ldots,2n\}$ by $f(u_i)=n+i$ if $1 \le i \le n$ and $f(v_i)=i$ if $1 \le i \le n$. And f induces that B_f: $E(C_n \Theta K_1) \rightarrow N$ by $B_f(uv) = \frac{[f(u)+f(v)]!}{f(u)!f(v)!}$, for every edges uv of $C_n \Theta K_1$ and are all distinct.
Example:2.13 The beta combination labeling of $C_8 \Theta K_1$ is shown in the Fig-5.

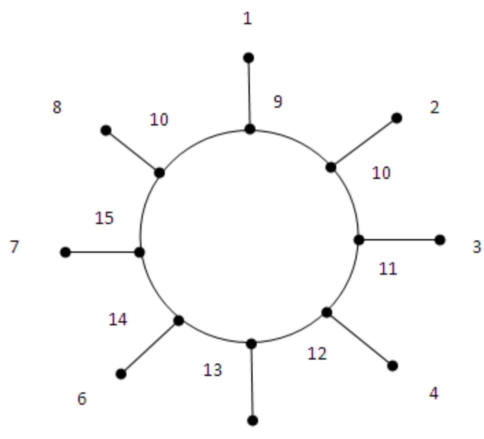

Fig-5.

Theorem :2.14 Every diamond graph is a Beta Combination graph.
Proof: Let G be a diamond graph with 4 vertices u_1, u_2, u_3, u_4. $E(G) = \{u_iu_{i+1}$ if $1 \le i \le n-1$; u_1u_4 ; $u_2u_4\}$ and $|E(G)| = 5$. Define $f: V(G) \rightarrow \{1,2,3,4\}$ is defined by $f(u_i)= i$, if $1 \le i \le 4$. And f induces that $B_f: E(G) \rightarrow N$ by $B_f(uv) = \frac{[f(u)+f(v)]!}{f(u)!f(v)!}$ for every edges uv in G and are all distinct.

3. CONCLUSION

We have planned to find applications of beta combination graphs.

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