Design of Wideband Microwave Integrator using Optimization Technique

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ABSTRACT
This paper presents a simple approach for the design of a wideband microwave integrator. The design consist of two steps; in step one, accurate and optimized representation of stable and causal discrete-time integrator system function is obtained using optimization algorithm. In step two, chain scattering parameters or T-parameters are used to represent microwave equivalent of transmission line system function. Upon equating and optimizing the error between the discrete-time and chain scattering parameter system function, the optimized value of characteristic impedance of transmission line network is obtained. Here optimization technique like pattern search has been used to ameliorate error.

Keywords
Wideband, Integrator, Chain Scattering, Optimization, Pattern Search

1. INTRODUCTION
The operation of a time integral of a signal is represented by an inverse of complex-frequency variable in the Laplace transform representation (1). Neglecting the attenuation factor, transfer function is equal to 1/ jω, where, ω is the signal angular frequency. As a result, an integrator is a low-pass filter and the amplitude of its system function is inversely proportional to signal frequency. Integrator has been used extensively in many areas of digital communication and digital signal processing such as correlation estimation, coherent detector, accumulator analysis, and waveform shaping. In the Fourier spectral analysis, the spectral of a measured signal is the output of an integrator that takes the time integration of the multiplication of the measured signal by harmonic signals [2].
The frequency response of an ideal integrator is given by

$$H_{INT}(\omega) = \frac{K_i}{j\omega} \qquad (1)$$

where, $j = \sqrt{-1}$, K_i is the proportional constant of an integrator and ω is the angular frequency in radians.
Almost all the traditional integrators are derived by taking Z-transform of the class of Newton-cotes interpolation formulas [1, 3-4]. Newton-cotes interpolation formula is basically a technique of computing a definite integral/curve by replacing that curve by a more integrable and simpler curve, thus introducing some error in the equation, but still approximating the result to a great extent. Several integrators has been proposed [2, 5] in the study of discrete-time signal processing (DSP), however their magnitude response can only approximate that of ideal response for a fraction of full band nyquist frequency range. Few of the classical integrators that are worth mentioning are Trapezoidal integrator, Simpson 1/3 integrator, Simpson 3/8 integrator and Bool's integrator.

$$H_{mp}(z) = \frac{T\sum_{i=0}^{m} a_i(p)\Delta^i B(z)}{(1-z^{-p})} \qquad (2)$$

where,
$$a_i(p) = \int_0^p \binom{\eta}{i}$$

and
$$\Delta^i B(z) = (-1)^i (1-z^{-1})^i$$

A new class of integrators has been developed in the recent past known as recursive digital integrators. Several methods have been used for their design. This include putting different values of *m* and *n* for Newton-Cotes digital integrators (2), which is basically arrived at by applying Z-transform to closed form Newton-Cotes integration formula [1], use of linear interpolation between the magnitude responses of the classical rectangular, trapezoidal and Simpson digital integrators [6-7] and use of linear programming approach in the design [8]. Every design has its own advantages and limitations. Newton-Cotes digital integrators, although applicable over a wideband and a maximum error margin of 6.5% compared to ideal analog integrators, are fit for use in higher frequency ranges. On the other hand, two integrator designs proposed by Al-Alaoui based on linear interpolation between magnitude responses of basic integrators [6-7] have lower error than Newton-Cotes Integrators but also have a lower operational bandwidth where the error margin is negligible. The design proposed by Papamarkos and Chamzas [8] based on the linear optimization techniques again suffer from a narrow bandwidth problem. However none of the above stated integrators and differentiator are near the ideal integrator and differentiator as far as the magnitude response is concerned.

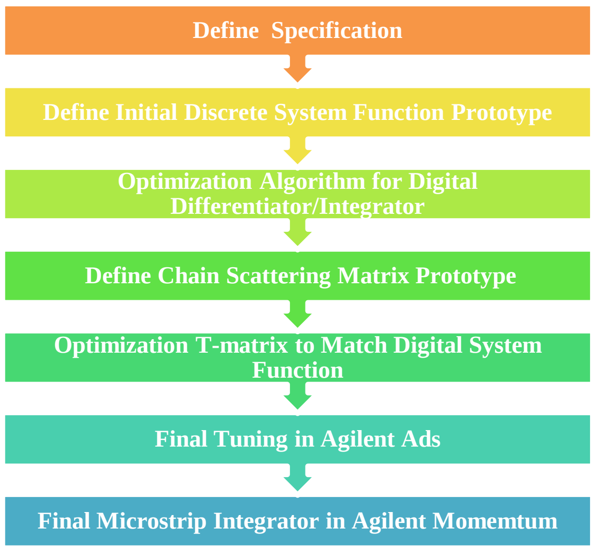


Fig 1: Design process block diagram

To overcome these limitations a new discrete-time integrator designing approach is proposed in this paper. The design procedure for integrator involves first obtaining multivariable system function and then modifying its system function approximately to obtain a wideband microwave integrator. The proposed design technique comprises several steps sketched in block diagram figure1.

2. DISCRETE-TIME INTEGRATOR

2.1 Design of Stable Discrete-time Integrator

The system function (3) for linear, time-invariant, causal digital filter can be expressed in the Z-domain in the form [2]:

$$H(z) = \frac{\sum_{i=0}^{N} b_i z^{-i}}{1+\sum_{j=1}^{M} a_j z^{-j}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_M z^{-M}} \quad (3)$$

For optimization of the discrete-time system function prototype, an error function is defined as:

$$Err = Min\left[\int_0^{\pi} (H_{Ideal}(z) - H_{INT}(z))^2 d\Omega\right] \quad (4)$$

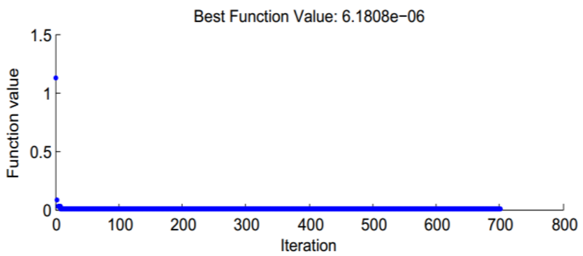
Error function (4) is basically an objective function for optimization algorithm, which is calculated by integrating the error between ideal response and proposed system function response. The integration is performed over normalized frequency Ω over the range of 0 to π.

An infinite impulse response system function is selected for designing first order discrete time integrator. The design specifications include designing an integrator with time constant K_i = 2 sec and maximum value of magnitude response equals to unity. The proposed digital integrator prototype is given as:

$$H_{Int}(z) = \frac{1}{x_1 + x_2 z^{-1} + x_3 z^{-2} + x_4 z^{-3} + x_5 z^{-4} + x_6 z^{-5} + x_7 z^{-6}} \quad (5)$$

The order of system (5) is selected on the basis of minimum order for stable response. The defined system function (4) has six order zeros at z = 0. The location of zeros is pre-assigned in order to make the discrete-time system function compatible with chain scattering parameter transfer function of cascaded network of serial transmission line as shown in Table 2. The poles of system function are optimization variable coefficients that are determined by multivariable optimization algorithm scheme. The pole location is selected between the ranges of $0 \leq z \leq 1$, so that system function obtained after optimization is stable and causal.

Upon using optimization, the error between ideal and proposed integrator is given as Err = 6.18×10^{-6} as shown in figure 2.



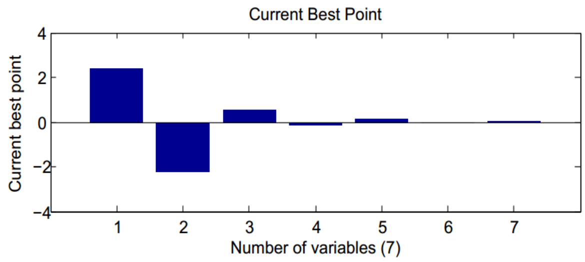


Fig 2: Err function plot using pattern search optimization process

The optimized value of system function coefficient obtained by pattern search optimization is tabulated in Table 1.

Table 1. Optimized coefficient values of proposed first order integrator

First order Discrete-Time Integrator		
S. No.	Coefficient	Value
1	x_1	2.378
2	x_2	-2.221
3	x_3	0.524
4	x_4	-0.156
5	x_5	0.123
6	x_6	-0.036
7	x_7	0.036

System function obtained after substituting the optimized coefficient is given by:

$$H_{Int}(z) = \frac{1}{2.378 - 2.221z^{-1} + 0.524z^{-2} - 0.156z^{-3} + 0.123z^{-4} - 0.036z^{-5} + 0.036z^{-6}} \quad (6)$$

The ideal integrator is assumed to have amplitude response inversely proportional to all frequency as shown in figure 3. The poles obtained (6) are p_1 = 0.6416 + 0.1870j, p_2 = 0.6416 - 0.1870j, p_3 = -0.2974 + 0.3109i, p_4 = -0.2974 - 0.3109i, p_5 = 0.1226 + 0.4099j and p_6 = 0.1226 - 0.4099j which shows the stability of designed discrete-time integrator. The proposed integrator (6) is suitable to be adopted as the system function of a wide-band integrator as shown in figure 3.

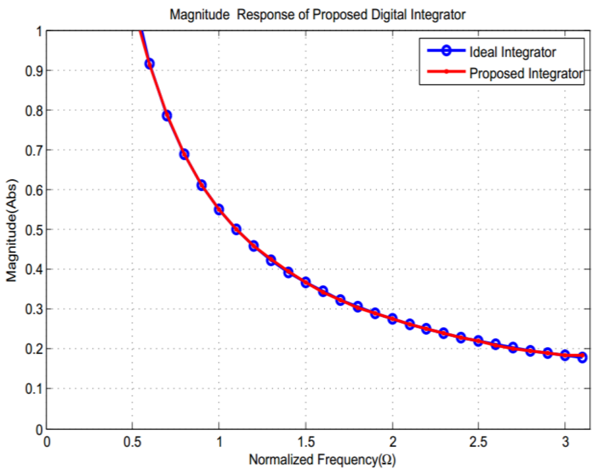


Fig 3: Frequency response of proposed integrator for sampling frequency normalized to π radian

Upon using (6), the integrator implemented by using transmission line, the maximum value of the transfer function $H_{int}(z)$ is given as unity for the frequency range $0 \le \Omega \le 0.6$. The rest part of the transfer function in the range $0.5 \le \Omega \le \pi$ satisfies (6). Under such circumstance, the circuit thus obtained behaves as an integrator over the frequency range $0.5 \le \Omega \le \pi$.

3. WIDEBAND MICROWAVE INTEGRATOR

3.1 Chain Scattering Parameter

For design of an integrator that have the operating frequencies up to 10GHz using microstrip configuration, a network consist of six sections of serial transmission lines has been used. It is to be noted that if a network obtained by cascading four section of serial transmission line, there is practical limitation of a maximum frequency operation of 5GHz. The number of sections or device length of microstrip network determines the time constant and frequency response of an integrator. The number of sections of integrators is determined by the optimization process that involves the curve fitting of transfer function of transmission line to the amplitude response of the discrete-time integrator (6) which represents a good approximation of an ideal integrator.

Table 2. Basic Transmission-Line Element's Chain Scattering-Parameter Matrices

Transmiss-ion Line	**Chain Scattering Parameter or T- parameter**
Serial Transmi-ssion Line	$\frac{1}{z^{-1/2}(1-\Gamma^2)}\begin{bmatrix} 1-\Gamma^2 z^{-1} & -(\Gamma-\Gamma z^{-1}) \\ \Gamma-\Gamma z^{-1} & -\Gamma^2+z^{-1} \end{bmatrix}$ where , $\Gamma = \frac{Z_2-Z_o}{Z_2+Z_o}$
Shunt Short Stub	$\frac{1}{1-z^{-1}}\begin{bmatrix} (1+a)-(1-a)z^{-1} & a+az^{-1} \\ -a-az^{-1} & (1-a)-(1+a)z^{-1} \end{bmatrix}$ where, $a = \frac{Z_O}{2Z_1}$
Shunt Open Stub	$\frac{1}{1+z^{-1}}\begin{bmatrix} (1+a)+(1-a)z^{-1} & a-az^{-1} \\ -a+az^{-1} & (1-a)+(1+a)z^{-1} \end{bmatrix}$ where, $a = \frac{Z_O}{2Z_1}$

The system function is obtained by cascading the chain scattering matrix [10] (Table2) and choosing the network element in order to compatible with (6), given as

$$T_{11} = \frac{\beta}{\alpha_1 + \alpha_2 z^{-1} + \alpha_3 z^{-2} + \alpha_4 z^{-3} + \alpha_5 z^{-4} + \alpha_6 z^{-5} + \alpha_7 z^{-6}} \quad (7)$$

$$\Gamma_n = \frac{(Z_n - Z_0)}{(Z_n + Z_0)} \quad (8)$$

where, α_1, α_2, α_3, α_4, α_5, α_6, and α_7 coefficients are functions of reflection coefficient of each serial transmission lines given by Γ_1, Γ_2, Γ_3, Γ_4, Γ_5 and Γ_6 respectively. The design variables Γ_1, Γ_2, Γ_3, Γ_4, Γ_5 and Γ_6 are optimization variables. The system function (7) obtained for six sections serial transmission line network is optimized with respect to discrete-time integrator system function obtained in (8).

The error function (4) minimization has been performed using pattern search optimization. Upon using optimization algorithm, the error between ideal and proposed integrator is given as Err = 2.2 x 10^{-3}. The figure 4 presents the progress of error function minimization with respect to increasing iteration. The optimized values of design variables i.e. Γ_1, Γ_2, Γ_3, Γ_4, Γ_5 and Γ_6 are plotted.

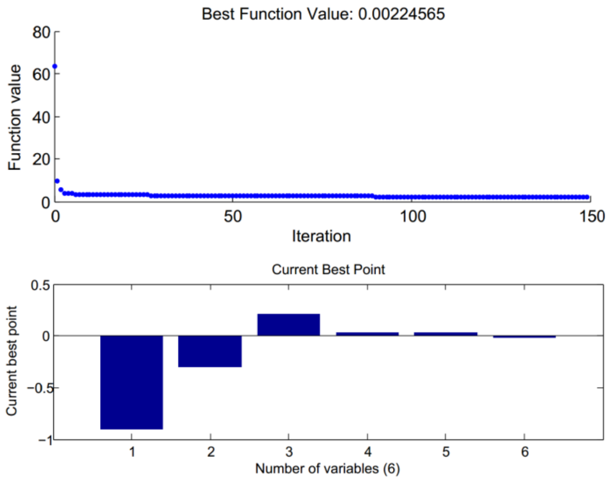


Fig 4: Error function plot versus iteration using pattern search optimization

The value of designed variables of system function (6) obtained by pattern search optimization algorithm is tabulated in Table 3.

Table 3. Optimized values of reflection coefficients obtained using pattern search algorithm

First order Discrete-Time Integrator		
S. No.	Reflection coefficient	Value
1	Γ_1	-0.700
2	Γ_2	-0.321
3	Γ_3	0.365
4	Γ_4	0.022
5	Γ_5	0.039
6	Γ_6	-0.021

Corresponding to reflection coefficients values as mentioned in table 3, the value of characteristic impedance of each sections of transmission line can be calculated using (8). The characteristic impedance Z_o of 50Ω has been used as standard transmission line. For implementation of microstrip integrator, RT/duroid® 5870 is used as dielectric substrate having a thickness of 30mil (0.762mm), dielectric loss tangent of 0.0009 and relative dielectric constant of ε = 2.4.

3.2 Schematic of Microwave Integrator using Agilent ADS

Figure 5, represents the schematic for microwave integrator using Agilent ADS. Six sections of microstrip transmission line named as TL_1 through TL_6 are used to design integrator. The microstrip line named as TL_8 and TL_9 represents the 50Ω characteristic impedance transmission line. RT/duroid® 5870 with dielectric substrate having a thickness of 30mil (0.762mm), dielectric loss tangent of 0.0009 and relative dielectric constant of ε = 2.4 are defined under MSUB palette. S_Parameter palette is used for S-parameter analysis of proposed wideband microwave integrator. The simulation is performed with linear sweep of frequency from DC to 10GHz. The figure 6 represents the substrate definition for microstrip under Agilent ADS software.

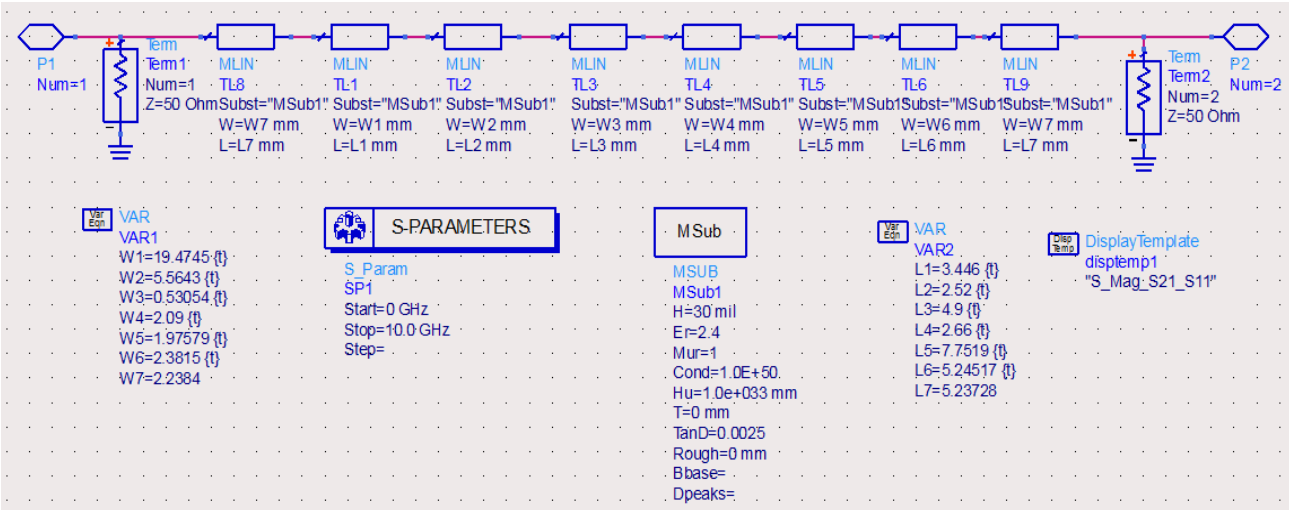


Fig 5: Agilent ADS Schematic of six element transmission line 1st Order Integrator

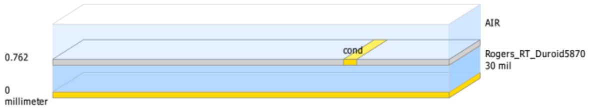


Fig 6: RT/duroid® 5870 Substrate

3.3 Layout of Microwave Integrator using Agilent Momentum

Based on the schematic diagram the layout for six element integrator is generated in Agilent ADS. For characterization of designed integrator meshing is performed and analysis is done under FEM simulator. An FEM simulation mesh is a part of the entire 3D problem domain, which is divided into a set of tetrahedra (or cells). The pattern of cells is based on the geometry of a layout so each layout has a unique mesh calculated for it. The mesh is then applied to the geometry to compute the electric fields within each cell. It also helps to identify any coupling effects in the layout during simulation. From these calculations, S-parameters are then calculated for the layout.

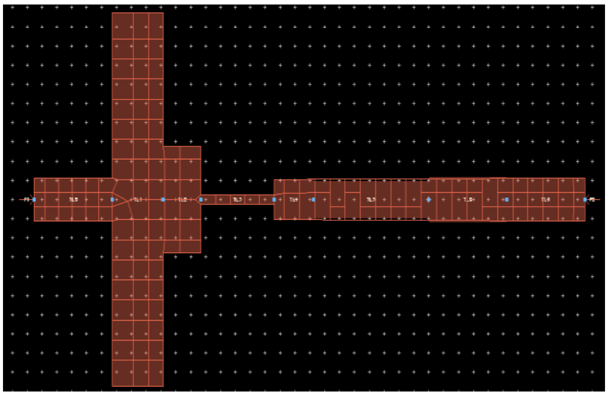


Fig 7: Layout diagram of proposed integrator with six sections of microstrip line

Figure 8 represents the final optimized six section microstrip integrator prototype build in Agilent EMPro software.

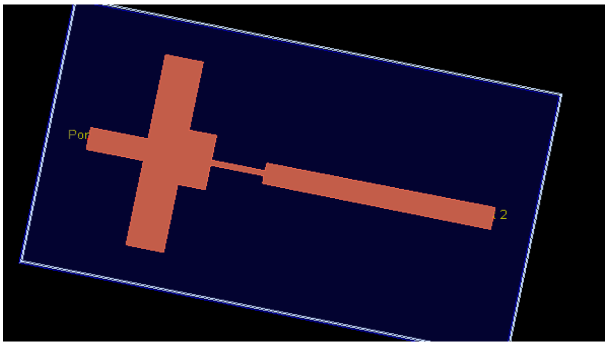


Fig 8: Agilent EMPro layout of first order integrator

3.4 Magnitude Response of Proposed Integrator

For characterization of the designed integrator, reflection coefficient S_{11} and transmission coefficient S_{21} parameters are plotted together with ideal integrator response to show to good degree of agreement between proposed and ideal integrator. It can be notice that the designed microwave integrator follows well the ideal integrator characteristic between the frequency range 2.5GHz to 10GHz.

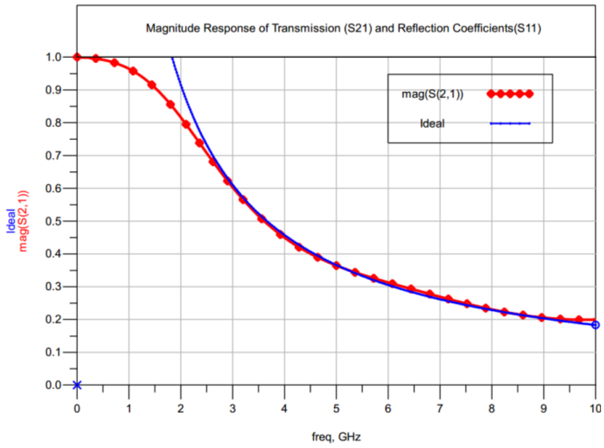


Fig 8: Frequency Response of the proposed 1st order integrator and ideal integrator

4. CONCLUSION

In this paper a simple and novel approach has been proposed for the design of a wideband microwave integrator. The proposed method is fast and eases the design complexity of wideband microwave integrator. In particular, the Z-domain representations of scattering characteristics of equal length non-uniform transmission lines facilitate the implementation of discrete domain integrators in the microwave frequency range. The integrator has been implemented by using multi-section microstrip transmission lines with appropriate characteristic impedance obtained by optimization algorithm. The designed microwave integrator is in good agreement with the ideal integrator characteristic for the frequency range 2.5GHz to 10GHz. The proposed integrator can be employed to measure the delay times of microwave transistors [11] or it can be used to implement high-frequency active filters [12].

5. REFERENCES

[1] N. Q. Ngo, "A New Approach for the Design of Wideband Digital Integrator and Differentiator", IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 53, no. 9, pp. 936–940, 2006.

[2] A. V. Oppenheim and R. W. Shafer, Discrete-Time Signal Processing. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[3] S. C. Chapra and R. P. Canale, "Numerical Methods for Engineers", 2nd ed. Singapore: McGraw-Hill, 1989.

[4] Joe D. Hoffman, "Numerical Methods for Engineers and Scientists", 2nd ed. US McGraw-Hill, 1992.

[5] J. Le Bihan, "Novel class of digital integrators and differentiators", Electron. Lett., vol. 28, no. 15, pp. 1376–1378, 1992.

[6] M. A. Al-Alaoui, "A class of second-order integrators and lowpass differentiators", IEEE Trans. Circuits Syst. I, vol. 42, no. 4, pp. 220–223, Apr. 1995.

[7] M. Al-Alaoui, "Novel digital integrator and differen-tiator", Electronics Letters, vol. 29, no. 4, pp. 376–378, 1993.

[8] N. Papamarkos and C. Chamzas, "A new approach for the design of digital integrators", IEEE Trans. Circuits Syst. I, Fundam. Theory Appl., vol. 43, no. 9, pp. 785–791, Sep. 1996

[9] B. Kumar, D. Choudhury, and A. Kumar, "On the design of linear phase, FIR integrators for midband frequencies", IEEE Trans. Signal Process., vol. 44, no. 10, pp. 2391–2395, 1996.

[10] D. M. Pozar, "Microwave and RF Design of Wireless System", 3rd Edition: John Wiley & Sons Inc, 2001.

[11] D. D. Cohen and R. A. Zakarevivius, "Operational amplifier integrators for the measurement of the delay times of microwave transistors", IEEE J. Solid-State Circuits, vol. SC-10, no. 1, pp. 19–27, Feb. 1975.

[12] R. L. Geiger and G. Bailey, "Integrator design for high-frequency active filter applications", IEEE Trans. Circuits Syst., vol. CAS-29, no. 9, pp. 595–603, Sep. 1982.