International Journal of Computer Applications (0975 8887)
Volume 75 - No. 1, August 2013

Critical Path Problem under Fuzzy Environment

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ABSTRACT

In this paper, a novel approach has been made to find the critical path in a directed acyclic graph, where for each arc lengths fuzzy numbers are assigned instead of crisp numbers. Procedures are designed to find the optimal path, and finally illustrative examples are provided to demonstrate the proposed approach.

Keywords:

Network (Graph), Trapezoidal fuzzy numbers, α-cut interval numbers, Signed distance measure, Centroid measure, Magnitude measure, Area measure, Metric distance, Ranking degree, Mean-Width notation of α-cut interval numbers, Critical path.

1. INTRODUCTION

Network diagram plays a vital role to determine project completion time. Normally a project will consist of a number of activities and some activities can be started only after finishing some other activities. There may be activities which are independent of some activities. Network Analysis is a technique which determines the various sequences of activities concerning a project and the project completion time. It is successfully used in wide range of significant management problems for evaluating certain types of projects [7]. The popular methods of this technique which is widely used are the Critical Path Method (CPM) and Program Evaluation and Review Techniques (PERT). Since the activities in the network can be carried out in parallel, the minimum time to complete the project is the length of the longest path from the start of project to its finish. The longest path is the critical path of the network. The main purpose of CPM is thus to identify critical activities on the critical path. However, the vagueness of the time parameters in the problem has led to the development of fuzzy CPM. The unknown problem that could occur in practical situation can be very well managed using this fuzzy CPM. Chanas and Kamburowski [1] introduced FPERT, they used fuzzy numbers to represent activity durations in project networks. Mon et al. [12], assumed the duration of each activity as a positive fuzzy numbers and using the α-cut of each fuzzy duration they exploited a linear combination of the duration bounds to represent the operation time of each activity and to determine the critical activities and paths by use of the traditional (crisp) PERT technique. However, based on the α values different critical activities and paths are obtained. Liberatore and Connelly [10], proposed a new straight forward method for applying fuzzy logic to assess uncertainty in critical path analysis. Chanas and Zielinski [2], assumed that the operation time of each activity can be represented as a crisp value, interval or a fuzzy number and discussed the complexity of criticality. Chen and Huang [3], proposed a new model that combines fuzzy set theory with the PERT technique to determine the critical degrees of activities (tasks) and paths, latest and earliest starting time and floats. Ghoseiri and Moghadam [6], developed an algorithm by the use of fuzzy sets, PERT technique and Bellman algorithm to specify the critical path and the fuzzy earliest and latest starting time and floats of activities in the continuous fuzzy network. Thus numerous papers have been published in Fuzzy Critical Path Problem [9, 14].
This paper is organized as follows: In section 2, Basic concepts and definitions are given for α-cut fuzzy interval numbers and some new definitions are coined for the same. New procedures for finding the fuzzy critical path in a network are presented with numerical examples in Section 3. We conclude in Section 4.

2. PRE-REQUISITES

DEFINITION 1 [4]. *A trapezoidal fuzzy number*

$$\tilde{A} = (a_1, a_2, a_3, a_4) \tag{1}$$

can be approximated as a fuzzy number $S(\mu, \sigma)$, μ *denotes the mean of* $\tilde{A}$, σ *denotes the standard deviation of* $\tilde{A}$, *and the membership function of* $\tilde{A}$ *in terms of Mean and Standard Deviation is defined as follows:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-(\mu-\sigma)}{\sigma} & \text{if } \mu-\sigma \le x \le \mu \\ \dfrac{(\mu+\sigma)-x}{\sigma} & \text{if } \mu \le x \le \mu+\sigma \end{cases} \tag{2}$$

where μ *and* σ *are calculated as follows:*

$$\sigma = \frac{2(a_4-a_1)+a_3-a_2}{4}, \;\; \mu = \frac{a_1+a_2+a_3+a_4}{4}$$

DEFINITION 2 [4]. *The* α*-cut interval number for fuzzy number* $\tilde{A}$ *in terms of Mean and Standard Deviation is denoted by*

$$\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)] = [(\mu-\sigma)+\sigma\alpha, (\mu+\sigma)-\sigma\alpha],$$

$\alpha \in [0,1]$. *It is obtained as follows:*
$\alpha = \dfrac{A_L(\alpha)-(\mu-\sigma)}{\sigma}$ *(from eqn. (2))*
$\Rightarrow A_L(\alpha) = (\mu-\sigma)+\sigma\alpha$
$\alpha = \dfrac{(\mu+\sigma)-A_R(\alpha)}{\sigma}$ *(from eqn. (2))*
$\Rightarrow A_R(\alpha) = (\mu+\sigma)-\sigma\alpha$

DEFINITION 3 [8]. *Let $\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)]$ and $\tilde{B}_\alpha = [B_L(\alpha), B_R(\alpha)]$ be two α-cut interval numbers for fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively, then the operations on α-cut interval numbers are given as follows:*
Addition Operation (+):

$$\begin{aligned}\tilde{A}_\alpha(+)\tilde{B}_\alpha &= [A_L(\alpha), A_R(\alpha)](+)[B_L(\alpha), B_R(\alpha)]\\ &= [A_L(\alpha) + B_L(\alpha), A_R(\alpha) + B_R(\alpha)]\end{aligned}$$

Subtraction Operation (-):

$$\begin{aligned}\tilde{A}_\alpha(-)\tilde{B}_\alpha &= [A_L(\alpha), A_R(\alpha)](-)[B_L(\alpha), B_R(\alpha)]\\ &= [A_L(\alpha) - B_R(\alpha), A_R(\alpha) - B_L(\alpha)]\end{aligned}$$

DEFINITION 4 [8]. *Let $\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)]$ and $\tilde{B}_\alpha = [B_L(\alpha), B_R(\alpha)]$ be two α-cut interval numbers for fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively, then the maximum and minimum operation on α-cut interval numbers are given as follows:*

$$\begin{aligned}\tilde{L}_{max}(\tilde{A}_\alpha, \tilde{B}_\alpha) &= [max(A_L(\alpha), B_L(\alpha)),\\ &\quad max(A_R(\alpha), B_R(\alpha))]\\ \tilde{L}_{min}(\tilde{A}_\alpha, \tilde{B}_\alpha) &= [min(A_L(\alpha), B_L(\alpha)),\\ &\quad min(A_R(\alpha), B_R(\alpha))]\end{aligned}$$

DEFINITION 5 [11]. *$d(b, 0)$ means the signed distance of b measured from '0' and is defined as $d(b, 0) = b$*

DEFINITION 6 [11]. *For any closed interval $[a, b]$, the signed distance measure of $[a, b]$ measured from '0' is defined as $d([a, b], 0) = \frac{1}{2}(a + b)$.*

DEFINITION 7 [13]. *Let $\tilde{A} = [a_1, b_1]$ be an interval number. The interval number in terms of mean-width notation is given by $\tilde{A} = (m(\tilde{A}), w(\tilde{A}))$ where $m(\tilde{A})$ = midpoint of $\tilde{A} = \dfrac{a_1 + b_1}{2}$ and $w(\tilde{A})$ = half width of $\tilde{A} = \dfrac{b_1 - a_1}{2}$.*

DEFINITION 8 [13]. *Let $\tilde{A} = (m_1, w_1)$ and $\tilde{B} = (m_2, w_2)$ be two interval numbers in terms of mean-width notation, then the addition operation is given by $\tilde{A}(+)\tilde{B} = (m_1 + m_2, w_1 + w_2)$*

The following definitions are introduced in this paper.
Minimum operation for two interval numbers in mean-width notation is given in [15].

DEFINITION 9. *Let $\tilde{L}_1 = (m_1, w_1)$ and $\tilde{L}_2 = (m_2, w_2)$ be two interval numbers in terms of mean-width notation, then the maximum operation is given by*

$$\tilde{L}_{max}(\tilde{L}_1, \tilde{L}_2) = (max(m_1, m_2), min(w_1, w_2))$$

The same concept holds for α-cut interval numbers.

DEFINITION 10. *Let $\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)]$ be a α-cut interval number, then the signed distance of $[A_L(\alpha), A_R(\alpha)]$ which is measured from zero is defined as*

$$d([A_L(\alpha), A_R(\alpha)], 0) = \frac{1}{2}(A_L(\alpha) + A_R(\alpha))$$

(by Definition 6)
As the function is continuous over the interval $0 \le \alpha \le 1$, we use the integration to obtain the mean value of signed distance as follows:

$$\begin{aligned}\int_0^1 d([A_L(\alpha), A_R(\alpha)], 0)d\alpha &= \int_0^1 \frac{(A_L(\alpha) + A_R(\alpha))}{2}d\alpha\\ &= \frac{1}{2}(A_L(\alpha) + A_R(\alpha)) = \mu\end{aligned}$$

DEFINITION 11. *Let $\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)]$ be a α-cut interval number, then the mean μ and standard deviation σ are calculated as follows:*

$$\mu = \frac{1}{2}\int_0^1 (A_L(\alpha) + A_R(\alpha))d\alpha,$$
$$\sigma = \frac{\int_0^1 (A_R(\alpha) - A_L(\alpha))d\alpha}{2\int_0^1 (1 - \alpha)d\alpha}$$

DEFINITION 12. *Let $\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)]$ be a α-cut interval number, then the Centroid measure of*

$$(\tilde{A}_\alpha) = \frac{\displaystyle\int_{(\mu-\sigma)+\sigma\alpha}^{(\mu+\sigma)-\sigma\alpha} \alpha x dx}{\displaystyle\int_{(\mu-\sigma)+\sigma\alpha}^{(\mu+\sigma)-\sigma\alpha} \alpha dx} = \mu = \frac{1}{2}(A_L(\alpha) + A_R(\alpha))$$

DEFINITION 13. *The metric distance between $\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)]$ and '0' is calculated as follows:*

$$\begin{aligned}D(\tilde{A}_\alpha, 0) &= \left[\int_0^1 [A_L(\alpha)]^2 d\alpha + \int_0^1 [A_R(\alpha)]^2 d\alpha\right]^{\frac{1}{2}}\\ &= \sqrt{2\mu^2 + \frac{2\sigma^2}{3}}.\end{aligned}$$

Here, $\tilde{A}_\alpha \ge \tilde{B}_\alpha$ if and only if $D(\tilde{A}_\alpha, 0) \ge D(\tilde{B}_\alpha, 0)$.

DEFINITION 14. *Let*

$$\begin{aligned}\tilde{A}_\alpha &= [A_L(\alpha), A_R(\alpha)]\\ &= [(\mu_1 - \sigma_1) + \sigma_1\alpha, (\mu_1 + \sigma_1) - \sigma_1\alpha]\end{aligned}$$

and

$$\begin{aligned}\tilde{B}_\alpha &= [B_L(\alpha), B_R(\alpha)]\\ &= [(\mu_2 - \sigma_2) + \sigma_2\alpha, (\mu_2 + \sigma_2) - \sigma_2\alpha]\end{aligned}$$

be two α-cut interval numbers. The ranking degree to which $\tilde{A}_\alpha \ge \tilde{B}_\alpha$ is denoted by $R(\tilde{A}_\alpha \ge \tilde{B}_\alpha)$ and calculated as follows:
$R(\tilde{A}_\alpha \ge \tilde{B}_\alpha) = \frac{1}{2}((A_L(\alpha) + A_R(\alpha)) - (B_L(\alpha) + B_R(\alpha)))$
using this ranking degree, we define two relations between $\tilde{A}_\alpha$ and $\tilde{B}_\alpha$:

$$\tilde{A}_\alpha = \tilde{B}_\alpha \text{ if and only if } R(\tilde{A}_\alpha \ge \tilde{B}_\alpha) = 0$$

$$\tilde{A}_\alpha > \tilde{B}_\alpha \text{ if and only if } R(\tilde{A}_\alpha \ge \tilde{B}_\alpha) > 0$$

DEFINITION 15. *Let $\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)]$ be a α-cut interval number, then the α-cut interval number in terms of mean-width notation is given by $\tilde{A}_\alpha = (m(\tilde{A}_\alpha), w(\tilde{A}_\alpha))$ where*

$$m(\tilde{A}_\alpha) = \frac{A_L(\alpha) + A_R(\alpha)}{2} = \mu$$

and

$$w(\tilde{A}_\alpha) = \frac{A_R(\alpha) - A_L(\alpha)}{2} = \sigma(1 - \alpha), \quad \alpha \in [0, 1]$$

(using Definition 7)
If $\tilde{A}_\alpha = (m_1, w_1)$ *and* $\tilde{B}_\alpha = (m_2, w_2)$ *then*

$$\tilde{A}_\alpha(+)\tilde{B}_\alpha = (m_1 + m_2, w_1 + w_2)$$

(using Definition 8)

DEFINITION 16. $\tilde{L}_i = (m_i, w_i)$ *be the* i^{th} *fuzzy path length in terms of mean-width notation and let* $\tilde{L}_{max} = (m, w)$ *be the fuzzy longest length in terms of mean-width notation then the acceptability index between* $\tilde{L}_i$ *and* $\tilde{L}_{max}$ *for interval numbers in terms of mean-width notation is given as*

$$A(\tilde{L}_i < \tilde{L}_{max}) = \frac{m - m_i}{w + w_i}.$$

DEFINITION 17. *Let* $\tilde{A} = (a, b, c, d)$ *be a trapezoidal fuzzy number such that* $a < b < c < d$. *It is converted to triangular fuzzy number as* $\tilde{A} = \left(a, b_1 = \frac{b+c}{2}, d\right)$ *such that* $a < b_1 < d$. *The magnitude measure of the triangular fuzzy number* $\tilde{A} = (a, b_1, d)$ *with parametric form*

$$\tilde{A}_\alpha = [A_L(\alpha), A_R(\alpha)] = [(b_1 - a)\alpha + a, d - (d - b_1)\alpha]$$

is given by

$$Mag(\tilde{A}) = \int_0^1 \frac{(A_L(\alpha) + A_R(\alpha) + b_1)}{2}\alpha d\alpha, \ \ \alpha \in [0, 1]$$
$$= \frac{a + 7b_1 + d}{12}.$$

DEFINITION 18 [5]. *Let* $\tilde{A} = (a, b, c, d; \lambda)$ *be a level* λ *trapezoidal fuzzy number such that* $a < b < c < d$, $0 < \lambda \leq 1$.
If $(c - b) \neq \lambda$, *the Area Measure of* $\tilde{A} = \dfrac{\lambda(c - b - a + d)}{2}$ *and*
If $(c - b) = \lambda$, *the Area Measure of* $\tilde{A} = \dfrac{\lambda(b - a + 2\lambda + d - c)}{2}$.

3. PROCEDURES FOR FUZZY CRITICAL PATH (FCP) PROBLEM

Forward Pass Calculation:
Forward pass calculations are employed to calculate the Earliest starting time $(E\tilde{S}T)$ in the project network.

$$\tilde{E}_j = Max_i(\tilde{E}_i(+)\tilde{t}_{ij}), i = \text{ number of preceding nodes} \quad (3)$$

$$\text{Earliest finishing time} = E\tilde{F}T = E\tilde{S}T(+)\text{Fuzzy activity time} \quad (4)$$

Backward Pass Calculation:
Backward pass calculations are employed to calculate the Latest finishing time $(L\tilde{F}T)$ in the project network.

$$\tilde{L}_i = Min_j(\tilde{L}_j(-)\tilde{t}_{ij}), j = \text{ number of succeeding nodes} \quad (5)$$

$$\text{Latest starting time } = L\tilde{S}T = L\tilde{F}T(-) \text{ Fuzzy activity time} \quad (6)$$

Total Float ($\tilde{T}F$):

$$\tilde{T}F = L\tilde{F}T(-)E\tilde{F}T(or)\tilde{T}F = L\tilde{S}T(-)E\tilde{S}T \quad (7)$$

Procedure 1: FCP based on Centroid measure
Step 1: Construct a network $G(V, E)$ where V is the set vertices and E is the set of edges. Here G is an acyclic digraph and arc lengths or edge weights are taken as trapezoidal fuzzy numbers which in turn converted in terms of α-cut interval numbers using Definition 2.
Step 2: Calculate Earliest starting time $(E\tilde{S}T)$ according to forward pass calculation given in eqn. (3).
Step 3: Calculate Earliest finishing time $(E\tilde{F}T)$ using eqn. (4).
Step 4: Calculate Latest finishing time $(L\tilde{F}T)$ according to backward pass calculation, given in eqn. (5).
Step 5: Calculate Latest starting time $(L\tilde{S}T)$ using eqn. (6).
Step 6: Calculate Total Float $(\tilde{T}F)$ using eqn. (7).
Step 7: Calculate Centroid measure or Signed distance measure for each activity using Definition 12 or Definition 6.
Step 8: If Centroid measure = Signed distance measure = 0, those activities are called as fuzzy critical activities and the corresponding path is the fuzzy critical path.

Numerical Example 1:
Consider, a Civil building construction project as given in Fig. 1.
Node 1 - Excavation and Foundation, Node 2 - Columns and Beams, Node 3 - Brick Work, Node 4 - Flooring, Node 5 - Roof concrete, Node 6 - Plastering, Node 7 - Painting.

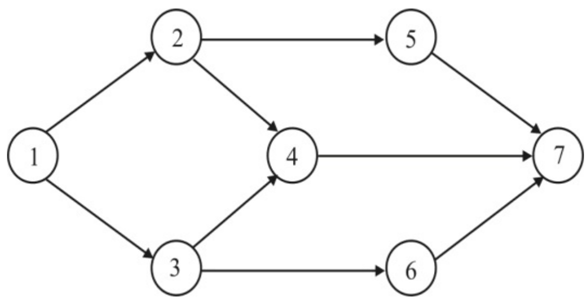

Fig. 1. Network of Civil Project

Results of the network based on Centroid measure is given in Table 1.

Procedure 2: FCP based on Magnitude measure
Expected time in terms of trapezoidal fuzzy numbers are converted to triangular fuzzy numbers and defuzzified using Definition 17 for all activities in the network diagram. Then the usual crisp critical path method is applied to identify the fuzzy critical path.

Numerical Example 2:
Consider the network given in Fig. 1 and assume the arc lengths as trapezoidal fuzzy numbers as given in the Numerical Example 1. (Refer Table 2).

Verification using Area Measure:
For the sake of verification, we consider the network diagram (Fig. 1) and assume the arc lengths as trapezoidal fuzzy numbers as given in the Numerical Example 1. Now, the expected time in terms of trapezoidal fuzzy numbers are defuzzified using Definition 18 (let $\lambda = 1$) for all activities in the network diagram. Then the usual crisp critical path method is followed to identify the fuzzy critical path. (Refer Table 3).

Procedure 3: FCP based on Metric distance
Step 1 is same as in Procedure 1.
Step 2: Calculate all possible paths p_i, $i = 1$ to n from source

Table 1. Results of the Network based on Signed distance measure (or) Centroid measure

Activity	Fuzzy Activity time (Trapezoidal Fuzzy Number)	Fuzzy Activity time (α-cut interval number) $\alpha = 0.5$	$\tilde{T}F$	Signed distance measure (or) Centroid measure
1-2	(25, 28, 32, 35)	[27, 33]	[-26, 46]	10
1-3	(40, 55, 65, 70)	[48.75, 66.25]	[-45, 45]	0
2-4	(32, 37, 43, 48)	[35.25, 44.75]	[-20.25, 55.25]	17.5
3-4	(20, 25, 35, 40)	[23.75, 36.25]	[-45, 45]	0
2-5	(35, 38, 42, 45)	[37, 43]	[-26, 46]	10
3-6	(42, 45, 55, 60)	[44.75, 56.25]	[-9.75, 70.75]	30.5
4-7	(60, 65, 75, 85)	[63.75, 78.75]	[-45, 45]	0
5-7	(65, 75, 85, 90)	[71.25, 86.25]	[-26, 46]	10
6-7	(15, 18, 22, 26)	[17, 23.5]	[-9.75, 70.75]	30.5

Here path 1-3-4-7 is identified as the fuzzy critical path.

Table 2. Results of the Network based on Magnitude measure

Activity	Fuzzy Activity time (Trapezoidal Fuzzy Number)	Fuzzy Activity time converted in terms of triangular fuzzy number using Definition 17	Defuzzified Activity Time using Definition 17	TF
1-2	(25, 28, 32, 35)	(25, 30, 35)	22.5	7.51
1-3	(40, 55, 65, 70)	(40, 60, 70)	44.17	0
2-4	(32, 37, 43, 48)	(32, 40, 48)	30	14.17
3-4	(20, 25, 35, 40)	(20, 30, 40)	22.5	0
2-5	(35, 38, 42, 45)	(35, 40, 45)	30	7.51
3-6	(42, 45, 55, 60)	(42, 50, 60)	37.67	22.67
4-7	(60, 65, 75, 85)	(60, 70, 85)	52.92	0
5-7	(65, 75, 85, 90)	(65, 80, 90)	59.58	7.51
6-7	(15, 18, 22, 26)	(15, 20, 26)	15.08	22.67

Here path 1-3-4-7 is identified as the fuzzy critical path.

Table 3. Results of the Network based on Area measure

Activity	Fuzzy Activity time (Trapezoidal Fuzzy Number)	Defuzzified Activity Time using Definition 18	TF
1-2	(25,28,32,35)	7	17
1-3	(40,55,65,70)	20	0
2-4	(32,37,43,48)	11	17
3-4	(20,25,35,40)	15	0
2-5	(35,38,42,45)	7	21
3-6	(42,45,55,60)	14	11
4-7	(60,65,75,85)	17.5	0
5-7	(65,75,85,90)	17.5	21
6-7	(15,18,22,26)	7.5	11

Here path 1-3-4-7 is identified as the fuzzy critical path.

vertex 's' to the destination vertex 'd' and the corresponding path lengths $\tilde{L}_i, i = 1$ to n using addition operation given in Definition 3 and set $\tilde{L}_i = [A_{L_i}(\alpha), A_{R_i}(\alpha)]$.
Step 3: Calculate metric distance for each possible path lengths that is $D(\tilde{L}_i, 0)$ for $i = 1$ to n using Definition 13.
Step 4: The path having the maximum metric distance is identified as the fuzzy critical path.

Numerical Example 3:
Consider the network given in Fig. 1 and assume the arc lengths as trapezoidal fuzzy numbers as given in the Numerical Example 1. (Refer Table 4).

Table 4. Results of the Network based on Metric distance

Paths	$D(\tilde{L}_i, 0)$	Ranking
$p_1 : 1-2-4-7$	201.3	3
$p_2 : 1-2-5-7$	211.5	2
$p_3 : 1-3-4-7$	227.5	1
$p_4 : 1-3-6-7$	183.7	4

Here path $p_3 : 1-3-4-7$ is identified as the fuzzy critical path.

Procedure 4: FCP based on Ranking degree
Step 1 and Step 2 are same as in Procedure 3.
Step 3: Calculate $\tilde{L}_{max}$ using Definition 4 and set $\tilde{L}_{max} = [A_L(\alpha), A_R(\alpha)]$.
Step 4: Calculate Ranking degree using Definition 14 for each possible path lengths that is $R(\tilde{L}_{max} \geq \tilde{L}_i), i = 1$ to n.
Step 5: The path having the minimum Ranking degree is identified

as the fuzzy critical path.

Numerical Example 4:
Consider the network given in Fig. 1 and assume the arc lengths as trapezoidal fuzzy numbers as given in the Numerical Example 1. (Refer Table 5).

Table 5. Results of the Network based on Ranking degree

Paths	$R(\tilde{L}_{max} \geq \tilde{L}_i)$	Ranking
$p_1 : 1-2-4-7$	17.5	3
$p_2 : 1-2-5-7$	10	2
$p_3 : 1-3-4-7$	0	1
$p_4 : 1-3-6-7$	30.5	4

Here path $p_3 : 1-3-4-7$ is identified as the fuzzy critical path.

Procedure 5: FCP based on Acceptability Index
Step 1 and Step 2 are same as in Procedure 3.
Step 3 : The path lengths $\tilde{L}_i$, $i = 1$ to n given in terms of α-cut interval numbers are converted into mean-width notation using Definition 15 and set $\tilde{L}_i = (m_i, w_i)$.
Step 4: Calculate $\tilde{L}_{max}$ in terms of mean-width notation using Definition 9 and set $\tilde{L}_{max} = (m, w)$.
Step 5: Calculate Acceptability index between $\tilde{L}_i$ and $\tilde{L}_{max}$ that is $A(\tilde{L}_i < \tilde{L}_{max})$ using Definition 16.
Step 6: The path having the minimum acceptability index is identified as the fuzzy critical path.

Numerical Example 5:
Consider the network given in Fig. 1 and assume the arc lengths as trapezoidal fuzzy numbers as given in the Numerical Example 1. (Refer Table 6).

Table 6. Results of the Network based on Acceptability Index

Paths	$A(\tilde{L}_i < \tilde{L}_{max})$	Ranking
$p_1 : 1-2-4-7$	0.61	3
$p_2 : 1-2-5-7$	0.37	2
$p_3 : 1-3-4-7$	0	1
$p_4 : 1-3-6-7$	0.98	4

Here path $p_3 : 1-3-4-7$ is identified as the fuzzy critical path.

Results and Discussion

In this paper, for Fig. 1, various ranking methods are applied to identify the critical path under fuzzy environment. It was found that the results obtained through five different procedures remains the same and also the fuzzy critical path problem obtained in this paper coincides with the existing earlier result [14]. Hence the procedures developed in this paper are alternative methods to get the fuzzy critical path.

4. CONCLUSION

Fuzzy critical path and critical path length are useful informations for the decision makers in planning and controlling the complex projects. The main advantage of fuzzy model is, the decision makers can model their project based on various linguistic variables such as, “maybe”, “in between”, “approximately”, etc., whereas this specification do not exist in crisp models. Hence fuzzy models are more effective in determining the critical path in a real project network. Thus in this paper, some procedures are developed to find the optimal path in a fuzzy weighted graph (network) and the results are presented in the form of ranking measures which helps the decision makers to decide the best possible critical path in fuzzy environment.

5. ACKNOWLEDGMENT

Our Sincere thanks to the reviewers for their valuable suggestions to improve the quality of the paper.

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