(i,j) - r^g Closed Sets in Bitopological Spaces

[1]C. Janaki
L.R.G. Govt Arts College (W), Tirupur.

[2]D. Savithiri,
Sree Narayana Guru College, Coimbatore.

ABSTRACT
The aim of this paper is to introduce a new class of sets called (i,j) - r^g closed sets and a new class of maps called D^(i,j) continuous maps and D^(i,j)- irresolute maps in bitopological spaces. Also we introduce some new spaces called (i,j) – $T^\wedge_{1/2}$, (i,j) - $^\wedge T_{1/2}$,*$T^\wedge_{1/2}$, $^\wedge T^*_{1/2}$ and $^\wedge T_{rg}$ and obtain their basic properties.

Mathematics Subject Classification: 54A10
Keywords
(i,j) - r^g closed sets, (i,j) - r^g open sets, (i,j)-$T^\wedge_{1/2}$, (i,j) - $^\wedge T_{1/2}$, (i,j) - *$T^\wedge_{1/2}$, (i,j)- $^\wedge T^*_{1/2}$, $^\wedge T_{rg}$ spaces, D^(i,j)-continuity.

1. INTRODUCTION
A triplet (X,τ_1,τ_2) , where X is a non-empty set and τ_1 , τ_2 are topologies on X, is called a bitopological space and Kelly [5] has initiated the study of such spaces. In 1985, Fututake [3] introduced the concepts of g - closed sets in bitopological spaces. Extensive research on the generalization of various concepts of topology by considering bitopological spaces was done by several authors. Later on N.Palaniappan [9] has investigated the concept of regular generalized closed sets in topological spaces. The purpose of this paper is to introduce the concepts of r^g closed sets , $T^\wedge_{1/2}$ spaces, $^\wedge T_{1/2}$ spaces and r^g continuity for bitopological spaces and investigate some of their properties.

2. PRELIMINARIES
If A is a subset of X with a topology τ , then the closure of A is denoted by τ-cl(A) or cl(A) , the interior of A is denoted by τ-int(A) or int(A) and the complement of A in X is denoted by A^c.

DEFINITIONS 2.1:
Definition 2.1.1: A subset A of a space (X, τ) is called an

(1) (i,j)-preopen[7] set if A $\subseteq$ τj-int(τi- cl(A)) and (i,j)-preclosed[7]set ifτj- cl(τi-int(A)) $\subseteq$ A.

(2) (i,j) semi-open[6] set if A $\subseteq$ τj-cl(τi-int(A)) and (i,j) semi-closed[6]set if τj -int(τi-cl(A)) $\subseteq$ A.

(3) (i,j) α-open set[10] if A$\subseteq$$\tau$j-int($\tau$i-cl($\tau$j-int(A))) and (i,j) α-closed[10] set if τj-cl(τi-int(τj-cl(A)))$\subseteq$ A.

The semi-closure (resp. α-closure, semi pre-closure) of a subset A of (X, τ) is denoted by τj-scl(A) (resp. τj -αcl(A) and τj -spcl(A)) and is the intersection of all semi-closed (resp. τj -α-closed and τj semi-preclosed) sets containing A.

Definition 2.1.2: The intersection of all g-closed sets containing A is called the g-closure of A and it is denoted by τ-gcl (A) or gcl (A).

Throughout this paper X and Y always represent nonempty bitopological spaces (X,τ1,τ2) and (Y,σ1,σ2) on which no separation assumed unless explicitly mentioned and the integers i, j, k $\in$ {1,2}. For a subset A of X, τi-cl(A) (resp. τi-int(A), τi-gcl(A)) denote the closure(resp. interior, g-closure) of A with respect to the topology τi . The family of all regular open sets of X with respect to the topology τi is represented by RO(X,τi) and the family of all τj- closed sets by Fj . The pair of topologies is denoted by (τi,τj).

Definition 2.1.3: A subset A of a topological space (X, τ_1, τ_2) is said to be

(i) (i,j)-g-closed [2] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U $\in$ τ_i.

(ii) (i,j)-g*-closed [10] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open τ_i.

(iii) (i,j)-rg-closed[9] if τ_j-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in τ_i.

(iv) (i,j)-gpr-closed[4] if τ_j-pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in τ_i.

(v) (i,j)-wg closed [3] if τ_j-cl(τ_i-int(A)) $\subseteq$ U whenever A $\subseteq$ U and U $\in$ τ_i.

The family of all (i,j) – g-closed (resp. (i,j) – rg closed,(i,j)-gpr – closed and (i,j)-wg-closed) subsets of a bitopological space (X, τ_1 , τ_2) is denoted by D (i,j) (resp. D_r (i,j), ς (i,j) and W(i,j).

Definition 2.1.4: A bitopological space (X , τ_1, τ_2) is said to be

(i) an (i,j)- $T_{1/2}$ space if every (i,j)-g-closed set is τ_j-closed.

(ii) a strongly pairwise $T_{1/2}$ space if it is both (1,2)- $T_{1/2}$ and (2,1)- $T_{1/2}$.

(iii) an (i,j)-T_b space if every (i,j)-gs-closed set is τ_j - closed.

(iv) an (i,j) – $T^*_{1/2}$ space if every (i,j) – g* closed set is τ_j - closed.

Definition 2.1.5: A map f: (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is called

(i) τ_j -- σ_k –continuous if $f^{-1}(V)$ $\in$ τ_j for every V $\in$ σ_k .

(ii) D(i,j) - σ_k- continuous (resp. D_r(i,j)- σ_k - continuous, ς(i,j)-σ_k- continuous, W(i,j)-σ_k- continuous) if the inverse image of every σ_k - closed set is (i,j)-g-closed (resp. (i,j)-rg closed, (i,j)-gpr-closed , (i,j)-wg-closed) set in (X ,τ_1 ,τ_2).

(iii) rwg- σ_k-continuous (resp. gs-σ_k-continuous and swg-σ_k-continuous) if the inverse image of every σ_k -closed set is (i,j)-rwg-closed(resp. (i,j)-gs-closed and (i,j)-swg closed) set in (X,τ_1 ,τ_2).

3. (i,j)-r^g closed sets

In this section we introduce the concept of (i,j) – r^g closed sets in bitopological spaces.

Definition 3.1: A subset A of a topological space (X,τ_1,τ_2) is said to be an (i,j)-r^g closed set if τ_j-gcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U $\in$ RO(X,τ_i).

We denote the family of all (i,j)-r^g closed sets of (X,τ_1,τ_2) by D^(i,j).

Remark 3.2: By setting $\tau_1 = \tau_2$ in definition 3.1, (i,j)-r^g-closed set is an r^g closed set.

Theorem 3.3:

(i) Every τ_j-closed set is (i,j)-r^g closed.
(ii) Every (i,j)-g-closed set is (i,j)-r^g closed.
(iii) Every (i,j)-rg-closed set is (i,j)-r^g closed.
(iv) Every (i,j)-g*-closed set is (i,j)-r^g closed.

Proof: Straight Forward.

Remark 3.4:
The converse of the above theorem is not true as seen from the following examples.

Example 3.5:
Let X = {a,b,c} , τ_1 = {X,φ,{a},{b},{a,b}}, τ_2 = {X,φ,{a},{c},{a,b},{a,c}}. Then the subset {a,c} is (1,2)-r^g closed but not τ_2-closed in (X,τ_1,τ_2).

Example 3.6:
Let X = {a,b,c,d}, τ_1 = {X,φ,{a},{b,c},{a,b,c}}, τ_2 = {X,φ,{a,b},{d},{a,b,d}}. Then the subset {b} is (1,2)-r^g closed but it is not (1,2)-g-closed , (1,2)-rg closed. The subset {a,b} is (1,2)-r^g closed but it is not (1,2)-g*-closed.

Theorem 3.7:
Every (i,j)-gpr closed , (i,j)-ω closed , τ_j- g-closed set is (i,j)-r^g closed.

Proof: Straight Forward.

Remark 3.8:
The following example shows that the converse of the above theorem need not be true.

Example 3.9: Let X = {a,b,c,d}, τ_1 = {X,φ,{a},{c},{a,c},{c,d},{a,c,d}}, τ_2 = {X,φ,{a} ,{c}, {a,c},{a,b,c}}

1. Let A = {c} , then A is (1,2)-r^g closed but it is not (1,2)-gpr closed set in (X,τ_1,τ_2).

2. Let B = {b}, then B is (1,2)-r^g closed but it is not (1,2)-ω closed in (X,τ_1,τ_2).

3. The subset {a,b} is (1,2)-r^g closed but it is not τ_2-g-closed.

Remark 3.10:
(i,j)-r^g closed sets and (i,j)-wg closed sets are independent.

Example 3.11:
Let X = {a,b,c,d} , τ_1 = {X,φ,{a},{c,d},{a,c,d}} , τ_2 = {X,φ,{a},{b},{a,b},{a,b,c}}. Let A = {c,d} , then A is (1,2)-r^g closed but it is not (1,2)-wg closed.

Example 3.12:
Let X = {a,b,c,d} , τ_1 = {X,φ,{a},{a,b},{c,d},{a,c,d}} , τ_2 = {X,φ,{a},{c},{a,c},{c,d}, {a,c,d}}. Let A = {c}, then A is (1,2)-wg closed set but it is not (1,2)- r^g closed.

Remark 3.13:
The concepts of (i,j)-preclosed sets and (i,j)-r^g closed sets are independent as seen in the following example.

Example 3.14:
In example 3.11, the subset {a} of (X,τ_1,τ_2) is (1,2) - r^g closed but it is not an (1,2)-preclosed, the subset {c} is (1,2)-preclosed but it is not an (1,2)-r^g closed set.

Remark 3.15: The concepts of (i,j)-gp closed sets and (i,j)-r^g closed sets are independent as seen in the following example.

Example 3.16:
Let X = {a,b,c}, τ_1 = {X,φ,{a},{b},{a,b},{b,c}}, τ_2 = {X,φ,{a},{c},{a,c}. Let A = {c}, then A is (1,2)- r^g closed but it is not (1,2)- gp closed. Let B = {c}, then B is (1,2) gp closed but not (1,2) – r^g closed.

Remark 3.17:
The concepts of (i,j)-gs closed sets and (i,j)-r^g closed sets are independent as seen in the following example.

Example 3.18:

- Let X = {a,b,c,d}, τ_1 = {X,φ,{a},{b},{a,b},{b,c}}, τ_2 = {X,φ,{a},{c},{a,c}, {c,d},{a,c,d}}. The subset {a} is (1,2) – gs closed but it is not (1,2) – r^g closed.
- Let X = {a,b,c}, τ_1 = {X,φ,{a},{b},{a,b}}, τ_2 = {X,φ,{a},{b,c}}, the subset {a,b} is (1,2)- r^g closed but it is not (1,2) - gs closed.

Remark 3.19:
(i,j) - sg closed sets and (i,j)-r^g closed sets are independent as seen in the following example.

Example 3.20:

- Let X = {a,b,c,d}, τ_1 = {X,φ,{a},{b},{a,b},{a,b,c}}, τ_2 = {X,φ,{c},{d},{b,d},{c,d},{a,c,d}, {b,c,d}}. In the space (X,τ_1,τ_2), the subset {d} is (1,2) - r^g closed but not (1,2) - sg closed.
- Let X = {a,b,c,d}, τ_1 = {X,φ,{a},{b},{a,b},{a,b,c}}, τ_2 = {X,φ,{a},{c},{d},{a,c},{c,d}, {a,c,d}}. Let A = {a}. Then A is (1,2) - sg closed but it is not (1,2) - r^g closed.

The above discussions are summarized in the following diagram 3.1.

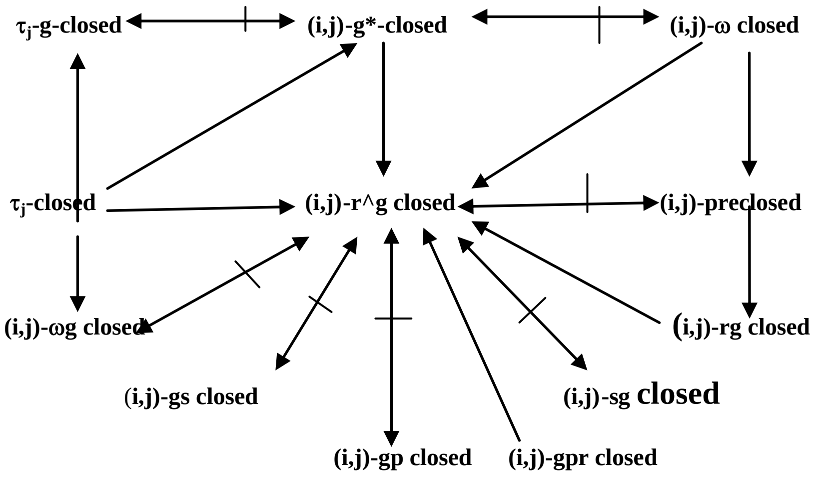

Fig 3.1

where A ⟶ B represents A implies B but not conversely, and A ←|→ B represents A and B are independent.

Theorem 3.21:
If A, B $\in$ D^(i,j) , then A$\cup$B $\in$ D^(i,j).

Remark 3.22:
The intersection of two (i,j)-r^g closed sets need not be (i,j)-r^g closed as seen in the following example.

Example 3.23:
Let X = {a,b,c,d} ,τ_1 = {X,φ,{a},{a,b},{c,d},{a,c,d}} ,τ_2 = {X,φ,{b},{a,b},{c,d},{b,c,d}}. Let A = {a,c} , B = {b,c}, then A and B are (1,2)-r^g closed sets. But A $\cap$ B = {c} is not (1,2)-r^g closed set.

Remark 3.24:
D^ (1,2) is generally not equal to D^(2,1).

Example 3.25:
In example 3.10. , D^(1,2) $\neq$ D^(2,1)

Theorem 3.26:
If $\tau_1 \subseteq \tau_2$ in (X,τ_1,τ_2) , then D^(2,1) $\subseteq$ D^(1,2).

Proof: Straight Forward.

The converse of the above theorem is not true as seen in the following example.

Example 3.27:
Let X = {a,b,c},τ_1 = {X,φ,{b},{c},{a,c},{b,c}}, τ_2 = {X,φ,{a},{b,c}}. Then D^(2,1) $\subseteq$ D^(1,2) but τ_1 is not contained in τ_2.

Theorem 3.28:
For each element x of (X,τ_1,τ_2) , {x} is either τ_i- regular closed or $\{x\}^c$ is (i,j)-r^g closed.

Proof:
If {x} is not τ_i-regular closed, then the only τ_i-regular open set containing $\{x\}^c$ is X. Thus $\{x\}^c$ is (i,j)-r^g closed .

Theorem 3.29:
If A is (i,j)-r^g closed , then τ_j-gcl(A) – A contains no non-empty τ_i- regular closed set.

Proof:
Let A be an (i,j)-r^g closed set and F be a τ_i- regular closed set such that F $\subseteq$ τ_j-gcl(A)-A i.e., F $\subseteq$ τ_j-gcl(A). Since A $\in$ D^(i,j), we have τ_j-gcl(A) $\subseteq$ F^c , this implies F $\subseteq$ $[\tau_j\text{-gcl(A)}]^c$. Thus F $\subseteq$ τ_j-gcl(A) $\cap$ $[\tau_j\text{-gcl(A)}]^c$ = φ. Therefore τ_j-gcl(A) –A contains no non-empty τ_i-regular closed set.

Corollary 3.30:
If A is (i,j)-r^g closed then A is τ_j-g-closed iff τ_j-gcl(A) – A is τ_i regular closed.

Proof:
Necessity: If A is τ_j-g-closed, then τ_j-gcl(A) = A i.e., τ_j-gcl(A) – A = φ and hence τ_j-gcl(A) – A is τ_i-regular closed.

Sufficiency: If τ_j-gcl(A) – A is τ_i- regular closed then by theorem 3.22, τ_j-gcl(A) – A = φ i.e., τ_j-gcl(A) = A . Hence A is τ_j-g-closed.

Theorem 3.31:
If A is (i,j)-r^g closed set such that A $\subseteq$ B $\subseteq$ τ_j-gcl(A) then B is also (i,j)-r^g closed set.

Proof:
Let U be τ_i-regular open set such that B $\subseteq$ U. Since A is (i,j)-r^g closed, τ_j-gcl(A) $\subseteq$ U. Now B $\subseteq$ τ_j-gcl(A) implies τ_j-gcl(B) $\subseteq$ τ_j-gcl(τ_j-gcl(A)) = τ_j-gcl(A) $\subseteq$ U implies τ_j-gcl(B) $\subseteq$ U. Hence B is also (i,j)-r^g closed set.

Theorem 3.32:
If A is an τ_i-regular open and (i,j)-r^g closed set of (X,τ_1,τ_2), then A is τ_j-g-closed.

Proof:
Let A be τ_i-regular open and (i,j)-r^g closed. Since A is (i,j)-r^g closed, we have τ_j-gcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-regular open . This implies τ_j-gcl(A) = A. Hence A is τ_j-g-closed.

Theorem 3.33:
In a bitopological space (X,τ_1,τ_2) , $RO(X,\tau_i) \subseteq GC(X,\tau_j)$ iff every subset of X is an (i,j)-r^g closed set.

Proof:
Suppose that $RO(X,\tau_i) \subseteq GC(X,\tau_j)$. Let A be a subset of X such that $A \subseteq U$ where $U \in RO(X,\tau_i)$. Then τ_j-gcl(A) $\subseteq \tau_j$-gcl(U) = U and hence A is (i,j)-r^g closed.

Conversely, suppose that every subset of X is (i,j)-r^g closed set. Let $U \in RO(X,\tau_i)$. Since U is (i,j)-r^g closed, we have τ_j-gcl(U) $\subseteq$ U. Therefore $U \in GC(X,\tau_j)$ and hence $RO(X,\tau_i) \subseteq GC(X,\tau_j)$.

4. (i,j) – r^g open sets:
Definition 4.1:
A subset A of a bitopological space (X,τ_1,τ_2) is called (i,j) – r^g open if A^c is (i,j) – r^g closed.

Theorem 4.2:
In a bitopological space $(X,\tau_{1,}\tau_2)$,

(i) Every τ_i – open set is (i,j)-r^g open but not conversely.

(ii) Every (i,j) – g-open and (i,j) – g*-open sets are (i,j) – r^g open.

Proof: Obvious.

Theorem 4.3:
If A and B are (i,j) - r^g open sets then $A \cap B$ is also an (i,j) - r^g open set in (X,τ_1,τ_2).

Proof:
Let A and B be two (i,j) - r^g open sets. Then Ac and Bc are (i,j) - r^g closed sets. By theorem 3.14, Ac ∪ Bc = (A∩B)c is (i,j) - r^g closed .Therefore A∩B is (i,j) - r^g open in (X,τ1,τ2).

Theorem 4.4:
If (i,j) - gintA ⊂ B ⊂ A and if A is (i,j) - r^g open then B is (i,j) -r^g open.

Proof: Given (i,j) - gintA ⊂ B ⊂ A, then X – A ⊂ X – B ⊂ (i,j) - gcl(X – A). Since A is (i,j) - r^g open, X –A is (i,j) - r^g closed. This implies X – B is (i,j) - r^g closed. Hence B is (i,j) -r^g open.

5. (i,j) – $T^\wedge_{1/2}$ spaces:
In this section we introduce four new spaces in bitopological spaces.

Definition 5.1:
A bitopological space (X,τ_1,τ_2) is said to be

(a) an (i,j) – $T^\wedge_{1/2}$ space if every (i,j) - r^g closed set is τ_j-g-closed.

(b) a strongly pairwise (1,2) - $T^\wedge_{1/2}$ space if it is both (1,2) – $T^\wedge_{1/2}$ and (2,1) – $T^\wedge_{1/2}$.

(c) an (i,j) - $^\wedge T_{1/2}$ space if every (i,j) - r^g closed set is (i,j) – g-closed.

(d) a strongly pairwise (1,2) - $^\wedge T_{1/2}$ if it is both (1,2) - $^\wedge T_{1/2}$ and (2,1) - $^\wedge T_{1/2}$ spaces.

(e) an (i,j) - $*T^\wedge_{1/2}$ space if every (i,j) - r^g closed set is τ_j – g*-closed.

(f) a strongly pairwise (1,2) - $*T^\wedge_{1/2}$ if it is both (1,2) – $*T^\wedge_{1/2}$ and (2,1) - $*T^\wedge_{1/2}$ spaces.

(g) an (i,j) - $^\wedge T*_{1/2}$ space if every (i,j) - r^g closed set is (i,j) - g*- closed.

(h) a strongly pairwise (1,2) - $^\wedge T*_{1/2}$ if it is both (1,2) - $^\wedge T*_{1/2}$ and (2,1) - $*T^\wedge_{1/2}$ spaces.

(i) an (i,j) - $^\wedge T_{rg}$ space if every (i,j) - r^g closed is (i,j) - rg closed.

(j) a strongly pairwise (1,2) - $^\wedge T_{rg}$ space if it is both (1,2) - $^\wedge T_{rg}$ and (2,1) - $^\wedge T_{rg}$.

Example 5.2:
Let X = {a,b,c} , τ_1 = {X,φ,{a},{b,c}}, τ_2 = {X,φ,{b},{c},{a,c},{b,c}}. Then $(X,\tau_{1,}\tau_2)$ is both (1,2) - $T^\wedge_{1/2}$ space and (2,1) – $T^\wedge_{1/2}$ space hence strongly pairwise –$T^\wedge_{1/2}$ space.

Example 5.3:
Let X = {a,b,c}, τ_1 = {X,φ,{a},{b,c}}, τ_2 = {X,φ,{a},{b},{a,b}}. Then $(X,\tau_{1,}\tau_2)$ is (1,2) - $^\wedge T_{1/2}$ space.

Example 5.4:
Let X = {a,b,c} , τ_1 = {X,φ,{a},{c},{a,b},{a,c}}, τ_2 = {X,φ,{a},{b},{a,b}}. Then (X,τ_1,τ_2) is both (1,2) - $^\wedge T_{1/2}$ and (2,1) - $^\wedge T_{1/2}$ spaces. Hence it is strongly pair wise (1,2) - $^\wedge T_{1/2}$ space.

Example 5.5:
Let X = {a,b,c}, τ_1 = {X,φ,{a},{c},{a,c},{a,b}}, τ_2 = {X,φ,{a},{b},{a,b}}. Then (X,τ_1,τ_2) is an (i,j) - $*T^\wedge_{1/2}$ space.

Theorem 5.6:
A bitopological space (X,τ_1,τ_2) is an (i,j) – $T^\wedge_{1/2}$ space iff {x} is τ_j-g-open or τ_i-regular closed for each $x \in X$.

Proof: Suppose that {x} is not τ_i-regular closed, then by preposition 3.21, it is trivially $\{x\}^c$ is (i,j) – r^g closed set. Since X is (i,j) – $T^\wedge_{1/2}$ space, $\{x\}^c$ is τ_j – g-closed and thus {x} is τ_j-g-open.

Conversely, let $A \subseteq X$ be (i,j) – r^g closed. Let $x \in \tau_j$-gcl(A). To show $x \in A$.

Case (i): Suppose {x} is τ_j-g-open, since $x \in \tau_j$-gcl(A) , then $\{x\} \cap A \neq \varphi$ implies $x \in A$.

Case (ii): Suppose {x} is τ_i-regular closed. If $x \notin A$, then $A \subseteq X$-{x}. Since A is (i,j) – r^g closed and X-{x} is regular open , τ_j-gcl(A) $\subseteq$ X – {x}. Hence $x \notin \tau_j$-gcl(A) which is a contradiction. Therefore $x \in A$.

Thus in both the cases, A = τ_j-gcl (A) or equivalently A is τ_j-g-closed. Hence $(X,\tau_{1,}\tau_2)$ is an (i,j)-$T^\wedge_{1/2}$ space.

Remark 5.7:
(X,τ_1) space is not generally $T^\wedge_{1/2}$ space even if (X,τ_1,τ_2) is (i,j) – $T^\wedge_{1/2}$ space as seen in the following example.

Example 5.8:
Let X = {a,b,c}, τ_1 = {X,φ,{a},{b,c}}, τ_2 = {X,φ,{b},{c},{a,c},{b,c}}. Then $(X,\tau_{1,}\tau_2)$ is $T^\wedge_{1/2}$ space but (X,τ_1) is not $T^\wedge_{1/2}$ space.

Theorem 5.9:
If (X,τ_1,τ_2) is strongly pair wise $T^\wedge_{1/2}$ space then it is strongly pair wise $T_{1/2}$ space but not conversely.

Proof: Straight Forward

The converse of the above theorem is not true as seen in the following example.

Example 5.10:
Let $X = \{a,b,c\}$, $\tau_1 = \{X,\varphi,\{a\},\{b\},\{a,b\},\{b,c\}\}$, $\tau_2 = \{X,\varphi,\{a\},\{b\},\{a,b\}\}$. Then (X,τ_1,τ_2) is strongly pair wise $T_{1/2}$ space but it is not strongly pair wise $T^\wedge_{1/2}$ space.

Theorem 5.11:
If A bitopological space (X,τ_1,τ_2) is both (i,j) - $^\wedge T_{1/2}$ space and (i,j) – $T_{1/2}$ space then it is (i,j)- $T^\wedge_{1/2}$ space.

Proof:
Let $A \subseteq X$ be an (i,j) – r^g closed set. Since X is (i,j) - $^\wedge T_{1/2}$ space , A is (i,j) – g-closed.

This implies that A is τ_j- closed, since X is (i,j) – $T_{1/2}$ space. Every τ_j- closed set is τ_j – g-closed. Hence (X,τ_1,τ_2) is $T^\wedge_{1/2}$ space.

Theorem 5.12:
(i) Every (i,j) - $T^\wedge_{1/2}$ space is (i,j) - $*T^\wedge_{1/2}$ space.

(ii) Every (i,j) - $^\wedge T_{1/2}$ space is (i,j) - $^\wedge T^*_{1/2}$ space.

(iii) Every (i,j) - $^\wedge T_{1/2}$ space is (i,j) - $^\wedge T_{rg}$ space.

(iv) Every (i,j) - $^\wedge T^*_{1/2}$ space is (i,j) - $^\wedge T_{rg}$ space.

Proof: Straight forward.

Theorem 5.13:
If a bitopological space (X,τ_1,τ_2) is both (i,j)-$^\wedge T_{1/2}$ and (i,j)-$T_{1/2}$ then it is (i,j)-$^\wedge T^*_{1/2}$ space.

Proof:
Let (X,τ_1,τ_2) be both (i,j)-$^\wedge T_{1/2}$ and (i,j)-$T_{1/2}$ space . Let A be an (i,j) – r^g closed set in X. By hypothesis A is (i,j) – g-closed, since X is (i,j)-$T^\wedge_{1/2}$. This implies that A is τ_j- g-closed, since it is (i,j)-$T_{1/2}$. Every τ_j-closed set is (i,j) – g*-closed. Hence (X,τ_1,τ_2) is (i,j)-$^\wedge T^*_{1/2}$ space.

Example 5.14:
1. Let $X = \{a,b,c\}$, $\tau_1 = \{X,\varphi,\{a\},\{b\},\{a,b\}\}$, $\tau_2 = \{X,\varphi,\{a\},\{b,c\}\}$. Then (X,τ_1,τ_2) is (1,2) – $T^\wedge_{1/2}$ space but it is not an (1,2) - $^\wedge T^*_{1/2}$ space

2. Let $= \{a,b,c\}$, $\tau_1 = \{X,\varphi,\{a\},\{c\},\{a,b\},\{a,c\}\}$, $\tau_2 = \{X,\varphi,\{a\},\{b\},\{a,b\}\}$. The space (X,τ_1,τ_2) is (2,1) -$^\wedge T^*_{1/2}$ space but it is not an (2,1) – $T^\wedge_{1/2}$ space.

Theorem 5.15:
A bitopological space (X,τ_1,τ_2) is both (i,j) - $^\wedge T_{1/2}$ space and $T^*_{1/2}$ space then it is $T^\wedge_{1/2}$ space.

Proof:
Let X be both (i,j) - $^\wedge T^*_{1/2}$ and $T^*_{1/2}$ spaces. Let $A \subseteq X$ be an (i,j) - r^g closed set. Since X is (i,j) - $^\wedge T^*_{1/2}$ space, A is (i,j) - g* closed. By hypothesis, A is τ_j- closed. Every τ_j- closed set is τ_j - g-closed. Hence (X,τ_1,τ_2) is $T^\wedge_{1/2}$ space.

Remark 5.16:
(i,j) - $^\wedge T_{rg}$ spaces and (i,j)- $*T^\wedge_{1/2}$ spaces are independent to each other as seen in the following example.

Example 5.17:
- Let $X = \{a,b,c\}$, $\tau_1 = \{X,\varphi,\{a\},\{c\},\{a,b\},\{a,c\}\}$, $\tau_2 = \{X,\varphi,\{a,c\}\}$. Then (X,τ_1,τ_2) is (1,2)-$*T^\wedge_{1/2}$space but it is not (1,2)-T_{rg} space.
- Let $X = \{a,b,c\}$, $\tau_1 = \{X,\varphi\{a\},\{b\},\{a,b\},\{b,c\}\}$, $\tau_2 = \{X,\varphi, \{a\},\{c\},\{a,c\}\}$. Then (X,τ_1,τ_2) is (1,2) - $^\wedge T_{rg}$ space but it is not (1,2)- $*T^\wedge_{1/2}$ space.

The above discussions are summarized as shown in the following figure.

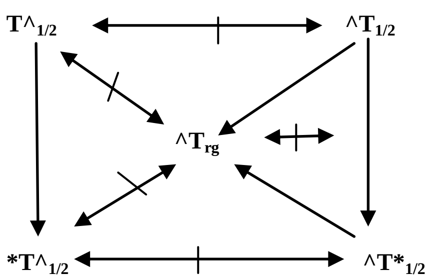

Fig 5.1

where A ⟶ B represents A implies B but not conversely, and A B represents A and B are independent.

6. D^(i,j) - continuous and D^(i,j) - irresolute functions:
In this section we introduce D^(i,j) continuous and D^(i,j) irresolute functions in bitopological spaces.

6.1. D^(i,j) - Continuous functions:
Definition 6.1.1:
A map $f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is called D^(i,j) - σ_k - continuous if the inverse image of every σ_k – closed set is an (i,j) - r^g closed in (X,τ_1,τ_2).

Theorem 6.1.2:
If f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is (i) τ_j-σ_k continuous (ii) D(i,j)-σ_k continuous (iii) D*(i,j)-σ_k-continuous then it is D^(i,j) - σ_k-continuous.

Proof: Straight Forward.

The converse of the above theorem need not be true as seen in the following example.

Example 6.1.3:
Let X = {a,b,c}, τ_1 = {X,φ,{a},{b},{a,b}}, τ_2 = {X,φ,{a,b}}. Y = {p,q},σ_1 = {Y,φ,{p}}, σ_2 = {Y,φ,{q}}. Define f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ by f(a) = f(b) = q, f(c) = p. Then f is D^ (1,2) - σ_k- continuous but it is not τ_2-σ_1 continuous.

Example 6.1.4:
In the example 6.3, the map f is D^(1,2)-σ_1 continuous but it is not D(1,2)-σ_1 continuous.

Example 6.1.5:
Let X = Y = {a,b,c}, τ_1 = {X,φ,{a},{b},{a,b}}, τ_2 = {X,φ,{a,b}}. σ_1 = {Y,φ{a},{b,c}}, σ_2 = {Y,φ,{b},{c},{b,c},{a,c}}. Define a map f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ by f(a) = c, f(b) = b, f(c) = a. Then f is D^ (1,2) - σ_1 continuous but it is not D*(1,2) - σ_1 continuous.

Theorem 6.1.6:
If f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is (i,j) – rwg - σ_k continuous, then it is D^(i,j) - σ_k-continuous.

Proof: Follows from the definition.

The converse of the above theorem is not true as seen in the following example.

Example 6.1.7:
Let X = {a,b,c} = Y. τ_1 = {X,φ,{a},[c},{a,b},{a,c}}, τ_2 = {X,φ,{a,c}}, σ_1 = {Y,φ,{a}}, σ_2 = {Y,φ,{c}}. Define f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$, the identity mapping, then f is D^(1,2) - σ_2 continuous but it is not an (1,2) – rwg- σ_2 continuous.

Remark 6.1.8:
D^(i,j) - σ_k – continuous maps are independent with (i) (i,j) - σ_k –wg continuous (ii) (i,j) – gs - σ_k continuous (iii) (i,j) - swg- σ_k continuous maps as seen in the following examples.

Example 6.1.9:
1. Let X = Y = {a,b,c,}. τ_1 = {X,φ,{a},{c},{a,b},{a,c}}, τ_2 = {X,φ,{a,c}}, σ_1 = {Y,φ, {a,b}}, σ_2 = {Y,φ,{a,c}}. Define f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$, the identity map, then f is (1,2) - σ_1-wg continuous but it is not D^(1,2) - σ_k continuous.

2. Let X = Y = {a,b,c}. τ_1 = {X,φ,{a},{b},{a,b},{a,c}}, τ_2 = {X,φ,{a},{c},{a,c}}, σ_1 = {Y,φ, {c}}, σ_2 = {Y,φ,{a,c}}. Then the identity map f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is D^(1,2) - σ_2-continuous but it is not (1,2) – wg-σ_2 continuous.

Example 6.1.10:
1. Let X = {a,b,c,} = Y. τ_1 = {X,φ,{a},{b},{a,b}}, τ_2 = {X,φ,{a},{b,c}}, σ_1 = {Y,φ,{a,c}}, σ_2 = {Y,φ,{b}}. Define an identity map f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$. Then f is D^(1,2) - σ_1 – continuous but it is not (1,2) – gs - σ_1 continuous.

2. Let X = {a,b,c} = Y, τ_1 = {X,φ,{a},{c},{a,b},{a,c}}, τ_2 = {X,φ,{a},{b},{a,b}}, σ_1 = {Y,φ, {b}}, σ_2 = {Y,φ,{b,c}}. Define f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ by f(a) = b, f(b) = a, f(c) = c, then f is (1,2)- gs - σ_2 – continuous but it is not D^(1,2) - σ_2 – continuous.

Example 6.1.11:
1. Let X = {a,b,c} = Y, τ_1 = {X,φ,{a},{b},{a,b},{b,c}}, τ_2 = {X,φ,{a},{c},{a,c}}, σ_1 = {Y,φ,{b,c}} , σ_2 = Y,φ,{c}}. Define the identity map f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$. Then f is D^(1,2) - σ_1 continuous but it is not (1,2) - σ_1 continuous.

2. Let X = Y = {a,b,c}, τ_1 = {X,φ,{a},{c},{a,c},{a,b}}, τ_2 = {X,φ,{a,c}}, σ_1 = {Y,φ, {a,c}}, σ_2 = {Y,φ,{a,b}}. Define f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ by f(a) = a, f(b) = c, f(c) = b. Then f is (1,2) –swg- σ_2 continuous but it is not D^(1,2) - σ_2 continuous.

6.2 D^(i,j) - irresolute functions:
Definition 6.2.1:
A function f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is called D^(i,j) – irresolute map if $f^{-1}(V)$ is (i,j) – r^g closed set of (X,τ_1,τ_2) for every (i,j) – r^g closed set V of (Y,σ_1,σ_2).

Theorem 6.2.2:
Every D^(i,j) – irresolute map is D^(i,j) - σ_k continuous.

Proof:
Let f be D^(i,j) – irresolute. Let V be a σ_k- closed set. Then $f^{-1}(V)$ is (i,j) – r^g closed, since f is D^(i,j) – irresolute. Hence f is D^(i,j) - σ_k continuous.

Remark 6.2.3:
The converse of the above theorem need not be true as seen in the following example.

Example 6.2.4:
Let X = Y = {a,b,c}. τ_1 = {X,φ,{a},{b},{a,b}}, τ_2 = {X,φ,{a},{b,c}}, σ_1 = {X,φ,{a}, {b,c}}, σ_2 = {X,φ,{a},{b},{a,b}}. Define an identity map f : $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$. Then f is D^(1,2) continuous but it is not an D^(1,2) irresolute.

The above discussions are summarized as shown below.

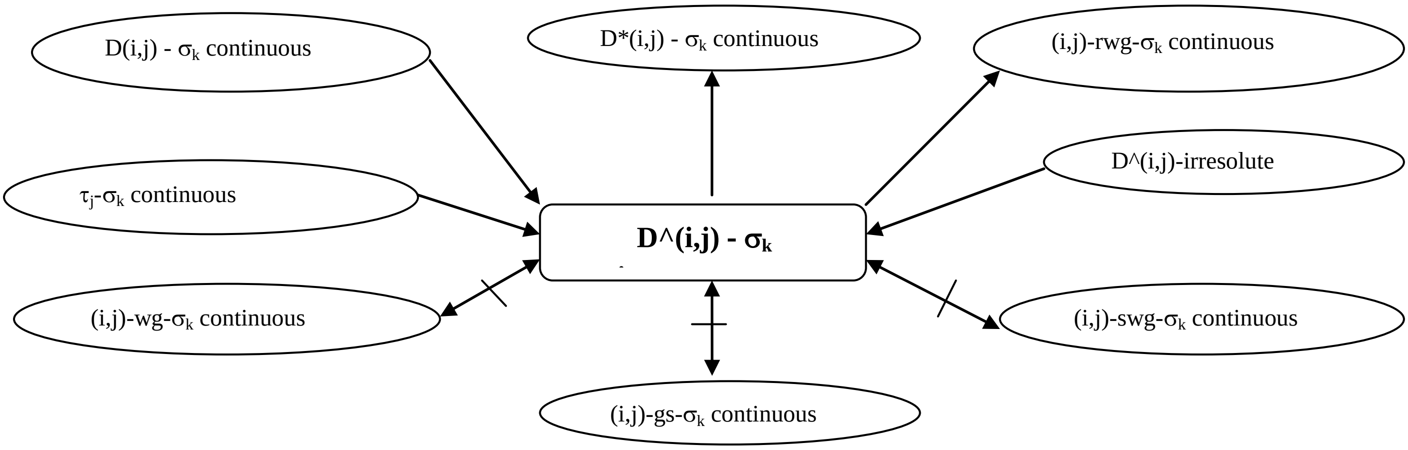

Fig 6.1

REFERENCES:

[1] Arockiarani, Studies on generalizations of generalized closed sets and maps in topological spaces, Ph.D., thesis, Bharathiar Univ., Coimbatore, 1997.

[2] T. Fututake On generalized closed sets in bitopological spaces, Fukuka Univ. Ed. Part III 35(1985) 19-28.

[3] T. Fututake, P. Sundaram and Sheik John , Bull. Fukuoka Univ. Ed. Part III 51 (2002), 1-9.

[4] 4. Y. Gnanambal, On Generalized pre-regular closed sets in topological spaces, Indian J. Pure. App. Math, 28(1997), 351-360.

[5] Kelly.J.C, Bitopological spaces, proceedings, London, Math.Soc. , Vol.13, pp-71-89, 1983.

[6] N. Levine, semi open sets and semi-continuity in topological spaces, Amer.Math.Monthly,70(1963),36-41.

[7] N. Levine, Generalized closed sets in topology, Rend. Circ Mat. Palermo,**19**(2)(1970), 89-96.

[8] H. Maki, J.Umehara and T. Noiri, Every topological space is pre-$T_{1/2}$,Mem.Fac. Sci. Kochi univ.Ser.A. Math.,17(1996),33-42.

[9] N. Palaniappan & K.C.Rao, Regular generalized closed sets, KyungpookMath.3(2)(1993),211

[10] M.Sheik John and P.Sundaram, g*-closed sets in bitopological Spaces, Indian Jour. Of Pure appl.Math.,35(1):71-80,January 2004.

[11] M.Stone, Applications of the theory of Boolean rings to general topology, Trans.Amer. Math.Soc,41(1937),374-481.

[12] M.K.R.S. Veera Kumar, Between closed sets and g closed sets, Mem. Fac. Sci Kochi Univ.(math) , 21(2000), 1-19.