Analysis of Stochastic Model on a Two-Unit Hot Standby Combined Hardware-Software System

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ABSTRACT
This paper deals with a stochastic model for a two-unit hot standby combined hardware-software system in which one unit is operative and the other is hot standby. The operative unit may have hardware or software failures and goes to repair. This leads to degradation of the system and then hot standby unit is under operation. Further on hardware or software failures of the hot standby unit the system goes to complete failures. Various measures of performance of the system are obtained using semi-Markov process and regenerative point techniques. Numerical results and graphs pertaining to a particular case are also included.

General Terms
Stochastic Process

Keywords
Two-Unit Hot Standby System, Combined hardware-software system, reliability, mean up time, profit, semi-Markov and Regenerative point techniques.

1. INTRODUCTION
In the recent years, a large number of hardware-software systems have been developed and adopted due to their practical applications and common man's affordability and inherent reliability. A hardware-software system consists of hardware and software subsystems. Most of the modern systems like mobiles, computers, robots, missiles, rockets, electronic meters, washing machines etc. are improvement over hardware-software systems. Today society and its Institutions are becoming more and more depend on many such systems. A few of the numerous applications of the hardware and software systems are control of communications and transport systems, automated plants operations and even in routine activities e.g. reservation of tickets, computation of various bills and in business. All these things demands high reliability since failure of any can be costly and hazardous. At present it is a matter of great importance to produce and provide the cost-effective, efficient, user friendly hardware-software system having higher reliability.

For effective performance of the hardware-software system, both its hardware and software components must function with considerable reliability. As the hardware-software systems are more complex and the demand on their reliability has increased exponentially. For the reliability analysis of the one-unit hardware-software system, combined reliability model i.e. considering both hardware and software components were discussed by some researchers such as Boyd and Monahan [1],

Friedman and Tran [2], Hecht and Hecht [3], Huang et al.[4], Iyer and Velardi [5], Kanoun and Ortalo-Borrel [6], Kumar and Malik [7], Kumar and Kumar [8], Martin and

Mathur [9], Teng et al. [10], Trivedi [11], Trivedi et al. [12], Welke et al. [13] etc. However, analysis of a two-unit hot standby hardware-software system with respect to its reliability, availability and profit has not been reported in the literature of reliability modeling.

Keeping this in view, in the present paper, a stochastic model for two identical units hot standby combined hardware-software system is analyzed. Initially, one unit is operative and other is hot standby and any one of the units may have a hardware or software component failure. On failure of a unit either due to hardware or software component failure, the system goes to degraded state and when both the units fails the system goes to complete failure. It is assumed here that hardware and software failures are independent of each other and occurs only due to hardware and software components, respectively. Further, it is assumed that there is single repair facility/engineer that handles all types of hardware or software failures and reaches the system in negligible time.

Other assumptions of the model are:

1. After each hardware/software repair, the system is as good as new.
2. If a unit is under repair, it does not work for the system.
3. The times to failures are exponentially distributed whereas recovery/repair time distributions are general.
4. All random variables are mutually independent.
5. Switching is perfect and instantaneous.

Various measures of system performance are obtained using semi Markov Process and regenerative point techniques. The expressions for various measures of system performance are obtained such as mean time to system failure, mean up time, expected number of software repairs, expected number of hardware repairs, expected number of visits by repairman. Expected profit incurred for the system is also computed using above measures. Various conclusions regarding reliability and profit of the system are drawn on the basis of graphical studies.

2. NOTATIONS

O	:	Operative unit.
F_{sr}	:	Software failure is not recovered and unit is under manual repair.
F_{hr}	:	Hardware failure is not recovered and unit is under manual repair.
F_{sR}	:	Software repair is continuing from the previous state.

F_{hR}	:	Hardware repair is continuing from the previous state.
F_{sW}	:	Unit is under software failure and it is waiting for repair.
F_{hW}	:	Unit is under hardware failure and it is waiting for repair.
λ_s	:	Software failure rate.
λ_h	:	Hardware failure rate.
α_s	:	Software repair rate.
α_h	:	Hardware repair rate.
$g_s(t)$	:	P.d.f. of time to software repair.
$g_h(t)$	:	P.d.f. of time to hardware repair.

The transition diagram depicting the various states of the system is shown in the fig.1.The epochs of entry into the states 0, 1, 2 are regenerative points and thus the states 0, 1, 2 are regenerative states and 3, 4, 5, 6 are failed states.

3. MEASURES OF THE SYSTEM PERFORMANCE

3.1 Transition Probabilities

The transition probabilities p_{ij} are given by

$$p_{01} = \lambda_s/S \qquad p_{02} = \lambda_h/S$$

$$p_{10} = \alpha_s/S_1 \qquad p_{13} = \lambda_s/S_1$$
$$p_{14} = \lambda_h/S_1 \qquad p_{20} = \alpha_h/S_2$$
$$p_{25} = \lambda_s/S_2 \qquad p_{26} = \lambda_h/S_2$$

$$p_{11}^{(3)} = \lambda_s/S_1 \qquad p_{12}^{(4)} = \lambda_h/S_1$$

$$p_{21}^{(5)} = \lambda_s/S_2 \qquad p_{22}^{(6)} = \lambda_h/S_2$$

By these transition probabilities it can be verified that

$$p_{01} + p_{02} = 1 \qquad p_{10} + p_{13} + p_{14} = 1$$

$$p_{20} + p_{25} + p_{26} = 1 \qquad p_{10} + p_{11}^{(3)} + p_{12}^{(4)} = 1$$

$$p_{20} + p_{21}^{(5)} + p_{22}^{(6)} = 1$$

where

$$S = \lambda_s + \lambda_h, \quad S_1 = \lambda_s + \lambda_h + \alpha_s, \quad S_2 = \lambda_s + \lambda_h + \alpha_h,$$

3.2 Mean Sojourn Time

The mean sojourn time (μ_i) in the ith state is the expected first passage time taken by the unit at the ith state before transiting to any other state.

$$\mu_i = \int_0^\infty \Pr(T_i > t)\, dt$$

where T_i is the p.d.f. of device life time.

Therefore,

$$\mu_0 = 1/2S \qquad ; \qquad \mu_1 = 1/S_1 \qquad ;$$

$$\mu_2 = 1/S_2 \qquad ; \qquad K_1 = 1/\alpha_s \qquad ;$$

$$K_2 = 1/\alpha_h$$

3.3. Unconditional Mean Time

The unconditional mean time (m_{ij}) taken by the system to transition for any regenerative state j, when it (time) is counted from epoch of entrance into that state i is, mathematically, stated as:

$$m_{ij} = \int_0^\infty t q_{ij}(t)dt = -q_{ij}^{*'}(0)$$

Thus,

$$m_{01} = \lambda_s/2S^2 \quad ; \qquad m_{02} = \frac{\lambda_h}{2S^2} \quad ; \qquad m_{10} = \alpha_s/S_1^2 \quad ;$$

$$m_{13} = \lambda_s[1 - \alpha_s S/S_1^2 - \alpha_s/S_1^2]/S^2$$

4. OTHER MEASURES OF THE SYSTEM PERFORMANCE

By probabilistic arguments for the regenerative process, the recursive relations for various measures of the system performance are obtained. Then on solving the relations using Laplace-Stieltjes transforms, we get the following measures:

Mean Time to System Failure $\qquad T_0 = \frac{N}{D}$

Mean Up Time $\qquad A_0 = \frac{N_1}{D_1}$

Expected Number of Hardware Repair $\qquad H_{R0} = \frac{N_3}{D_1}$

Expected Number of Software Repairs $\qquad S_{R0} = \frac{N_4}{D_1}$

Expected Number of Visits by the Repairman $\quad V_0 = \frac{N_5}{D_1}$

where

$$D = 1 - p_{01}p_{10} - p_{02}p_{20}$$

$$D_1 = \mu_0\left[p_{10}p_{21}^{(5)} + p_{20}\left(1 - p_{11}^{(3)}\right)\right] + \left(p_{01}p_{20} + p_{21}^{(5)}\right)K_1 + (1 - p_{01}p_{10} - p_{11}^{(3)})K_2$$

$$N = \mu_0 + \mu_1 p_{01} + \mu_2 p_{02}$$

$$N_1 = \mu_0[p_{20}(1 - p_{11}^{(3)}) + p_{10}p_{21}^{(5)}]$$

$$N_2 = \mu_1[p_{01}p_{20} + p_{21}^{(5)}] + \mu_2[1 - p_{01}p_{10} - p_{11}^{(3)}]$$

$$N_3 = [1 - p_{01}]p_{10} + p_{12}^{(4)}$$

$$N_4 = p_{01}p_{20} + p_{21}^{(5)}$$

$$N_5 = (p_{10} + p_{12}^{(4)})p_{20} + p_{10}p_{21}^{(5)}$$

5. PROFIT ANALYSIS

The expected total profit (P_0) incurred to the system in steady state is given by

$$P_0 = C_0 A_0 + C_1 D_0 - C_2 H_{R0} - C_3 S_{R0} - C_4 V_0 - C_5$$

where

C_0 = revenue per unit up time of the system.

C_1 = revenue per unit degradation time of the system.

C_2 = cost per unit of hardware repair.

C_3 = cost per unit of software repair.

C_4 = cost per visit of the repairman.

C_5 = installation cost.

6. PARTICULAR CASE

For the analysis purpose, the following particular case is taken:

$$g_s(t) = \alpha_s e^{-\alpha_s t} \qquad ; \qquad g_h(t) = \alpha_h e^{-\alpha_h t}$$

The values of the various failure rates and repair rates are assumed as, i.e $\lambda_h = .01$, $\lambda_h = .001$, $\lambda_h = .002$, $\lambda_s = .0002$, $\lambda_s = .002$, $\alpha_h = .2$, $\alpha_h = .6$, $\alpha_h = .8$, $\alpha_h = 1.8$, $\alpha_s = .3$, $\alpha_s = .5$, $\alpha_s = .6$, $\alpha_s = .7$, $C_0 = 30000$, $C_1 = 30000$, $C_2 = 500$, $C_3 = 300$,

$C_4 = 20$, $C_5 = 24000$ etc.

7. GRAPHICAL ANALYSIS

Using the above particular case and the assumed values of parameters, various graphs are plotted for mean time to system failure (T_0), mean up time (A_0), mean degradation time (D_0) and profit (P_0) of the system for different values of hardware and software failures rates (λ_h, λ_s) and repair rates (α_h, α_s), respectively.

Fig. 2 and fig. 3 depicts the behavior of mean time to system failure (T_0) with respect to various hardware and software failure rates (λ_h, λ_s), respectively for different values of hardware and software repair rates (α_h, α_s). It can be seen that mean time to system failure decreases as hardware and software failure rates increase and further mean time to system failure increases with higher values of repair rates.

Fig. 4 and fig. 5, shows the behavior of mean up time (A_0) with respect to various hardware and software failure rates (λ_h, λ_s), respectively for different values of hardware and software repair rates (α_h, α_s). From the graphs, it can be seen that mean up time decreases with the increase in the values of hardware and software failure rates and further mean up time increases with higher values of repair rates.

Fig. 6 and fig. 7 reveals the pattern of the profit (P_0) incurred to the system with respect to hardware and software failure rates (λ_h, λ_s). It can be seen that the profit (P_0) of the system decreases as the failure rates increases whereas increases for higher values of repair rates.

8. CONCLUSIONS

It can be observed that reliability, mean up time and profit of the two unit hot standby hardware-software system decrease as hardware and software failure rates increases and these increases with higher values of hardware and software repair rates. The limits of hardware and software failure/repair rates can be obtained for the system to give positive profit that may be quite useful for both the system developer and the system user.

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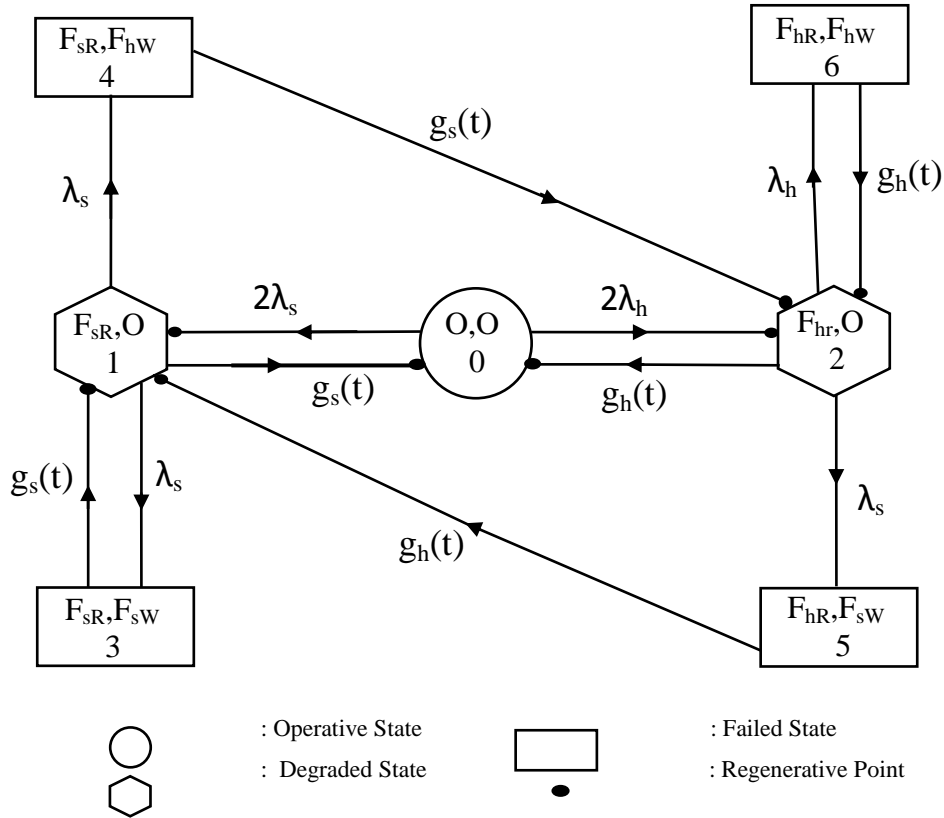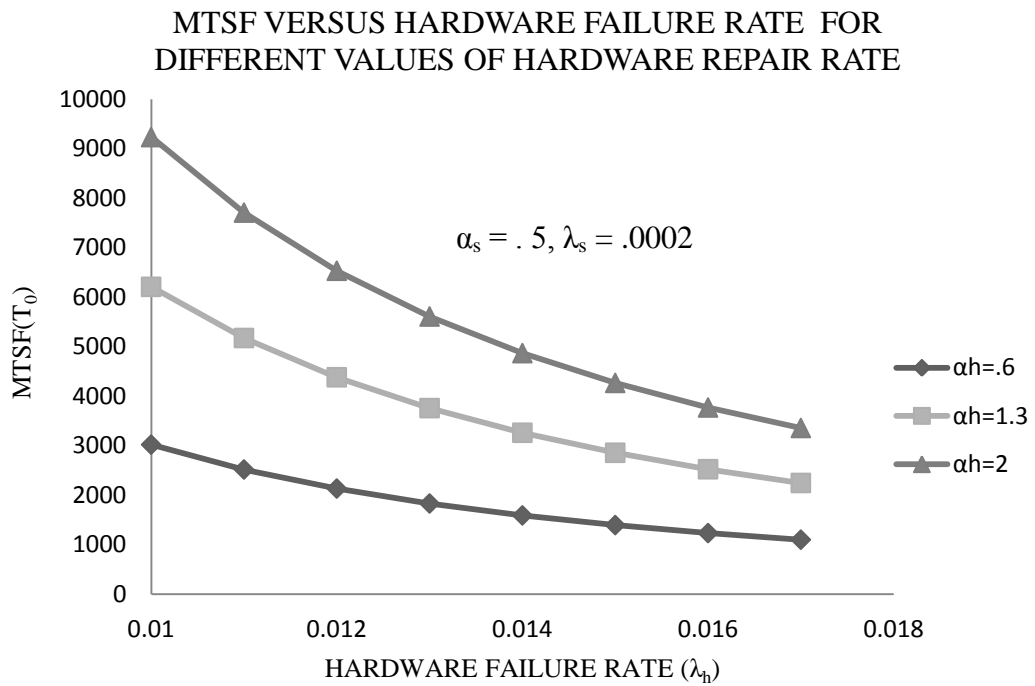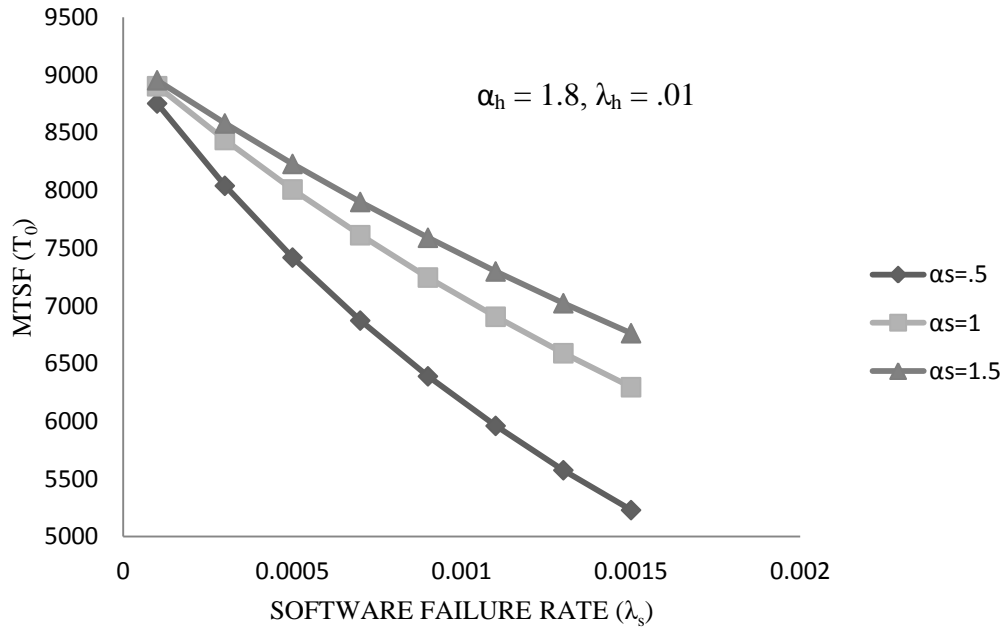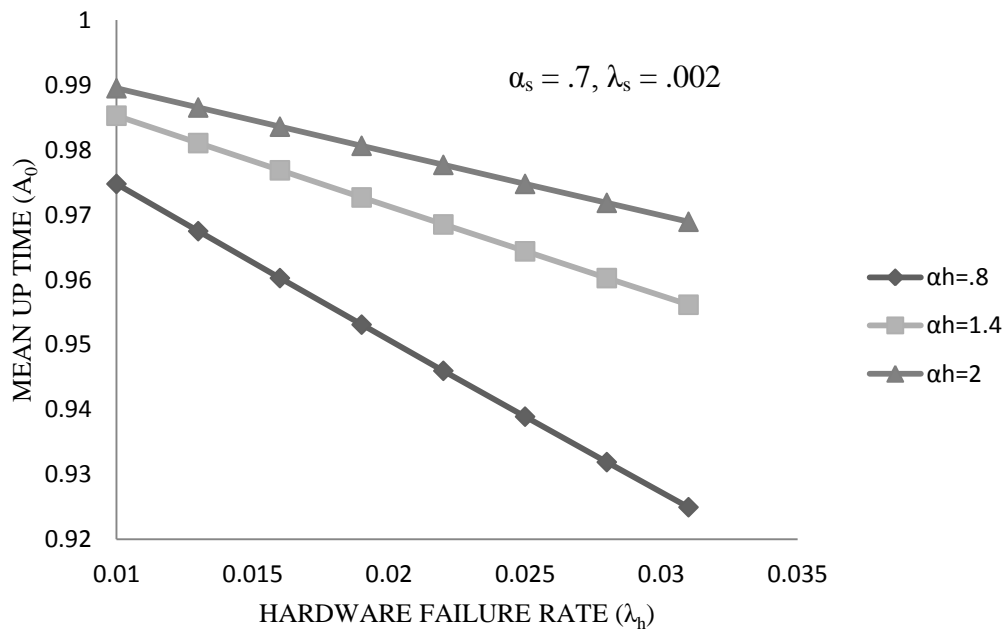
Fig. 1: State Transition Diagram

Fig. 2: MTSF versus Hardware Failure Rate

MTSF VERSUS SOFTWARE FAILURE RATE FOR DIFFERENT VALUES OF SOFTWARE REPAIR RATE
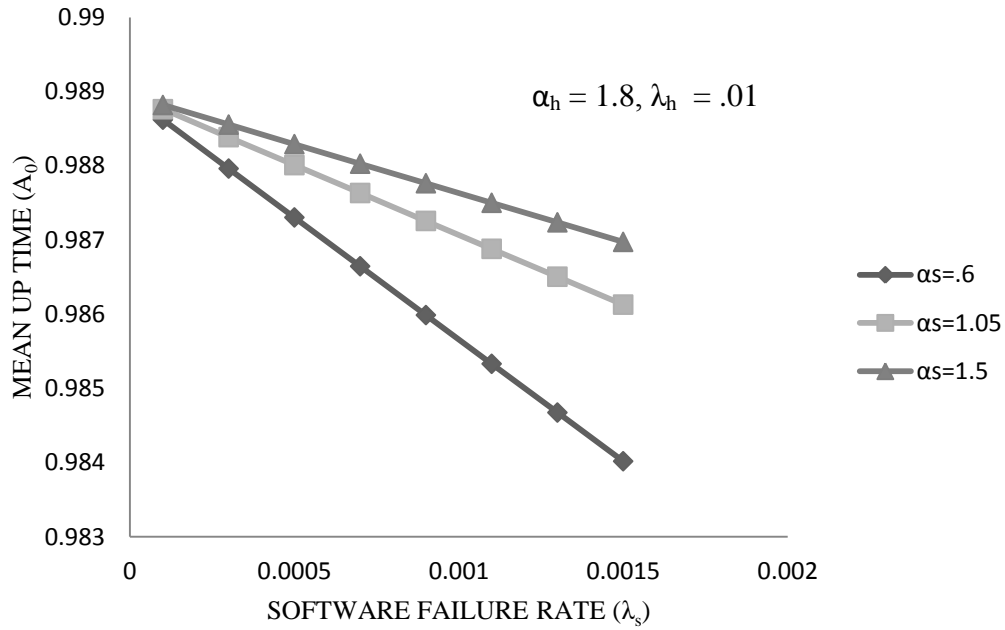

$\alpha_h = 1.8, \lambda_h = .01$

Fig. 3: MTSF Versus Software Failure Rate

MEAN UP TIME VERSUS HARDWARE FAILURE RATE FOR DIFFERENT VALUES OF HARDWARE REPAIR RATE
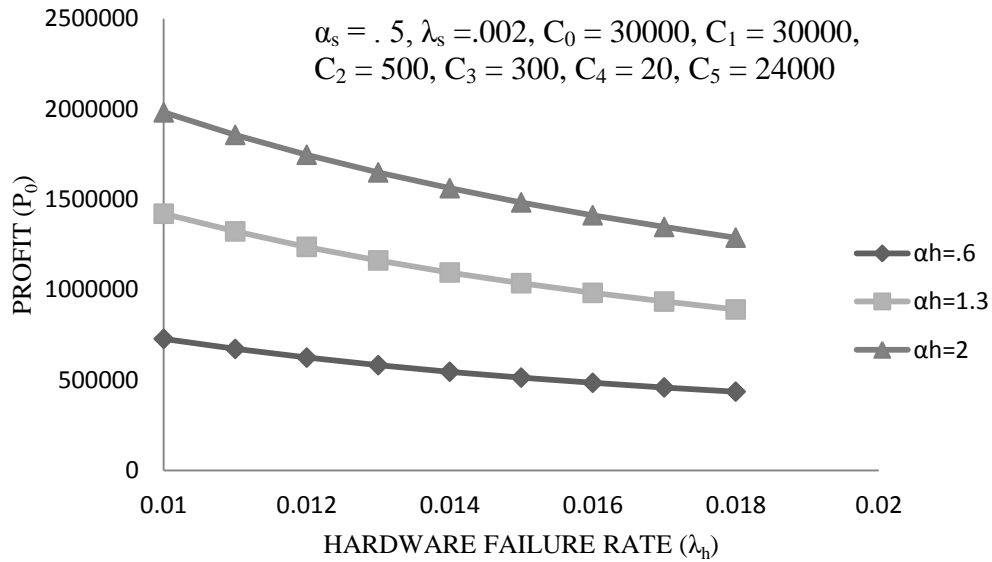

$\alpha_s = .7, \lambda_s = .002$

Fig. 4: Mean Up Time Versus Hardware Failure Rate

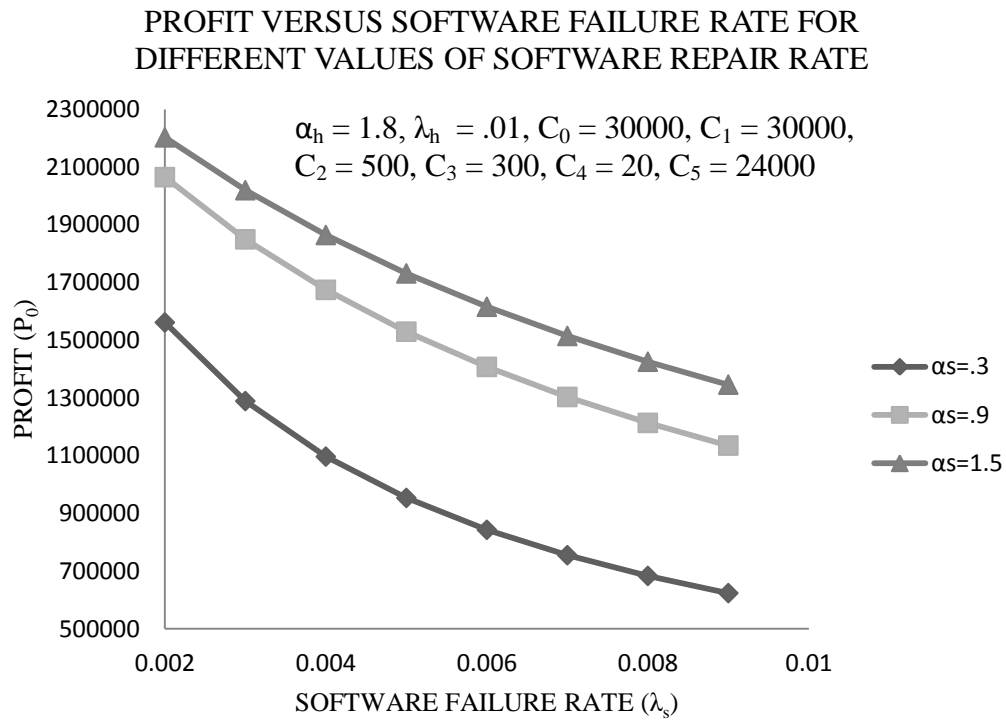## MEAN UP TIME VERSUS SOFTWARE FAILURE RATE FOR DIFFERENT VALUES OF SOFTWARE REPAIR RATE

Fig. 5: Mean Up Time Versus Software Failure Rate

PROFIT VERSUS HARDWARE FAILURE RATE FOR DIFFERENT VALUES OF HARDWARE REPAIR RATE

Fig. 6: Profit Versus Hardware Faifure Rate

PROFIT VERSUS SOFTWARE FAILURE RATE FOR
DIFFERENT VALUES OF SOFTWARE REPAIR RATE

Fig. 7: Profit Versus Software Failure Rate