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Sliding Mode Control with Predictive PID Sliding Surface for Improved Performance

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ABSTRACT

In this paper, a sliding mode control system with a predictive proportional-integral-derivative (PPID-SMC) sliding surface is proposed. A robust sliding mode controller is suggested to track the desired trajectory despite uncertainty, set point variations, and external disturbances. The proposed sliding mode controller is chosen to ensure the stability of overall dynamics during the reaching phase and sliding phase. The chattering problem is overcome using a hyperbolic tangent function for the sliding surface. Simulation example is given to illustrate the use of the proposed structure for better performance in terms of time domain specifications over some existing design methods.

General Terms:

Predictive control, Sliding mode control

Keywords:

Sliding Mode Control, Sliding surface, Predictive PID, GPCifx

1. INTRODUCTION

It is well known that physical systems are non-linear in nature. Model uncertainty as well as time varying has been a serious challenge to the control community [1]. Conventional controllers, such as PID, lead-lag or Smith predictors, are sometimes not sufficiently versatile to compensate for these effects. Thus, a SMC could be designed to control nonlinear systems with the assumption that the robustness of the controller will compensate for modeling errors arising from the linearization of the nonlinear model of the process.
Sliding mode control (SMC), first proposed in the early 1950s, has been proved to be able to tackle system uncertainties and external disturbances with good robustness [2, 3, 4]. The dynamic performance of the system under the SMC method can be shaped according to the system specification by an appropriate choice of switching function [5]. Robustness is the best advantage of a sliding mode control and systematic design procedures for sliding mode controllers are well known and available in the literature [2, 5, 6, 7, 8]. In SMC, the dynamic behavior of the system may be tailored by the particular choice of switching functions and the closed-loop response becomes totally insensitive to a particular class of uncertainty [9].
In this paper, a sliding mode controller is designed using a Predictive PID sliding surface. In order to validate the proposed approach, a numerical example is considered. The performance comparison between the proposed structure and the existing control structures is carried out by simulation using MATLAB SIMULINK. The results obtained are compared with the Predictive PID control and Generalized predictive control.

2. BASIC CONCEPT

2.1 Generalized Predictive Control

Generalized predictive control (GPC) is one of the most popular predictive control algorithms developed by Clarke [10]. For satisfying the control objectives, it makes the use of a controlled auto regressive and integrated moving average (CARIMA) model is used to obtain good output predictions and optimize a sequence of future control signals to minimize a multistage cost function defined over a prediction horizon. The inclusion of disturbance is necessary to deduce the correct controller structure.

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + C(z^{-1})\frac{e(t)}{\Delta} \tag{1}$$

where A, B, and C are the polynomials in the backward shift operator z^{-1} and y, and u are the predicted output and control input respectively. The derivation of optimal prediction can be obtained by recursion of Diophantine equation [11],

$$C = E_j A\Delta + z^{-j}F_j \tag{2}$$

$$E_j B = C\tilde{G}_j + z^{-j}\bar{G}_j \tag{3}$$

In GPC, the predictions are posed in terms of increments in control $(\Delta u(j); j \geq t)$. These assumptions are the cornerstone of the GPC approach [12].
The best prediction of $y(t+j)$ is,

$$\hat{y}(t+j|t) = G_j(z^{-1})\Delta u(t+j-1) + F_j(z^{-1})y(t) \tag{4}$$

The prediction in vector can be written as,

$$y = Gu + f \tag{5}$$

where f is the free response of output. The predicted output depends on previous values of output and previous and future values of the control signal. The control signals are used to achieve the objective in GPC by minimizing the cost function given as,

$$\begin{aligned} J(N_1, N_2, N_u) &= \sum_{j=N_1}^{N_2} \delta(j)[\hat{y}(t+j|t) - r(t+j)]^2 \\ &+ \sum_{j=1}^{N_u} \lambda(j)[\Delta u(t+j-1)]^2 \end{aligned} \tag{6}$$

where N_1, N_2 and N_u are the minimum costing horizon, maximum costing horizon and control horizon respectively. $\hat{y}(t+j|t)$ is the optimum j-step ahead prediction of system output, $r(t+j)$ is the future reference trajectory, $\lambda(j)$ and $\delta(j)$ are the weighting sequences. For no constraints, the future control for minimization of cost is,

$$u = (G^TG + \lambda I)^{-1}G^T(r - f) \tag{7}$$

The first element of the control signal u is,

$$\Delta u = K(r - f) \tag{8}$$

where K is the first row of matrix $(G^T G + \lambda I)^{-1} G^T$. The current control is,

$$u(t) = u(t-1) + K(r - f) \tag{9}$$

For $r - f = 0$, there is no control move.

2.2 GPC with steady state weighting

A terminal matching condition, defined as the weighted square of the steady state error, is included in the GPC cost function (equation (6)), to derive GPC with steady state weighting (denoted herein as GPC_{ssw}) [13, 14]. The following quadratic function to be minimized to achieve the control objective is,

$$\begin{aligned} J &= \gamma_y \sum_{j=N_1}^{N_2} [\hat{y}(t+j|t) - r(t+j)]^2 + \lambda \sum_{j=1}^{N_u} [\Delta u(t+j-1)]^2 \\ &+ \gamma \sum_{j=N_1}^{N_u} [\hat{y}(s|t+j-1) - r(s)]^2 \end{aligned} \tag{10}$$

where γ_y, γ, and s are the finite prediction weight, steady state weight, and the steady state value respectively. The first two terms on the right-hand side form the standard generalized predictive control (GPC) objective. The last term corresponds to the additional terms penalizing the squares of errors at the predicted steady state.

2.3 The Predictive PID control law

The PID control law is,

$$u(t) = K_P e(t) + K_I \sum_{i=0}^{t} e(i) + K_D[e(t) - e(t-1)] \tag{11}$$

where K_P, K_I, and K_D are the proportional, integral and derivative control gain respectively.
The incremental control law is determined by applying the differencing operator to the control output as,

$$\begin{aligned} \Delta u(t) &= [(K_P + K_I + K_D) \\ &+ (-K_P - 2K_D)z^{-1} + (K_D)z^{-2}]e(t) \end{aligned} \tag{12}$$

where $e(t)$=$r(t) - y(t)$ is the tracking error between the reference and the output.
The Predictive PID control law can be expressed as,

$$\begin{aligned} \Delta u(t) &= K_I r(t) - [(K_P + K_I + K_D) \\ &+ (-K_P - 2K_D)z^{-1} + (K_D)z^{-2}]y(t) \end{aligned} \tag{13}$$

2.4 Sliding Mode Control

The robustness to the uncertainties becomes an important aspect in designing any control system. Sliding mode control (SMC), originally studied by Utkin [2], is a robust and simple procedure for the control of linear and nonlinear processes based on principles of variable structure control (VSC). It is proved to be an appealing technique for controlling nonlinear systems with uncertainties. Figure 1 shows the graphical representation of SMC using phase-plane, which is made up of the error ($e(t)$) and its derivative ($\dot{e}(t)$). It can be seen that starting from any initial condition, the state trajectory reaches the surface in a finite time (reaching mode), and then slides along the surface towards the target (sliding mode).
The first step of the SMC design requires the design of a custom-made surface. On the sliding surface, the plants dynamics is restricted to the equations of the surface and is robust to match plant uncertainties and external disturbances [15]. At the second step, a feedback control law is required to be designed to provide convergence of a systems trajectory to the sliding surface; thus, the sliding surface should be reached in a finite time. The systems motion on the sliding surface is called the sliding mode.

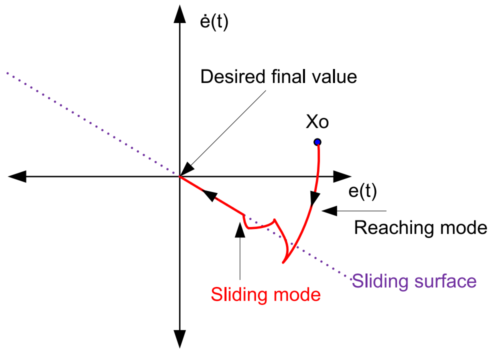

Fig. 1. Graphical interpretation of SMC.

The sliding surface, $S(t)$ [1, 16] depends on the tracking error, $e(t)$ and derivatives of the tracking error is,

$$S(t) = \left(\lambda + \frac{d}{dt}\right)^{n-1} e(t), \tag{14}$$

where n is the system order, and λ is a positive scalar, which helps to shape $S(t)$. λ is selected by the designer, and it determines the performance of the system on the sliding surface [17].
For the second order process ($n = 2$), the first time derivative of the sliding surface (equation (14)) is,

$$\dot{S}(t) = \lambda \dot{e}(t) + \ddot{e}(t), \tag{15}$$

Filippov's construction [18] of the equivalent dynamics is the method normally used to generate the equivalent SMC law. The control objective is to ensure that the controlled variable is driven to its reference value, i.e, in the stationary state, $e(t)$ and its derivatives must be zero. This condition is achieved by,

$$\frac{dS(t)}{dt} = \dot{S}(t) = 0, \tag{16}$$

and substituting it into the system dynamic equations; the control law is thereby obtained.
Once the sliding surface has been selected, a control law is designed so that it drives the controlled variable to its reference value and satisfies equation (16).
The SMC control law ($U_{SMC}(t)$), usually results in a fast motion to bring the state onto the sliding surface, and a slower motion to proceed until a desired state is reached.
The SMC control law consists of two additive parts; a continuous part, $U_c(t)$, and a discontinuous part, $U_d(t)$,

$$U_{SMC}(t) = U_c(t) + U_d(t). \tag{17}$$

In the proposed work, the sliding surface in SMC is designed with the predictive PID control. The design procedure of the proposed work is given in the section below.

3. PROPOSED METHOD

3.1 SMC with Predictive PID sliding surface

Let the tracking error between the reference and the output is $e(t) = r(t) - y(t)$, then a sliding surface in the space of error can be defined using the coefficients obtained for control law (12), called Predictive PID control law of as,

$$S(t) = K_P e(t) + K_I \int_0^t e(t)dt + K_D \frac{de(t)}{dt} \tag{18}$$

If the initial error at time $t = 0$ is $e(0) = 0$, then the tracking problem can be considered as the error remaining on the sliding surface $S(t) = 0$ for all $t \geq 0$. If the system trajectory has reached the sliding surface $S(t) = 0$, it remains on it while sliding into the origin $e(t) = 0$, $\dot{e}(t) = 0$ as shown in figure 1.
The purpose of sliding mode control law is to force error $e(t)$ to approach the sliding surface and then move along the sliding surface to the origin. Therefore it is required that the sliding surface is stable, which means

$$\lim_{t\to\infty} e(t) = 0 \tag{19}$$

This implies that the system dynamics will track the desired trajectory [1].
The control objective is to determine a control $u(t)$ such that the closed-loop system will follow the desired trajectory, that is, the tracking error $e(t)$ should converge to zero. The process of sliding mode control can be divided into two phases, that is, the sliding phase with $S(t) = 0$, $\dot{S}(t) = 0$, and the reaching phase with $S(t) \neq 0$. Corresponding to two phases, two types of control law can be derived separately [1, 19]. In sliding mode the equivalent control is described when the trajectory is near $S(t) = 0$, while the hitting control is determined in the case of $S(t) \neq 0$ [2].
The derivative of the sliding surface defined by equation (18) can be given as,

$$\dot{S}(t) = K_P\dot{e}(t) + K_I e(t) + K_D\ddot{e}(t) \tag{20}$$

A necessary condition for the output trajectory to remain on the sliding surface $S(t)$, is $\dot{S}(0) = 0$ [1, 20, 21],

$$K_P\dot{e}(t) + K_I e(t) + K_D\ddot{e}(t) = 0 \tag{21}$$

If the control gains K_P, K_I, and K_D are properly obtained by proper selection of the prediction horizon, control horizon and weights such that the characteristic polynomial in equation (21) is strictly Hurwitz, that is, a polynomial whose roots lie strictly in the open left half of the complex plane, it implies that,

$$\lim_{t\to\infty} e(t) = 0 \tag{22}$$

When equation (22) satisfies, it indicates that the closed-loop system is stable [22].
The error, $e(t) = r(t) - y(t)$ can be defined in terms of physical plant parameters, where $r(t)$ is the command signal and $y(t)$ is the measured output signal. The second derivative of the error $e(t)$ is,

$$\ddot{e}(t) = \ddot{r}(t) - \ddot{y}(t) \tag{23}$$

The equivalent control $U_c(t)$ [2], is obtained as the solution of the problem $\dot{S}(t) = 0$ which leads to,

$$U_c(t) = [K_P\dot{e} + K_I e(t) + K_D\ddot{r}(t) + K_D\dot{y}(t) + K_D y(t)] \tag{24}$$

In regulatory control, the reference values are constants or step changes. At the moment of transition the derivative control goes to infinite and hence an undesirable ‘kick’ appears in the controller output hence it should be eliminated (i.e the term $\ddot{r}(t) = 0$). The equivalent control or continuous part of the SMC control law becomes,

$$U_c(t) = [K_P\dot{e} + K_I e(t) + K_D\dot{y}(t) + K_D y(t)] \tag{25}$$

The controller must drive the output trajectory to the sliding modes $S(t) = 0$ in presence of disturbances. For this purpose, the Lyapunov function can be chosen as,

$$V(t) = \frac{1}{2}S^2(t) \tag{26}$$

with $V(0) = 0$ and $V(t) > 0$ for $S(t) \neq 0$.
A sufficient condition to guarantee that the trajectory of the error will translate from reaching phase to sliding phase is to select the control strategy, also known as the reaching condition [1],

$$\dot{V}(t) = S(t)\dot{S}(t) < 0, \qquad S(t) \neq 0 \tag{27}$$

To satisfy the above reaching condition, the SMC control law (17) needs to be determined.
The discontinuous part of SMC, ($U_d(t)$), generally incorporates a nonlinear element that includes the switching element of the control law. This part of the controller is discontinuous across the sliding surface, which is designed on the basis of a relay-like function, because it allows for changes between the structures with a hypothetical infinitely fast speed.
In practice, however, it is impossible to achieve the high switching control because of the presence of finite time delays for control computations or limitations of the physical actuators, thus causing chattering around of the sliding surface [1, 2].
Chattering is a high frequency oscillation around the desired equilibrium point. It is undesirable in practice, because it involves high control activity and can excite high frequency dynamics ignored in the modeling of the system [1]. The aggressiveness for reaching the sliding surface depends on the control gain, but if the controller is too aggressive it can collaborate with the chattering.
To reduce the chattering, different approaches can be used to replace the relay-like function. The system robustness is a function of the width of the boundary layer. A thin boundary layer can be introduced around the sliding surface for the hitting control or the discontinuous part of the SMC control law [1, 19] to be,

$$U_d = K_d sat\left(\frac{S}{\phi}\right), \tag{28}$$

where K_d is the positive constants, and ϕ is positive constant, defines the thickness of the boundary layer parameter to reduce chattering. The saturation factor is defined as,

$$\begin{aligned} sat\left(\frac{S}{\phi}\right) &= \left(\frac{S}{\phi}\right) && \text{if } |\frac{S}{\phi}| \leq 1 \\ &= sgn\left(\frac{S}{\phi}\right) && \text{if } |\frac{S}{\phi}| > 1 \end{aligned} \tag{29}$$

This controller is actually a continuous approximation of the ideal relay control [24, 25]. In the proposed work, a hyperbolic tangent function is used instead of a saturation function, to improve the hitting control effort and it is given as [8, 25],

$$U_d = K_d tanh\left(\frac{S}{\phi}\right), \tag{30}$$

where K_d is the tuning parameter responsible for the speed with which the sliding surface is reached.
Therefore the proposed control law becomes,

$$\begin{aligned} U_{SMC}(t) &= [K_P\dot{e} + K_I e(t) + K_D\ddot{r}(t) + K_D\dot{y}(t) + K_D y(t)] \\ &+ K_d tanh\left(\frac{S}{\phi}\right). \end{aligned} \tag{31}$$

4. SIMULATION RESULT

To illustrate the performance of the proposed controller, following second order unstable plant is considered.

$$G_p = \frac{-0.015}{1 - 1.9z^{-1} + 0.935z^{-2}} \tag{32}$$

The different controllers were tested for set point and disturbances changes, applied to the process. The performance of the proposed controller is compared against a predictive PID controller structure [13] and Generalized predicative controller.

Figures 2-4 shows the performance comparisons of the proposed method to Predictive-PID and GPC_{ssw}.

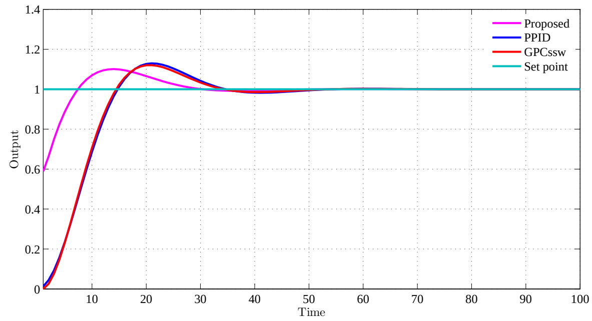

Fig. 2. Output response to step signal

Figures 2 and 3 shows the improvement of the system in terms of settling time and overshoot.
In Figure 4, a step disturbance of 0.1 is applied/removed at the 100 and 200 sampling instants, respectively. It shows that the proposed control law is robust to set point variations and presence of disturbances. Figure 5 shows the corresponding control signal.

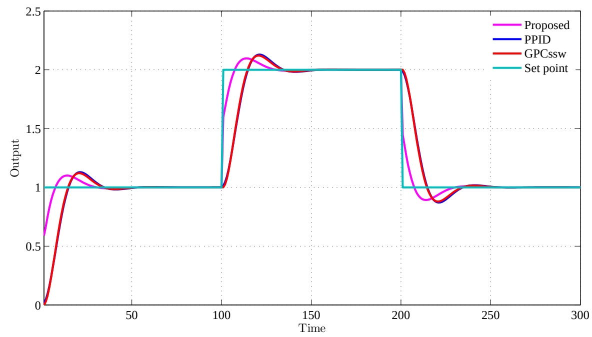

Fig. 3. Output response to set point variations

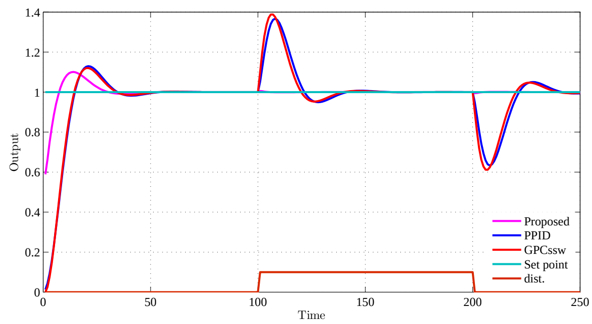

Fig. 4. Output response to 10 % disturbance

Figure 6 shows the comparison of output response of the proposed method to Predictive PID controller for 20% model parameter uncertainty. It proves that the proposed method is robust to model parameter uncertainty.
Table 1 indicates the performance analysis using indices like the

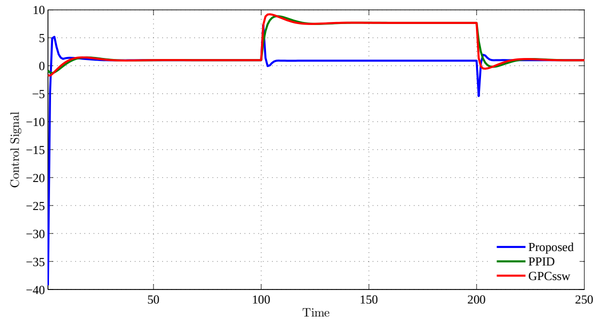

Fig. 5. Control Signal

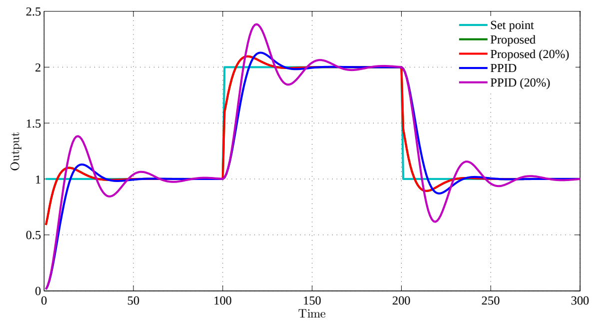

Fig. 6. Output response to 20% model uncertainty

integral of absolute error (IAE), the integral of time weighted absolute error (ITAE) and the integral of squared error (ISE).

Table 1. Performance analysis

Controller	IAE	ITAE	ISE
Proposed	2.537	26.15	0.4023
PPID	4.742	45.87	1.5570
GPC_{ssw}	5.671	41.57	3.8260

5. CONCLUSION

In this study, a sliding mode control with Predictive PID sliding surface has been proposed. An unstable plant is used for the performance analysis. Simulation was carried out using MATLAB to test the effectiveness of the proposed method. In the proposed method, a hyperbolic tangent function has been used in order to avoid the chattering phenomena. The proposed controller ensures the invariance property against parameter uncertainties, set point variations, and disturbances compared with Predictive PID controller and Generalized predictive controller.

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