Intuitionist Fuzzy almost Generalized Semi-pre Closed Mappings

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ABSTRACT
In this paper we introduce intuitionistic fuzzy almost generalized semi-pre closed mappings and intuitionistic fuzzy almost generalized semi-pre open mappings. We investigate some of their properties. Also we provide the relation between intuitionistic fuzzy almost generalized semi-pre closed mappings and other intuitionistic fuzzy closed mappings.

Keywords
Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi-pre $T_{1/2}$ space, intuitionistic fuzzy almost generalized semi-pre closed mappings.

1. INTRODUCTION
After the introduction of fuzzy sets by Zadeh [13], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. R. Santhi and D. Jayanthi [6] introduced the notion of intuitionistic fuzzy generalized semi-pre closed mappings and intuitionistic fuzzy generalized semi-pre open mappings. In this paper we introduce intuitionistic fuzzy almost generalized semi-pre closed mappings. We investigate some of its properties. Also we provide the relation between an intuitionistic fuzzy almost generalized semi-pre closed mapping and other intuitionistic fuzzy closed mappings.

2. PRELIMINARIES
Definition 2.1:[1] An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form

$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$.Then

(a)$A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b)$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c)$A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$

(d)$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$

(e)$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$.

The intuitionistic fuzzy sets $0\sim = \{\langle x, 0, 1 \rangle / x \in X\}$ and $1\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [11] The IFS $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta} \rangle$ where $\alpha \in (0, 1]$, $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point(IFP for short) in X.

Note that an IFP $c(\alpha, \beta)$ is said to belong to an IFS $A = \langle x, \mu_A, \nu_A \rangle$ of X denoted by $c(\alpha, \beta) \in A$ if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.4:[3] An intuitionistic fuzzy topology(IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0\sim, 1\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.5:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A \}$

$cl(A) = \cap\{ K / K$ is an IFCS in X and $A \subseteq K \}$

Note that for any IFS A in (X, τ), we have $cl(A^c) = [int(A)]^c$ and $int(A^c) = [cl(A)]^c$[11].

Definition 2.6:[5] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i)intuitionistic fuzzy semi closed set(IFSCS in short) if $int(cl(A)) \subseteq A$

(ii)intuitionistic fuzzy pre closed set(IFPCS in short) if $cl(int(A)) \subseteq A$

(iii)intuitionistic fuzzy αclosed set(IFαCS in short) if $cl(int(cl(A)) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs.The family of all IFSCSs, IFPCSs, and IFαCSs (respectively IFSOSs, IFPOSs and IFαOSs) of an IFTS (X,τ) are respectively denoted by IFSC(X), IFPC(X) and IFαC(X) (respectively IFSO(X), IFPO(X) and IFαO(X)).

Definition 2.7:[11] An IFS A = $\langle x, \mu_A, \nu_A \rangle$in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi-pre closed set (IFSPCS for short) if there exists an IFPCS B such that int(B) ⊆A ⊆B.

(ii) intuitionistic fuzzy semi-pre open set (IFSPOS for short) if there exists an intuitionistic fuzzy pre open set (IFPOS for short) B such that B ⊆A ⊆cl(B).

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X,τ) is denoted by IFSPC(X) (respectively IFSPO(X)).

Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not be true in general [11].

Note that an IFS A is an IFSPCS if and only if int(cl(int(A)))⊆A and an IFSPOS if and only if A ⊆cl(int(cl(A))) [7].

Definition 2.8:[7]Let A be an IFS in an IFTS (X, τ). Then the semi-pre interior and the semi- pre closure of A are defined by

spint (A) =∪{ G / G is an IFSPOS in X and G ⊆A }.

spcl (A) = ∩{ K / K is an IFSPCS in X and A ⊆K }.

Note that for any IFS A in (X, τ), we have spcl(A^c) = $[\text{spint}(A)]^c$ and spint(A^c) = $[\text{spcl}(A)]^c$[7].

Definition 2.9:[10] An IFS A is an

(i) intuitionistic fuzzy regular closed set(IFRCS for short) if A = cl(int(A)).

(ii) intuitionistic fuzzy generalized closed set(IFGCS for short) if cl(A) ⊆U whenever A⊆U and U is an IFOS.

The family of all IFRCSs and IFGCSs (respectively IFROSs and IFGOSs) of an IFTSb(X,τ) are respectively denoted by IFRC(X) and IFGC(X) (respectively IFRO(X) and IFGO(X)).

Definition 2.10:[7]An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy generalized semi-pre closed set(IFGSPCS for short) if spcl(A) ⊆U whenever A ⊆U and U is an IFOS in (X, τ).

Every IFCS, IFGCS, IFSCS, IFPCS, IFRCS, IFαCS and IFSPCS is an IFGSPCS but the separate converses may not be true in general [7].The family of all IFGSPCSs of an IFTS (X,τ) is denoted by IFGSPC(X).

Definition 2.11:[6]Let A be an IFS in an IFTS (X, τ). Then the generalized semi-pre interior and the generalized semi- pre closure of A are defined by

gspint (A) = ∪{ G / G is an IFGSPOS in X and G ⊆A }.

gspcl (A) = ∩{ K / K is an IFGSPCS in X and A⊆K }.

Note that for any IFS A in (X, τ), we have gspcl(A^c) = $[\text{gspint}(A)]^c$ and gspint(A^c) = $[\text{gspcl}(A)]^c$

Remark 2.12: If an IFS A in an IFTS (X, τ) is an IFGSPCS in X, then gspcl(A) = A. But the converse may not be true in general, since intersection does not exist in IFGSPCSs[7]

Remark 2.13: If an IFS A in an IFTS (X, τ) is an IFGSPOS in X, then gspint(A) = A. But the converse may not be true in general, since union does not exist in IFGSPOSs[7].

Definition 2.14:[7] The complement A^c of an IFGSPCS A in an IFTS (X,τ) is called an intuitionistic fuzzy generalized semi-pre open set (IFGSPOS for short) in X.

Every IFOS, IFGOS, IFSOS, IFPOS, IFROS, IFαOS and IFSPOS is an IFGSPOS but the separate converses may not be true in general [7].The family of all IFGSPOSs of an IFTS (X,τ) is denoted by IFGSPO(X).

Definition 2.15:[9] Let c(α, β) be an IFP of an IFTS (X, τ). An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of c(α, β) if there exists an IFOS B in X such that c(α,β) ∈B ⊆A.

Definition 2.16:[8]If every IFGSPCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an intuitionistic fuzzy semi- pre $T_{1/2}$space(IFSP$T_{1/2}$ space for short).

Definition 2.17:[5] A map f: X →Y is called an intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for each IFCS A in X.

Definition 2.18:[5] A map f: X→Y is called an

(i) intuitionistic fuzzy semi open mapping(IFSOM for short) if f(A) is an IFSOS in Y for each IFOS A in X.

(ii)intuitionistic fuzzyαopen mapping(IFαOM for short) if f(A) is an IFαOS in Y for each IFOS A in X.

(iii)intuitionistic fuzzy preopen mapping(IFPOM for short) if f(A) is an IFPOS in Y for each IFOS A in X.

Definition 2.19: The intuitionistic fuzzy semi closure and the intuitionistic fuzzyαclosure of an IFS A in an IFTS (X, τ) are defined by

scl(A) = ∩{ K / K is an IFSCS in X and A ⊆K }.

αcl(A) = ∩{ K / K is an IFαCS in X and A ⊆K }.

Definition 2.20:[6] Let c(α,β) be an IFP in (X, τ). An IFS A of X is called an intuitionistic fuzzy semi neighborhood(IFSN for short) of c(α, β) if there is an IFSPOS B in X such that c(α, β) ∈B ⊆A.

Definition 2.21:[6] A mapping f : X →Y is said to be an intuitionistic fuzzy generalized semi-pre closed mapping (IFGSPCM, for short) if f(A) is an IFGSPCS in Y for every IFCS A in X.

Definition 2.22:[6] A mapping f : X →Y is said to be an intuitionistic fuzzy M-generalized semi-pre closed mapping(IFMGSPCM, for short) if f(A) is an IFGSPCS in Y for every IFGSPCS A in X.

Definition 2.23:[6] A mapping f : X →Y is said to be an intuitionistic fuzzy almost generalized semi-pre continuous mapping(IFaGSP continuous mapping, for short) if $f^{-1}(A)$ is an IFGSPCS in X for every IFRCS A in Y.

Definition 2.24: An IFS A is said to be an intuitionistic fuzzy dense(IFD for short) in another IFS B in an IFTS (X, τ), if cl(A) = B.

3. INTUITIONISTIC FUZZY ALMOST GENERALIZED SEMI-PRE CLOSED MAPPINGS AND INTUTITIONISTIC FUZZY ALMOST GENERALIZED SEMI-PRE OPEN MAPPINGS

In this section we introduce intuitionistic fuzzy almost generalized semi-pre closed mappings and intuitionistic fuzzy almost generalized semi-pre open mappings. We study some of their properties.

Definition 3.1:A map f: X $\rightarrow$Y is called an intuitionistic fuzzy almost generalized semi-pre closed mapping (IFaGSPCM for short) if f(A) is an IFGSPCS in Y for each IFRCS A in X.

Example 3.2: Let X = { a, b }, Y = { u, v } and G_1= $\langle$x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)$\rangle$, G_2= $\langle$y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)$\rangle$. Then τ = {$0_\sim$, G_1, $1_\sim$}and σ= { $0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively.. Define a mapping f : (X, τ) $\rightarrow$(Y, σ) by f(a) = u and f(b) = v. Then f is an IFaGSPCM.

Theorem 3.3: Every IFCM is an IFaGSPCM but not conversely.

Proof: Let f: X $\rightarrow$Y be an IFCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. Then f(A) is an IFCS in Y. Since every IFCS is an IFGSPCS , f(A) is an IFGSPCS in Y. Hence f is an IFaGSPCM.

Example 3.4: In Example 3.2 f is an IFGSPCM but not an IFCM, since G_1^c= $\langle$x, (0.5a, 0.6_b), (0.5_a, 0.4_b)$\rangle$is an IFCS in X but f(G_1^c) = $\langle$y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)$\rangle$is not an IFCS in Y, since cl(f(G_1^c)) = $G_2^c \neq$f(G_1^c) .

Theorem 3.5: Every IFSCM is an IFaGSPCM but not conversely.

Proof: Let f: X $\rightarrow$Y be an IFSCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. Then f(A) is an IFSCS in Y. Since every IFSCS is an IFGSPCS , f(A) is an IFGSPCS in Y. Hence f is an IFaGSPCM.

Example 3.6: Let X = { a, b }, Y = { u, v } and G_1= $\langle$x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)$\rangle$, G_2= $\langle$y, (0.5_u, 0.4_v), (0.2_u, 0.3_v)$\rangle$. Then τ = {$0_\sim$, G_1, $1_\sim$}and σ= { $0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) $\rightarrow$(Y, σ) by f(a) = u and f(b) = v. Then f is an IFaGSPCM but not an IFSCM, since G_1^c= $\langle$x, (0.5a, 0.6_b), (0.5_a, 0.4_b)$\rangle$is an IFCS in X but f(G_1^c) = $\langle$y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)$\rangle$is not an IFSCS in Y, since int(cl(f(G_1^c))) = $1_\sim \subseteq$f(G_1^c).

Theorem 3.7: Every IFαCM is an IFaGSPCM but not conversely.

Proof:Let f: X $\rightarrow$Y be an IFαCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. Then f(A) is an IFαCS in Y. Since every IFαCS is an IFGSPCS , f(A) is an IFGSPCS in Y. Hence f is an IFaGSPCM.

Example 3.8: In Example 3.6 f is an IFaGSPCM but not an IFαCM, since G_1^c= $\langle$x, (0.5a, 0.6_b), (0.5_a, 0.4_b)$\rangle$ is an IFCS in X but f(G_1^c) = $\langle$y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)$\rangle$is not an IFαCS in Y, since cl(int(cl(f(G_1^c)))) = $1_\sim \subseteq$f(G_1^c).

Theorem 3.9: Every IFPCM is an IFaGSPCM but not conversely.

Proof: Let f: X $\rightarrow$Y be an IFPCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X Then f(A) is an IFPCS in Y. Since every IFPCS is an IFGSPCS, f(A) is an IFGSPCS in Y. Hence f is an IFaGSPCM.

Example 3.10: In Example 3.2 f is an IFaGSPCM but not an IFPCM, since G_1^c= $\langle$x, (0.5a, 0.6_b), (0.5_a, 0.4_b)$\rangle$ is an IFCS in X but f(G_1^c) = $\langle$y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)$\rangle$is not an IFPCS in Y, since cl(int(f(G_1^c))) = $G_2^c \subseteq$f(G_1^c) .

Theorem 3.11: Every IFGSPCM is an IFaGSPCM but not conversely.

Proof: Let f: X $\rightarrow$Y be an IFGSPCM. Let A be an IFRCS in X. Since every IFRCS is an IFCS, A is an IFCS in X Then f(A) is an IFGSPCS in Y. Hence f is an IFaGSPCM.

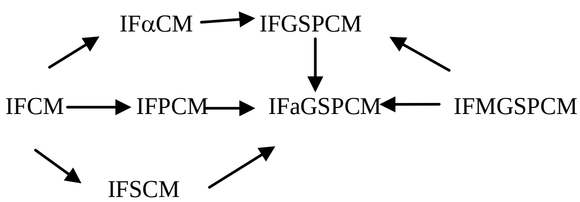
Example 3.12: Let X = { a, b }, Y = { u, v } and G_1= $\langle$x, (0.2_a, 0.2_b), (0.4_a, 0.4_b)$\rangle$, G_2 = $\langle$x, (0.2_a, 0_b), (0.5_a, 0.4_b)$\rangle$, G_3= $\langle$y, (0.5_u, 0.6_v), (0.2_u, 0_v)$\rangle$and G_4= $\langle$y, (0.4_u, 0.1_v), (0.2_u, 0.1_v)$\rangle$,. Then τ = {$0_\sim$, G_1, G_2,$1_\sim$}and σ= { $0_\sim$, G_3,G_4,$1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) $\rightarrow$(Y, σ) by f(a) = u andf(b) = v. Then f is an IFaGSPM but not an IFGSPCM, since G_2^c= $\langle$x, (0.5_a, 0.4_b), (0.2_a, 0_b)$\rangle$is an IFCS in X but f(G_2^c) = $\langle$y, (0.5_u, 0.4_v), (0.2_u, 0_v)$\rangle$is not an IFGSPCS in Y, since f(G_2^c) $\subseteq G_3$but spcl(f(G_2^c) = $1_\sim \subseteq G_3$.

Theorem 3.13 : Every IFMGSPCM is an IFaGSPCM but not conversely.

Proof: Let f: X $\rightarrow$Y be an IFMGSPCM. Let A be an IFRCS in X. Then A is an IFGSPCS in X. By hypothesis f (A) is an IFGSPCS in Y. Therefore f is an IFaGSPCM.

Example 3.14: In Example 3.12 f is an IFaGSPCM but not an IFMGSPCM, since A = $\langle$x, (0.4_a, 0.2_b), (0.2_a, 0_b)$\rangle$is an IFGSPCS in X but f(A) = $\langle$y, (0.4_u, 0.2_v), (0.2_u, 0_v)$\rangle$is not an IFGSPCS in Y, since f(A) $\subseteq$G3but spcl(f(A)) = $1_\sim \subseteq G_3$.

The relation between various types of intuitionistic fuzzy closedness is given in the following diagram.



The reverse implications are not true in general in the above diagram.

Definition 3.15:A map f: X $\rightarrow$Y is called an intuitionistic fuzzy almost generalized semi-pre open mapping (IFaGSPOM for short) if f(A) is an IFGSPOS in Y for each IFROS A in X.

Theorem 3.16: Let f: X $\rightarrow$Y be a mapping. Then the following are equivalent.

(i)f is an IFaGSPOM

(ii)f is an IFaGSPCM

Proof: Straightforward.

Theorem 3.17: Let c(α, β) be an IFP in X. A mapping f: X $\rightarrow$Y is an IFaGSPCM if for every IFOS A in X with f^{-1}(c(α, β)) $\in$A, there exists an IFOS B in Y with c(α, β) $\in$B such that f (A) is IFD in B.

Proof: Let A be an IFROS in X. Then A is an IFOS in X. Let f^{-1}(c(α, β)) $\in$A, then there exists an IFOS B in Y such that

$c(\alpha, \beta) \in B$ and cl(f (A)) = B. Since B is an IFOS, cl(f(A)) = B is also an IFOS in Y. Therefore int(cl(f(A))) = cl(f (A)). Now

f (A) $\subseteq$cl(f (A)) = int(cl(f(A))) $\subseteq$cl(int(cl(f(A)))). This implies f(A) is an IFSPOS in Y and hence an IFGSPOS in Y. Thus f is an IFaGSPCM.

Theorem 3.18: Let f: X $\rightarrow$Y be a mapping where Y is an IFSPT1/2space. Then the following are equivalent:

(i)f is an IFaGSPCM.

(ii)spcl(f (A)) $\subseteq$f (cl(A)) for every IFSPOS A in X

(iii)spcl(f (A)) $\subseteq$f (cl(A)) for every IFSOS A in X

(iv)f (A) $\subseteq$spint(f (int(cl(A)))) for every IFPOS A in X.

Proof:(i) $\Rightarrow$(ii)Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(A) is an IFGSPCS in Y and hence is an IFSPCS in Y, since Y is an IFSPT1/2 space. This implies spcl(f(cl(A))) = f (cl(A)). Now spcl(f(A)) $\subseteq$spcl(f(cl(A))) = f(cl(A)). Thus spcl(f(A)) $\subseteq$f(cl(A)).

(ii) $\Rightarrow$(iii) Since every IFSOS is an IFSPOS, the proof directly follows.

(iii) $\Rightarrow$(i)Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSOS in X. By hypothesis, spcl(f(A)) $\subseteq$f(cl(A)) = f(A) $\subseteq$spcl(f(A)). Hence f(A) is an IFSPCS and hence is an IFGSPCS in Y. Thus f is an IFaGSPCM.

(i) $\Rightarrow$(iv)Let A be an IFPOS in X. Then A $\subseteq$int(cl(A)). Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A))) is an IFGSPOS in Y. Since Y is an IFSPT1/2space, f(int(cl(A)))is an IFSPOS in Y. Therefore f(A) $\subseteq$f(int(cl(A))) $\subseteq$spint(f(int(cl(A)))).

(iv) $\Rightarrow$(i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A) $\subseteq$spint(f(int(cl(A)))) = spint(f(A)) $\subseteq$f(A). This implies f(A) is an IFSPOS in Y and hence is an IFGSPOS in Y. Therefore f is an IFaGSPCM.

Theorem 3.19: Let f: X $\rightarrow$Y be a mapping. Then f is an IFaGSPCM if for each IFP $c(\alpha, \beta) \in Y$ and for each IFSPOS B in X such that f-1($c(\alpha, \beta)$) $\in$B , spcl(f(B)) is an IFSN of $c(\alpha, \beta) \in Y$.

Proof: Let $c(\alpha, \beta) \in Y$ and let A be an IFROS in X. Then A is an IFSPOS in X. By hypothesis $f^{-1}(c(\alpha, \beta)) \in A$, that is $c(\alpha, \beta) \in f(A)$ in Y and spcl((f(A)) is an IFSN of $c(\alpha, \beta)$ in Y. Therefore there exists an IFSPOS B in Y such that $c(\alpha, \beta))$ $\in$B $\subseteq$spcl(f(A)). We have $c(\alpha, \beta) \in f(A) \subseteq$spcl(f(A)). Now B = $\cup\{ c(\alpha, \beta) / c(\alpha, \beta) \in B\}$ = f (A). Therefore f(A) is an IFSPOS in Y and hence an IFGSPOS in Y. Thus f is an IFaGSPOM. Hence by Theorem 3.16 f is an IFaGSPCM.

Theorem 3.20: Let f: X $\rightarrow$Y be a mapping. If f is an IFaGSPCM, then gspcl(f(A) $\subseteq$f(cl(A)) for every IFSPOS A in X.

Proof: Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFGSPCS in Y. Then gspcl(f(cl(A)) = f(cl(A)). Now gspcl(f(A)) $\subseteq$gspcl(f(cl(A))) f(cl(A)). That is gspcl(f(A) $\subseteq$f(cl(A)).

Corollary 3.21: Let f: X $\rightarrow$Y be a mapping. If f is an IFaGSPCM, then gspcl(f(A) $\subseteq$f(cl(A)) for every IFSOS A in X.

Proof: Since every IFSOS is an IFSPOS, the proof directly follows from the Theorem 3.20.

Corollary 3.22: Let f: X $\rightarrow$Y be a mapping. If f is an IFaGSPCM, then gspcl(f(A) $\subseteq$f cl(A)) for every IFPOS A in X.

Proof: Since every IFPOS is an IFSPOS, the proof directly follows from the Theorem 3.20.

Theorem 3.23: Let f: X $\rightarrow$Y be a mapping. If f is an IFaGSPCM, then gspcl(f(A)) $\subseteq$

f(cl(spint(A))) for every IFSPOS A in X.

Proof: Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis, f(cl(A)) is an IFGSPCS in Y. Then gspcl(f(A)) $\subseteq$gspcl(f(cl(A))) = f(cl(A)) $\subseteq$f(cl(spint(A))), since spint(A) = A.

Corollary 3.24: Let f: X $\rightarrow$Y be a mapping. If f is an IFaGSPCM, then gspcl(f(A)) $\subseteq$f(cl(spint(A))) for every IFSOS A in X.

Proof: Since every IFSOS is an IFSPOS, the proof directly follows from the Theorem 3.23.

Corollary 3.25: Let f: X $\rightarrow$Y be a mapping. If f is an IFaGSPCM, then gspcl(f(cl(A))) $\subseteq$f(cl(spint(A))) for every IFPOS A in X.

Proof: Since every IFPOS is an IFSPOS, the proof directly follows from the Theorem 3.23.

Theorem 3.26: Let f: X $\rightarrow$Y be a mapping. If f(spint(B)) $\subseteq$spint(f(B)) for every IFS B

in X, then f is an IFaGSPOM.

Proof: Let B $\subseteq$X be an IFROS. By hypothesis, f(spint(B)) $\subseteq$spint(f(B)). Since B is an IFROS, it is an IFSPOS in X. Therefore spint(B) = B. Hence f(B) = f(spint(B)) $\subseteq$spint(f(B)) $\subseteq$f(B). This implies f(B) is an IFSPOS and hence an IFGSPOS in Y. Thus f is an IFaGSPOM.

Theorem 3.27: Let f: X $\rightarrow$Y be a mapping. If spcl(f(B))$\subseteq$f(spcl(B)) for every IFS B in X, then f is an IFaGSPCM.

Proof: Let B $\subseteq$X be an IFRCS. By hypothesis, spcl(f(B))$\subseteq$f(spcl(B)). Since B is an IFRCS, it is an IFSPCS in X. Therefore spcl(B) = B. Hence f(B) = f(spcl(B)) $\supseteq$spcl(f(B)) $\supseteq$f(B). This implies f(B) is an IFSPCS and hence an IFGSPCS in Y. Thus f is an IFaGSPCM.

Theorem 3.28: Let f: X $\rightarrow$Y be a mapping where Y is an IFSPT$_{1/2}$space. Then the following are equivalent.

(i)f is an IFaGSPOM

(ii)for each IFP $c(\alpha, \beta)$ in Y and each IFROS B in X such that $f^{-1}(c(\alpha, \beta)) \in B$, cl(f(cl(B))) is an IFSN of $c(\alpha, \beta)$ in Y.

Proof:(i) $\Rightarrow$(ii) Let $c(\alpha, \beta) \in Y$ and let B be an IFROS in X such that $f^{-1}(c(\alpha, \beta)) \in B$. That is $c(\alpha, \beta) \in f(B)$. By hypothesis, f(B) is an IFGSPOS in Y. Since Y is an IFSPT$_{1/2}$ space, f(B) is an IFSPOS in Y. Now $c(\alpha, \beta) \in f(B) \subseteq$f(cl(B)) $\subseteq$cl(f(cl(B))). Hence cl(f(cl(B))) is an IFSN of $c(\alpha, \beta)$ in Y.

(ii) $\Rightarrow$(i)Let B be an IFROS in X. Then $f^{-1}(c(\alpha, \beta)) \in B$. This implies $c(\alpha, \beta) \in f(B)$. By hypothesis, cl(f(cl(B))) is an IFSN of $c(\alpha, \beta)$. Therefore there exists an IFSPOS A in Y such that $c(\alpha, \beta) \in A \subseteq$cl(f(cl(B))). Now A = $\cup\{ c(\alpha, \beta) / c(\alpha, \beta) \in A\}$ = f(B). Thereforef(B) is an IFSPOS and hence an IFGSPOS in Y. Thus f is an IFaGSPOM.

Theorem 3.29: The following are equivalent for a mapping f: X →Y where Y is an $IFSPT_{1/2}$space.

(i)f is an IFaGSPCM

(ii)spcl(f(A)) ⊆f(αcl(A)) for every IFSPOS A in X

(iii)spcl(f(A)) ⊆f(αcl(A)) for every IFSOS A in X

(iv)f(A) ⊆spint(f(scl(A))) for every IFPOS A in X.

Proof:(i) ⇒(ii) Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(A) is an IFGSPCS in Y and hence is an IFSPCS in Y, since Y is an $IFSPT_{1/2}$space. This implies spcl(f(cl(A))) = f(cl(A)). Now spcl(f(A)) ⊆spcl(f(cl(A))) = f(cl(A)). Since cl(A) is an IFRCS, cl(int(cl(A))) = cl(A). Therefore spcl(f(A)) ⊆f(cl(A)) = (cl(int(cl(A)))) ⊆f(A ∪cl(int(cl(A)))) ⊆f(αcl(A)). Hence spcl(f(A)) ⊆f(αcl(A)).

(ii) ⇒(iii) Let A be an IFSOS in X. Since every IFSOS is an IFSPOS, the proof is obvious.

(iii) ⇒(i) Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSOS in X. By hypothesis, spcl(f(A)) ⊆f(αcl(A)) ⊆f(cl(A)) = f(A) ⊆spcl(f(A)). That is spcl(f(A)) = f(A). Hence f(A) is an IFSPCS and hence is an IFGSPCS in Y. Thus f is an IFaGSPCM.

(i) ⇒(iv) Let A be an IFPOS in X. Then A ⊆int(cl(A)). Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A))) is an IFGSPOS in Y. Since Y is an $IFSPT_{1/2}$space,

f(int(cl(A))) is an IFSPOS in Y. Therefore f(A) ⊆f(int(cl(A))) ⊆spint(f(int(cl(A)))) = spint(f(A ∪int(cl(A)))) = spint(f(scl(A))). That is f(A) ⊆spint(f(scl(A))).

(iv) ⇒(i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A) ⊆spint(f(scl(A))). This implies f(A) ⊆spint(f(A ∪int(cl(A)))) ⊆spint(f(A ∪A)) = spint(f(A)) ⊆f(A). Therefore f(A) is an IFSPOS in Y and hence an IFGSPOS in Y. Thus f is an IFaGSPCM, by Theorem 3.16.

Theorem 3.30: Let f: X →Y be a mapping where Y is an $IFSPT_{1/2}$space. If f is an IFaGSPCM, then int(cl(int(f(B)))) ⊆f(spcl(B)) for every IFRCS B in X.

Proof: Let B ⊆X be an IFRCS. By hypothesis, f(B) is an IFGSPCS in Y. Since Y is an IFSPT1/2space, f(B) is an IFSPCS in Y. Therefore spcl(f(B)) = f(B). Now int(cl(int(f(B)))) ⊆f(B) ∪int(cl(int(f(B)))) = spcl(f(B)) = f(B) = f(spcl(B)). Hence int(cl(int(f(B)))) ⊆f(spcl(B)).

Theorem 3.31: Let f: X →Y be a mapping where Y is an IFSPT1/2space. If f is an IFaGSPCM, then f(spint(B)) ⊆cl(int(cl(f(B)))) for every IFROS B in X.

Proof: This theorem can be easily proved by taking complement in Theorem3.30.

Next we provide the characterization theorem for an IFaGSPOM.

Theorem 3.32: Let f: X →Y be a bijective mapping. Then the following are equivalent.

(i)f is an IFaGSPOM

(ii)f is an IFaGSPCM

(iii)f-1is an IFaGSP continuous mapping.

Proof:(i) ⇔(ii) is obvious from the Theorem 3.16.

(ii) ⇒(iii) Let A ⊆X be an IFRCS. Then by hypothesis, f(A) is an IFGSPCS in Y. That is $(f^{-1})^{-1}(A)$ is an IFGSPCS in Y. This implies $f^{-1}$is an IFaGSP continuous mapping.

(iii)⇒(ii)Let A ⊆X be an IFRCS.Then by hypothesis $(f^{-1})^{-1}(A)$ is an IFGSPCS in Y. That is f(A) is an IFGSPCS in Y. Hence f is an IFaGSPCM.

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