On (λ, μ)-Anti-Fuzzy Subrings

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ABSTRACT

In this paper we introduce the notions of (λ, μ)-anti-fuzzy subrings, studied some properties of them and discussed the product of them.

Keywords:

(λ, μ)-anti-fuzzy subring, (λ, μ)-anti-fuzzy ideal, product, homomorphism.

1. INTRODUCTION

Fuzzy sets was first introduced by Zadeh [11] and then the fuzzy sets have been used in the reconsideration of classical mathematics. W. Liu [5] defined fuzzy set and fuzzy ideals of a ring. Bhakat and Das introduced the concepts of $(\in, \in \vee q)$-fuzzy groups [1, 2] and $(\in, \in \vee q)$-fuzzy subring [3]. B. Yao introduced the concepts of (λ, μ)-fuzzy groups [8] and (λ, μ)-fuzzy subring [9]. Shen [7] researched anti-fuzzy subgroups and Dong [4] studied the product of anti-fuzzy subgroups. We introduce the notion of (λ, μ)-anti fuzzy subring, (λ, μ)-anti fuzzy ideals and product of (λ, μ)-anti fuzzy subrings.

2. PRELIMINARIES

DEFINITION 2.1. *A mapping $A : X \to [0, 1]$ is called a fuzzy subset of a non empty set X. If A is a fuzzy subset of X, then we denote $A_{(\alpha)} = \{x \in X | A(x) < \alpha\}$ for all $\alpha \in [0, 1]$.*

DEFINITION 2.2. *[3] A fuzzy subset A of a group G is said to be a fuzzy subgroup of G if for all $x, y \in G$,*
(i) $A(xy) \geq \min\{A(x), A(y)\}$
(ii) $A(x^{-1}) \geq A(x)$.

DEFINITION 2.3. *[10] A fuzzy set A of a group G is called a (λ, μ)-anti-fuzzy subgroup of G if $\forall a, b, c \in G$,*
(i) $A(ab) \wedge \mu \leq (A(a) \vee A(b)) \vee \lambda$
(ii) $A(c^{-1}) \wedge \mu \leq A(c) \vee \lambda$.

DEFINITION 2.4. *[5] A fuzzy subset A of a ring R is said to be a fuzzy subring of R if $\forall a, b \in R$,*
(i) $A(a - b) \geq A(a) \wedge A(b)$
(ii) $A(ab) \geq A(a) \wedge A(b)$

DEFINITION 2.5. *[5] A fuzzy subset A of a ring R is said to be a fuzzy ideal of R if $\forall a, b \in R$,*
(i) $A(a - b) \geq A(a) \wedge A(b)$
(ii) $A(ab) \geq A(a) \vee A(b)$

3. (λ, μ)-ANTI-FUZZY SUBRING

DEFINITION 3.1. *A fuzzy set A of a ring R is called a (λ, μ)-anti-fuzzy subring of R if $\forall a, b, c \in R$.*
$A(a + b) \wedge \mu \leq (A(a) \vee A(b)) \vee \lambda$
$A(-x) \wedge \mu \leq A(x) \vee \lambda$
and $A(ab) \wedge \mu \leq (A(a) \vee A(b)) \vee \lambda$.

PROPOSITION 3.2. *If A is a (λ, μ)-anti-fuzzy subring of a ring R, then $A(0) \wedge \mu \leq A(x) \vee \lambda$, for all $x \in R$, where 0 is the identity of R.*

Proof: $\forall x \in R$ and let $(-x)$ be the inverse element of x. Then
$A(0) \wedge \mu = A(x - x) \wedge \mu$
$= (A(x - x) \wedge \mu) \wedge \mu \leq \{(A(x) \vee A(-x)) \vee \lambda\} \wedge \mu$
$= (A(x) \wedge \mu) \vee (A(-x) \wedge \mu) \vee (\lambda \wedge \mu) \leq A(x) \vee (A(x) \vee \lambda) \vee \lambda$
$= A(x) \vee \lambda$.

THEOREM 3.3. *Let A be fuzzy subset of a ring R. Then A is a (λ, μ)-anti fuzzy subring of R iff. $A(x - y) \wedge \mu \leq (A(x) \vee A(y)) \vee \lambda$ and $A(xy) \wedge \mu \leq (A(x) \vee (A(y))) \vee \lambda$.*

Proof: Let A be a (λ, μ)-anti fuzzy subring of R, then
$A(x - y) \wedge \mu = A(x - y) \wedge \mu \wedge \mu \leq ((A(x) \vee A(y)) \vee \lambda) \wedge \mu$
$= (A(x) \wedge \mu) \vee (A(-y) \wedge \mu) \vee (\lambda \wedge \mu) \leq A(x) \vee (A(y) \vee \lambda) \vee \lambda$.
$= A(x) \vee A(y) \vee \lambda$
$A(xy) \wedge \mu \leq (A(x) \vee A(y)) \vee \lambda$ ($\because$ A is (λ, μ)-anti fuzzy subring)
Conversely, suppose
(i) $A(x - y) \wedge \mu \leq (A(x) \vee A(y)) \vee \lambda$ and
(ii) $A(xy) \wedge \mu \leq (A(x) \vee A(y)) \vee \lambda$.
then $A(0) \wedge \mu \leq A(x - x) \wedge \mu \leq A(x) \vee A(x) \vee \lambda$ (by (i))
$= A(x) \vee \lambda$.
So
$A(-x) \wedge \mu = A(0 - x) \wedge \mu$
$= A(0 - x) \wedge \mu \wedge \mu \leq [A(0) \vee A(x) \vee \lambda] \wedge \mu$
$= (A(0) \wedge \mu) \vee [(A(x) \vee \lambda) \wedge \mu] \leq (A(x) \vee \lambda) \vee (A(x) \vee \lambda)$
$= A(x) \vee \lambda$.

$$\begin{aligned} A(x + y) \wedge \mu &= [A(x - (-y)) \wedge \mu] \wedge \mu \leq [A(x) \vee A(-y) \vee \lambda] \wedge \mu \\ &= \{(A(x) \wedge \mu) \vee (A(-y) \wedge \mu) \vee (\lambda \wedge \mu)\} \\ &\leq (A(x)) \vee (A(y) \vee \lambda) \vee \lambda \\ &= A(x) \vee A(y) \vee \lambda \end{aligned}$$

Clearly $A(xy) \wedge \mu \leq (A(x) \vee A(y)) \vee \lambda$.
Therefore A is a (λ, μ)- anti-fuzzy subring of R.

THEOREM 3.4. *Let A be a fuzzy subset of a ring R. Then the following are equivalent:*
(1) A is a (λ, μ)-anti-fuzzy subring of R.
(2) A_α is a subring of R, for any $\alpha \in (\lambda, \mu)$.

Proof: (1) $\Rightarrow$ (2)
Let A be a (λ, μ)-anti-fuzzy subring of R. For any $\alpha \in (\lambda, \mu]$ such that $A_\alpha \neq \phi$, we need to show that (i) $x - y \in A_\alpha$ and (ii) $xy \in A_\alpha$ for all $x, y \in A_\alpha$. Since $A(x) < \alpha$ and $A(y) < \alpha$, then $A(x-y) \wedge \mu \leq A(x) \vee A(y) \vee \lambda < \alpha \vee \alpha \vee \lambda = \alpha \vee \lambda = \alpha, (\because \alpha > \lambda)$. $A(x-y) \wedge \mu < \alpha \Rightarrow A(x-y) \leq \alpha (\because \alpha \leq \mu)$. $\therefore (x-y) \in A_\alpha$. Consider $A(xy) \wedge \mu \leq A(x) \vee A(y) \vee \lambda < \alpha \vee \alpha \vee \lambda = \alpha \vee \lambda = \alpha$. $A(xy) \wedge \mu < \alpha \Rightarrow A(xy) < \alpha (\because \alpha \leq \mu)$. $\therefore xy \in A_\alpha$. Therefore A_α is a subring of R.
(2) $\Rightarrow$ (1)
Conversely, let A_α is a subring of R for any $\alpha \in (\lambda, \mu]$. We have to prove $A(x-y) \wedge \mu \leq A(x) \vee A(y) \vee \lambda$ and $A(xy) \wedge \mu \leq A(x) \vee A(y) \vee \lambda, \forall x \in R$. Suppose $A(x-y) \wedge \mu > A(x) \vee A(y) \vee \lambda = \alpha$ then $A(x-y) > \alpha$ (since $\alpha \leq \mu$) $\Rightarrow x-y \notin A_\alpha$ for $x, y \in A_\alpha$, which is a contradiction to that A_α is a subring of R. Hence $A(x-y) \wedge \mu \leq A(x) \vee A(y) \vee \lambda$.
Suppose $A(xy) \wedge \mu > A(x) \vee A(y) \vee \lambda = \alpha$ that is $A(xy) \wedge \mu > \alpha \Rightarrow A(xy) > \alpha$ (since $\alpha \leq \mu$). $\Rightarrow xy \notin A_\alpha$ for all $x, y \in A_\alpha$, which is a contradiction. So $A(xy) \wedge \mu \leq A(x) \vee A(y) \vee \lambda$.
Therefore A is a (λ, μ)-anti-fuzzy subring.

DEFINITION 3.5. *Let A be a fuzzy subset of R. Then A is called a (λ, μ)-anti-fuzzy ideal of R if for all $x, y \in R$,*
(i) $A(x-y) \wedge \mu \leq A(x) \vee A(y) \vee \lambda$.
(ii) $A(xy) \wedge \mu \leq (A(x) \wedge A(y)) \vee \lambda$.

THEOREM 3.6. *Let A be a fuzzy subset of a ring R. Then the following are equivalent.*
(i) A is a (λ, μ)-anti-fuzzy ideal of R.
(ii) A_α is an ideal of R, for any $\alpha \in (\lambda, \mu]$.

Proof: (i) $\Rightarrow$ (ii)
Let A be a (λ, μ)-anti-fuzzy ideal of R. We have to prove A_α is an ideal of R. Let $x, y \in A_\alpha$. Then $A(x) < \alpha$ and $A(y) < \alpha$.
Consider $A(x-y) \wedge \mu \leq A(x) \vee A(y) \vee \lambda < \alpha \vee \alpha \vee \lambda = \alpha$. (Since $\lambda < \alpha$). i.e. $A(x-y) \wedge \mu < \alpha \Rightarrow A(x-y) < \alpha$ (since $\alpha \leq \mu$).$\therefore x - y \in A_\alpha$.
Let $x \in A_\alpha, r \in R$. Then $A(xr) \wedge \mu \leq (A(x) \wedge A(r)) \vee \lambda < (\alpha \wedge A(r)) \vee \lambda < \alpha$. (Since $\lambda < \alpha$)
i.e. $A(xr) \wedge \mu < r \Rightarrow A(xr) < r$ (Since $\alpha \leq \mu$).
Similarly $rx \in A_\alpha$. Hence A_α is n ideal of R.
(ii) $\Rightarrow$ (i)
Conversely, let A_α be an ideal of R for any $\alpha \in (\lambda, \mu]$.
Suppose let us consider $A(x-y) \wedge \mu > A(x) \vee A(y) \vee \lambda = \alpha$ then
$A(x-y) > \alpha$ (since $\alpha \leq \mu$) $\Rightarrow x - y \notin A_\alpha$, for all $x, y \in A_\alpha$ which is a contradiction to that A_α is an ideal of R. Hence $A(x-y) \wedge \mu \leq A(x) \vee A(y) \vee \lambda$, for all $x, y \in R$. Suppose $A(xy) \wedge \mu > (A(x) \wedge A(y)) \vee \lambda = \alpha$, that is $A(xy) \wedge \mu > \alpha \Rightarrow A(xy) > \alpha$ (since $\alpha \leq \mu$)
$\Rightarrow xy \notin A_\alpha$ for all $x, y \in A_\alpha$, which is a contradiction to that A_α is an ideal. Hence A is a (λ, μ)-anti fuzzy ideal of R. Hence the theorem.

THEOREM 3.7. *Let $f : R_1 \to R_2$ be a homomorphism and let A be a (λ, μ)-anti fuzzy subring of R_1. Then $f(A)$ is a (λ, μ)-anti-fuzzy subring of R_2. If A is a (λ, μ)-anti-fuzzy ideal of R_1 and f is onto, then $f(A)$ is a (λ, μ)-anti-fuzzy ideal of R_2, where $f(A)(y) = \inf_{x \in R_1} \{A(x) | f(x) = y\}$, for all $y \in R_2$.*

Proof:

(1) Let A be a (λ, μ)-anti-fuzzy subring of R_1. To prove $f(A)$ is a (λ, μ)-anti-fuzzy subring of R_2. For this we have to show first
(i) for all $y_1, y_2 \in R_2$, we have
$f(A)(y_1 - y_2) \wedge \mu = \inf\{A(x_1 - x_2) | f(x_1 - x_2) = y_1 - y_2\} \wedge \mu$
$= \inf\{A(x_1 - x_2) \wedge \mu | f(x_1 - x_2) = y_1 - y_2\}$
$\leq \inf\{A(x_1) \vee A(x_2) \vee \lambda | f(x_1) = y_1, f(x_2) = y_2\}$ (Since A is (λ, μ) anti-fuzzy subring) $= \inf\{A(x_1) | f(x_1) = y_1\} \vee \inf\{A(x_2) | f(x_2) = y_2\} \vee \lambda$
$= f(A)(y_1) \vee f(A)(y_2) \vee \lambda$.
(ii) $f(A)(y_1 y_2) \wedge \mu = \inf\{A(x_1 x_2) | f(x_1 x_2) = y_1 y_2\} \wedge \mu$
$= \inf\{A(x_1 x_2) \wedge \mu | f(x_1).f(x_2) = y_1 y_2\}$
$\leq \inf\{A(x_1) \vee A(x_2) \vee \lambda | f(x_1) = y_1, f(x_2) = y_2\}$
$= \inf\{A(x_1) | f(x_1) = y_1\} \vee \inf\{A(x_2) | f(x_2 = y_2)\} \vee \lambda$
$= f(A)(y_1) \vee f(A)(y_2) \vee \lambda$.

(2) Now assume that A is a (λ, μ)-anti-fuzzy ideal of R_1. To prove $f(A)$ is a (λ, μ)-anti-fuzzy ideal of R_2.
(i) By part one the proof for $f(A)(y_1 - y_2) \wedge \mu \leq f(A)(y_1) \vee f(A)(y_2) \vee \lambda$ is obtained.
(ii) $f(A)(y_1 y_2) \wedge \mu = \inf\{A(x_1 x_2) | f(x_1 x_2) = y_1 y_2\} \wedge \mu$.
$= \inf\{A(x_1 x_2) \wedge \mu | f(x_1 x_2) = y_1 y_2\}$
$\leq \inf\{(A(x_1) \wedge A(x_2)) \vee \lambda | f(x_1) = y_1, f(x_2) = y_2\}$
$= (\inf\{A(x_1) | f(x_1) = y_1)\} \wedge \inf\{A(x_2) | f(x_2) = y_2\}) \vee \lambda$
$= (f(A)(y_1) \wedge f(A)(y_2)) \vee \lambda$.
Hence $f(A)$ is a (λ, μ)-anti-fuzzy ideal of R_2.

THEOREM 3.8. *Let $f : R_1 \to R_2$ be a homomorphism and let B be a (λ, μ)-anti fuzzy subring ((λ, μ)-anti-fuzzy ideal) of R_2. Then $f^{-1}(B)$ is a (λ, μ)-anti-fuzzy subring ((λ, μ)-anti-fuzzy ideal) of R_1 where*
$f^{-1}(B)(x) = B(f(x)); \forall x \in R_1$.

Proof:

(1) To prove $f^{-1}(B)$ is a (λ, μ)-anti-fuzzy subring. For $x_1, x_2 \in R_1$, we have
(i) $f^{-1}(B)(x_1 - x_2) \wedge \mu \leq B(f(x_1 - x_2)) \wedge \mu = B(f(x_1) - f(x_2)) \wedge \mu$.
$\leq B(f(x_1)) \vee B(f(x_2)) \vee \lambda = f^{-1}(B)(x_1) \vee f^{-1}(B)(x_2) \vee \lambda$.
(ii) Consider $f^{-1}(B)(x_1 x_2) \wedge \mu = B(f(x_1 x_2)) \wedge \mu = B(f(x_1) f(x_2)) \wedge \mu$
$\leq B(f(x_1)) \vee B(f(x_2)) \vee \lambda. = f^{-1}(B)(x_1) \vee f^{-1}(B)(x_2) \vee \lambda$.
Hence $f^{-1}(B)$ is a (λ, μ)-anti-fuzzy subring.

(2) To prove $f^{-1}(B)$ is a (λ, μ)-anti-fuzzy ideal.
(i) By part one we have proof for
$f^{-1}(B)(x_1 - x_2) \wedge \mu \leq f^{-1}(B)(x_1) \vee f^{-1}(B)(x_2) \vee \lambda$.
(ii) Consider $f^{-1}(B)(x_1 x_2) \wedge \mu = B(f(x_1 x_2)) \wedge \mu$.
$= B(f(x_1).f(x_2)) \wedge \mu \leq (B(f(x_1)) \wedge B(f(x_2))) \vee \lambda$.
$= (f^{-1}(B)(x_1) \wedge f^{-1}(B)(x_2)) \vee \lambda$.
Hence $f^{-1}(B)$ is a (λ, μ)-anti-fuzzy ideal.

Let R_1 be a ring with the identity 0 and R_2 be a ring with the identity $0'$, then $R_1 \times R_2$ is a ring with the identity $(0, 0')$ if we define $(x_1, y_1)(x_2, y_2) = (x_1x_2, y_1y_2)$ for all $(x_1, y_1), (x_2, y_2) \in R_1 \times R_2$. Moreover, the inverse element of any $(x, y) \in R_1 \times R_2$ is $(a, b) \in R_1 \times R_2$ if and only if a is the inverse of x in R_1 and b is the inverse element of y in R_2.

THEOREM 3.9. *Let A, B be two (λ, μ)-anti-fuzzy subrings R_1 and R_2 respectively. The product of A and B denoted by $A \times B$, is a (λ, μ)-anti-fuzzy subring of the ring $R_1 \times R_2$ where $(A \times B)(x, y) = A(x) \vee B(y), \forall (x, y) \in R_1 \times R_2$.*

Proof: Let $(x_1, y_1), (x_2, y_2) \in R_1 \times R_2$. Now,
$$\begin{aligned}(A\times B)((x_1, y_1)-(x_2, y_2))\wedge\mu &= (A\times B)(x_1-x_2, y_1-y_2)\wedge\mu.\\ &= (A(x_1 - x_2) \vee B(y_1 - y_2)) \wedge \mu\\ &= (A(x_1 - x_2) \wedge \mu) \vee (B(y_1 - y_2) \wedge \mu)\\ &\le (A(x_1) \vee A(x_2) \vee \lambda) \vee (B(y_1) \vee B(y_2) \vee \lambda)\\ &= (A(x_1) \vee B(y_1)) \vee (A(x_2) \vee B(y_2)) \vee \lambda\\ &= ((A \times B)(x_1, y_1)) \vee (A \times B)(y_1, y_2) \vee \lambda.\end{aligned}$$
Also
$$\begin{aligned}(A \times B)((x_1, y_1)(x_2, y_2)) \wedge \mu &= (A \times B)(x_1x_2, y_1y_2) \wedge \mu\\ &= (A(x_1x_2) \vee B(y_1y_2)) \wedge \mu\\ &= (A(x_1x_2) \wedge \mu) \vee (B(y_1y_2) \wedge \mu)\\ &\le (A(x_1) \vee A(x_2) \vee \lambda) \vee (B(y_1) \vee B(y_2) \vee \lambda)\\ &= (A(x_1) \vee B(y_1)) \vee (A(x_2) \vee B(y_2)) \vee \lambda\\ &= ((A \times B)(x_1, y_1)) \vee ((A \times B)(x_2, y_2)) \vee \lambda.\end{aligned}$$
Hence $(A \times B)$ is a (λ, μ)-fuzzy subring of $R_1 \times R_2$.

THEOREM 3.10. *Let A and B be two fuzzy subsets of rings R_1 and R_2 respectively. If $A \times B$ is a (λ, μ)-anti-fuzzy subring of $R_1 \times R_2$ then at least one of the following statements must hold.*
$A(0)\wedge\mu \le B(a)\vee\lambda, \forall a \in R_2$ and $B(0')\wedge\mu \le A(x)\vee\lambda, \forall x \in R_1$.

Proof: Let $A\times B$ be a (λ, μ)-anti-fuzzy subring of the ring $R_1 \times R_2$.
By contraposition, suppose that none of the statements hold. Then we can find $x \in R_1$ and $a \in R_2$ such that $A(x) \vee \lambda < B(0') \wedge \mu$ and $B(a) \vee \lambda < A(0) \wedge \mu$.
Now
$$\begin{aligned}(A \times B)(x, a) \vee \lambda &= (A(x) \vee B(a)) \vee \lambda\\ &= (A(x) \vee \lambda) \vee (B(a) \vee \lambda)\\ &< (B(0') \wedge \mu) \vee (A(0) \wedge \mu)\\ &= (A \times B)(0, 0') \wedge \mu.\end{aligned}$$
This is a contradiction with that $(0, 0')$ is the identity of $R_1\times R_2$.

THEOREM 3.11. *Let A and B be fuzzy subsets of R_1 and R_2 respectively, such that $B(0')\wedge\mu \le A(x)\vee\lambda$ for all $x \in R_1$. If $A \times B$ is a (λ, μ)-anti-fuzzy subring of $R_1 \times R_2$, then A is a (λ, μ)-anti-fuzzy subring of R_1.*

Proof: From $B(0')\wedge\mu \le A(x)\vee\lambda$ we obtained that $\mu \le A(x)\vee \lambda$ or $B(0') \le A(x) \vee \lambda$, for all $x \in R_1$. Let $x, y \in R_1$, then $(x, 0'), (y, 0') \in R_1 \times R_2$.
Two cases are possible:

(1) If $\mu \le A(x) \vee \lambda$ for all $x \in R_1$. Then
$A(x - y) \wedge \mu \le \mu \le A(x) \vee \lambda \le (A(x) \vee A(y)) \vee \lambda$ and
$A(xy) \wedge \mu \le \mu \le A(x) \vee \lambda \le (A(x) \vee A(y)) \vee \lambda$.

(2) If $B(0') \le A(x) \vee \lambda$ for all $x \in R_1$. Then
$$\begin{aligned}A(x - y) \wedge \mu &\le (A(x - y) \vee B(0' - 0')) \wedge \mu\\ &= ((A \times B)(x - y, 0' - 0')) \wedge \mu\\ &= ((A \times B)((x, 0') - (y, 0')) \wedge \mu)\\ &\le ((A \times B)(x, 0') \vee (A \times B)(y, 0')) \vee \lambda\\ &= A(x) \vee B(0') \vee A(y) \vee B(0') \vee \lambda\\ &= A(x) \vee A(y) \vee \lambda.\end{aligned}$$
$$\begin{aligned}A(xy) \wedge \mu &\le (A(xy) \vee B(0'0')) \wedge \mu\\ &= ((A \times B)(xy, 0'0')) \wedge \mu\\ &= ((A \times B)((x, 0')(y, 0')) \wedge \mu)\\ &\le ((A \times B)(x, 0') \vee (A \times B)(y, 0')) \vee \lambda\\ &= A(x) \vee B(0') \vee A(y) \vee B(0') \vee \lambda\\ &= A(x) \vee A(y) \vee \lambda.\end{aligned}$$
Hence A is a (λ, μ)-anti-fuzzy subring of R_1.

THEOREM 3.12. *Let A and B be fuzzy subsets of rings R_1 and R_2 respectively, such that $A(0) \wedge \mu \le B(a) \vee \lambda$ for all $a \in R_2$. If $A\times B$ is a (λ, μ)-anti-fuzzy subring of $R_1\times R_2$, then B is a (λ, μ)-anti-fuzzy subring of R_2.*

From the previous theorems, we have the following Corollary.

COROLLARY 3.13. *Let A and B be fuzzy subsets of rings R_1 and R_2 respectively. If $A \times B$ is a (λ, μ)-anti-fuzzy subring of $R_1 \times R_2$, then either A is a (λ, μ)-anti-fuzzy subring of R_1 or B is a (λ, μ)-anti-fuzzy subring of R_2.*

4. REFERENCES

[1] S. K. Bhakat and P. Das, $(\in, \in \vee q)$-fuzzy group *Fuzzy Sets and Systems* **80**(1996) 359-368.

[2] S. K. Bhakat and P. Das, $(\in, \in \vee q)$-fuzzy normal, quasi normal and maximal sub-groups *Fuzzy Sets and Systems* **112**(2000) 299-312.

[3] S. K. Bhakat, On the definitions of fuzzy group, *Fuzzy Sets and Systems* **51**(1992) 235-241.

[4] B. Dong, Direct product of anti-fuzzy subgroups, *J Shaoxing Teachers College* **5**(1992) in Chinese 29-34.

[5] W. Liu, Fuzzy invariant subgroups and fuzzy ideals, *Fuzzy Sets and Systems* **59**(1993) 205-210.

[6] Rajeshkumar, Fuzzy Algebra, *Publication Division, University of Delhi* **1**(1993).

[7] Z. Shen, The anti-fuzzy subgroup of a group, *J. Liaoning Normat University (Nat. Sci.)* **18(2)** (1995) in Chinese 99-101.

[8] B, Yao, (λ, μ)-fuzzy normal subgroups and (λ, μ)-fuzzy quotients subgroups, *The Journal of Fuzzy Mathematics* **13 (3)** (2005) 695-705.

[9] B, Yao, (λ, μ)-fuzzy subrings and (λ, μ)-fuzzy ideals, *The Journal of Fuzzy Mathematics* **15 (4)** (2007) 981-987.

[10] Yuming Feng and Bingxue Yao, On (λ, μ)-anti-fuzzy subgroups, *Journal of Inequalities and Applications* **10**, 1186/1029-242X-2012-78.

[11] L.A. Zadeh, Fuzzy Set, *Information and Control* **8** (1965) 338-353.