Reliability Analysis of a Standby Redundant System with Critical Human Error

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ABSTRACT

The author has considered a transient system composed of two indentical units in a standby mode which can fail due to hardware common cause failure. Initially, one unit is operational mode and other in standby mode. The operative unit may fail partially or totally. In case of total failure of first unit, standby unit becomes operational system works with full efficiency.

1. INTRODUCTION

In this paper, the author has considered a transient system composed of two identical units in a standby mode which can fail due to hardware common cause failure. Initially, one unit is operational mode and other in standby mode. The operative unit may fail partially or totally. In case of total failure of first unit, standby unit becomes operational system works with full efficiency. It is assumed that whenever there is a hardware failure, system goes partially or totally failure mode first and then total failure of either unit, system works with lesser efficiency. System may fail either due to failure of both units and common cause failure. The entire system can also fail due to critical human errors. The system can be repaired in degraded state, or in failed state due to the unit failures, but can not be repaired when it fails due to human errors.

2. NOTATIONS

(a)

$$D \, / \, Dt \, / \, Dx \, / \, Dy \, / \, Dz \, / \, Dw : \frac{d}{dt} \, / \, \frac{\partial}{\partial t} \, / \, \frac{\partial}{\partial x} \, / \, \frac{\partial}{\partial y} \, / \, \frac{\partial}{\partial z} \, / \, \frac{\partial}{\partial w}$$

(b)

$\alpha_1 \, / \, \alpha_2 \, / \, \alpha_3 \, / \, \alpha_4$: Constant failure due to minor/major errors,

(c) $\lambda_{hG} \, / \, \lambda_{hD}$: Constant failure rate due to critical/human errors when the system is in good state,

(d) $P_{0s} t : P$: (at time t the system is in state S_0),

(e) $P_{Ps} t(x, t)\square : P$: (the system is in state S_1 at time t due to minor failure and repair time lies in the interval $(x, x + \square)$,

(f) $P_{T0} t(y, t)\square : P$: (the system in in state S_2 at time t and elapsed repair time lies in the interval $(y, y + \square)$,

(g) $P_{TP} t(z, t)\square : P$: (the system is in the state S_3 at time t and elapsed repair time lies in interval $(z, z + \square)$,

(h) $P_{TT} t(w, t)\square : P$: (the system is state S_4 at time t and elapsed repair time lies in the interval $(w, w + \square)$,

(i) $P_C (x, t)\square : P$: (the system is in state S_5 at time t and elapsed repair time lies in interval $(x, x + \square)$

$$S_i^K (s) = K(x) e^{-\int_0^x Kxdx}, \qquad \text{where}$$

$$K = \beta_1, \psi_1, \phi_1, \phi_2, w_c$$

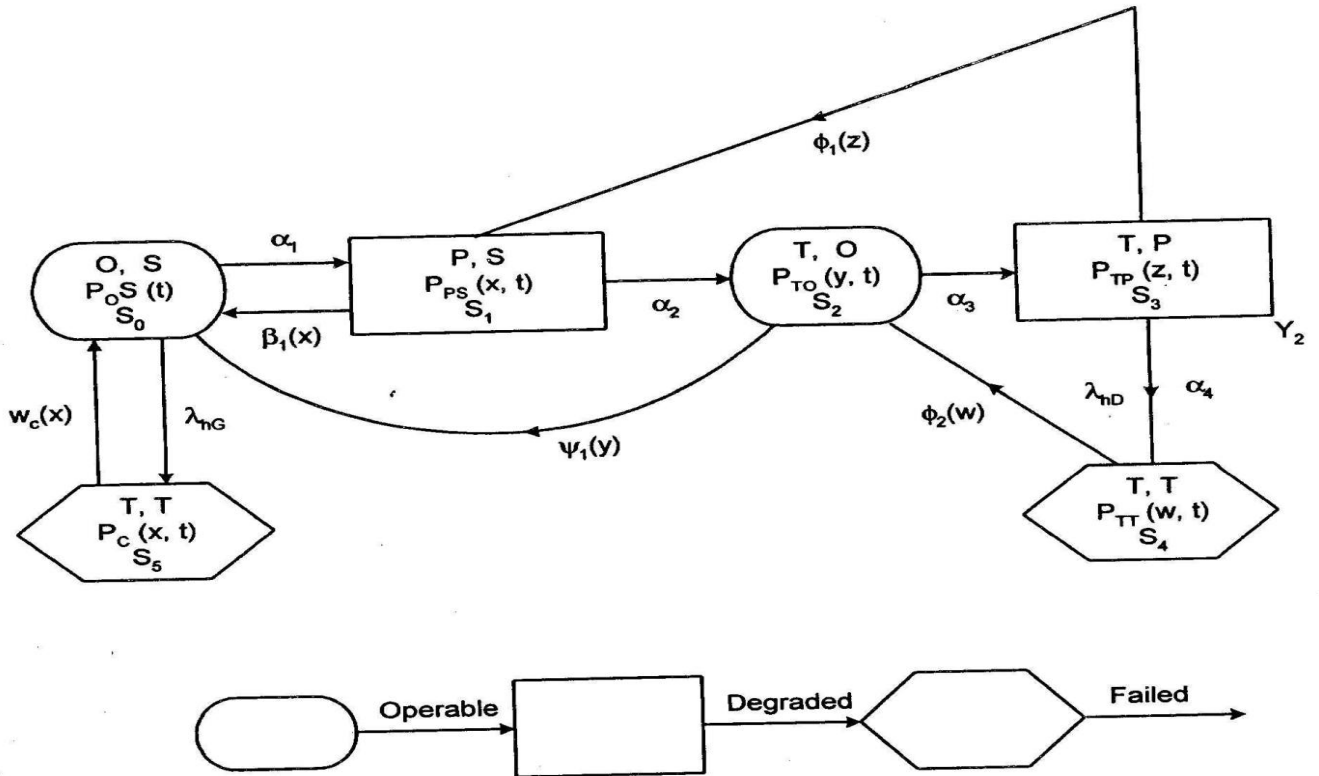$$\sum_i = \sum_{i=1}^n \quad , \, , \text{ unless otherwise stated.}$$

Fig 1: Transition diagram

3. FORMULAION OF THE MATHEMATICAL PROBLEM

Viewing the nature of this problem the following set of difference differential equations is obtained.

$$(D+\alpha_1+\lambda_{hG})P_{os}^{(t)} = \int \beta_1(x)P_{PS}(x,t)dx$$
$$+\int w_C(x)P_C(x,t)dx + \int \psi_1(y)P_{T0}(y,t)dy \quad (1)$$

$$[Dx+Dt+\alpha_2+\beta_1(x)]P_{PS}(x,t)=0 \quad (2)$$

$$[Dy+Dt+\alpha_3+\psi_1(y)]P_{T0}(y,t)=0 \quad (3)$$

$$[Dz+Dt+\alpha_4+\phi_1(z)+\lambda_{hD}]P_{TP}(z,t)=0 \quad (4)$$

$$[Dw+Dt+\phi_2(w)]P_{TT}(w,t)=\lambda_{hD}P_{TP}(z,t) \quad (5)$$

$$[Dx+Dt+w_C(x)]P_C(x,t)=\lambda_{hG}P_{0S}(t) \quad (6)$$

4. BOUNDARY CONDITIONS

$$P_{PS}(0,t)=\alpha_1 P_{0s}(t)+\int \phi_1(z)P_{TP}(z,t)dz \quad (7)$$

$$P_{T0}(0,t)=\alpha_2\int P_{Ps}(x,t)dt+\int \phi_2(w)P_{TT}(w,t)dw \quad (8)$$

$$P_{TP}(0,t)=\alpha_3\int P_{T0}(y,t)dy \quad (9)$$

$$P_{TT}(0,t)=(\lambda_{hD}+\alpha_4)P_{TP}(z,t) \quad (10)$$

$$P_C(0,t)=\lambda_{hG}P_{0s}(t) \quad (11)$$

5. INITIAL CONDITION:

$$P_{0S}(0)=1, \quad \text{otherwise} \quad 0.$$

SOLUTION OF THE PROBLEM: Taking laplace transform of equations (1) through (11) and using initial conditions, one may obtain

$$(s+\alpha_1+\lambda_{hG})P_{0S}^*(s)=1+\int \beta_1(x)P_{PS}^*(x,s)dx$$
$$+\int w_C(x)P_C^*(x,s)dx+\int \psi_1(y)P_{T0}^*(y,s)dy, \quad (12)$$

$$[Dx+s+\alpha_2+\beta_1(x)]P_{PS}^*(x,s)=0, \quad (13)$$

$$[Dy+s+\alpha_3+\psi_1(y)]P_{T0}^*(y,s)=0, \quad (14)$$

$$[Dz+s+\alpha_4+\phi_1(z)+\lambda_{hD}]P_{TP}^*(z,s)=0, \quad (15)$$

$$[Dw+s+\phi_2(w)]P_{TT}^*(w,s)=P_{TP}^*(z,s)(\lambda_{hD}+\alpha_4), \quad (16)$$

$$[Dx+s+w_c(x)]P_C^*(x,s)=\lambda_{hG}P_{0s}^*(s), \quad (17)$$

$$P_{Ps}^*(0,s)=\alpha_1 P_{0s}^*(s)+\int \phi_1(z)P_{TP}^*(z,s)dz, \quad (18)$$

$$P_{T0}^*(0,s)=\alpha_2\int P_{Ps}^*(x,s)ds+\int \phi_2(w)P_{TT}^*(w,s)dw, \quad (19)$$

$$P_{TP}^*(0,s)=\alpha_3\int P_{T0}^*(y,s)dy, \quad (20)$$

$$P_{TT}^{*}(0,s) = (\lambda_{hD} + \alpha_4) P_{TP}^{*}(z,s), \text{(21)}$$

$$P_{C}^{*}(O,s) = \lambda_{hG} P_{0s}^{*}(s). \text{(22)}$$

Integrating the above equations, we obtain

$$P_{PS}^{*}(x,s) = \left(\alpha_1 P_{0s}^{*}(s) + \int \phi_1(z) P_{TP}^{*}(z,s)dz\right) e^{-(s+\alpha_2)x} e^{-\int_0^x \beta_1(x)dx}, \text{(23)}$$

$$P_{T0}^{*}(y,s) = \left(\alpha_2 \int P_{Ps}^{*}(x,s)dx + \int \phi_2(w) P_{TT}^{*}(w,s)dw\right) \text{(24)}$$

$$P_{TP}^{*}(z,s) \square \left(\alpha_3 \int P_{T0}^{*}(y,s)dy\right) e^{-z(s+\lambda_{hG})} e^{-\int_0^z \phi_1(z)dz} \text{(25)}$$

$$P_{C}^{*}(x,s) \square \lambda_{hG} P_{0s}^{*}(s) e^{-sx} e^{-\int_0^x wc(x)dx} \text{(26)}$$

$$P_{TT}^{*}(w,s) \square e^{-sw} e^{-\int_0^w \phi_2 wdw} (\alpha_4 + \lambda_{hD}) P_{TP}^{*}(z,s) \text{(27)}$$

$$(s + \alpha_1 + \lambda_{hG}) P_{0S}^{*}(s) = 1 + \left[\alpha_1 P_{0S}^{*}(s) + \int \phi_1(z) P_{TP}^{*}(z,s)dz\right]$$
$$\int \beta_1(x) e^{-(s+\alpha_2)x} . e^{-\int_0^x \beta_1(x)dx} + \lambda_{hG} P_{0S}^{*}(s) \int w_c(x)dx\, e^{-s(x)} e^{-\int_0^x w_c(x)dx}$$
$$+ \left(\alpha_2 \int P_{PS}^{*}(x,s)dx + \int \phi_2(w) P_{TT}^{*}(w,s)dw\right) \int \psi_1(y)dy . e^{-(s+\alpha_3)y} . e^{-\int \psi_1(y)dy}$$

$$\text{(28)}$$

On solving above equation, we fin d

$$\bar{P}_{0s}(s) = \frac{1}{D(s)}, \qquad \text{(29)}$$

$$\bar{P}_{PS}(s) = \frac{B(s)}{D(s)}, \qquad \text{(30)}$$

$$\bar{P}_{T0}(s) = \frac{\alpha_1 \alpha_2}{A(s)} . \frac{1 - \bar{S}_{\beta_1}(s+\alpha_2)}{s+\alpha_2} . \frac{1 - \bar{S}_{\psi_1}(s+\alpha_3)}{s+\alpha_3} . \frac{1}{D(s)}, \qquad \text{(31)}$$

$$\bar{P}_{TP}(s) = \frac{C(s)}{D(s)}, \qquad \text{(32)}$$

$$\bar{P}_{TT}(s) = (\lambda_{hD} + \alpha_4) \frac{1 - \bar{S}_{\phi_2}(s)}{s} \frac{C(s)}{D(s)}, \qquad \text{(33)}$$

$$\bar{P}_{C}(s) = \lambda_{hG} \frac{1 - \bar{S}_{wc}(s)}{s} \frac{1}{D(s)}. \qquad \text{(34)}$$

Evaluation of Laplace transforms of up and down state Probabilities.

$$\bar{P}_{up}(s) = \bar{P}_{0s} + \bar{P}_{PS}(s) + \bar{P}_{T0}(s) + \bar{P}_{TP}(s)$$

$$= \left[1 + \beta(s) + \frac{\alpha_1 \alpha_2}{A(s)} . \frac{1 - \bar{S}_{\beta_1}(s+\alpha_2)}{s+\alpha_2} . \frac{1 - \bar{S}_{\psi_1}(s+\alpha_3)}{s+\alpha_3}\right] \frac{1}{D(s)}.$$

$$\text{(35)}$$

$$\bar{P}_{down}(s) = \bar{P}_{TT}(s) + \bar{P}_{C}(s)$$

$$= \left[(\lambda_{hD} + \alpha_4) C(s) \frac{1 - \bar{S}_{\phi_2}(s)}{s} + \lambda_C \frac{1 - \bar{S}_{wc}(s)}{s}\right] \frac{1}{D(s)}.$$

$$\text{(36)}$$

It is worth noticing that

$$\bar{P}_{up}(s) + \bar{P}_{down}(s) = 1$$

4.1 Particular Cases
When repair follows exponential time distribution setting :

$$\bar{S}^{\theta}(s) \frac{\theta}{(s+\theta)}$$

Where $\theta = \psi_1, \phi_1, \beta_1, \phi_2, w_c$

$$\bar{P}_{os}(s) = \frac{1}{E(s)},$$

$$\bar{P}_{Ps}(s) = \frac{F(s)}{E(s)},$$

$$\bar{P}_{T0}(s) = \frac{1}{G(s)} \alpha_1 \alpha_2 \frac{1}{s+\alpha_2+\beta_1} \frac{1}{s+\alpha_3+\psi_1} . \frac{1}{E(s)},$$

$$\bar{P}_{TP}(s) = \frac{H(s)}{E(s)},$$
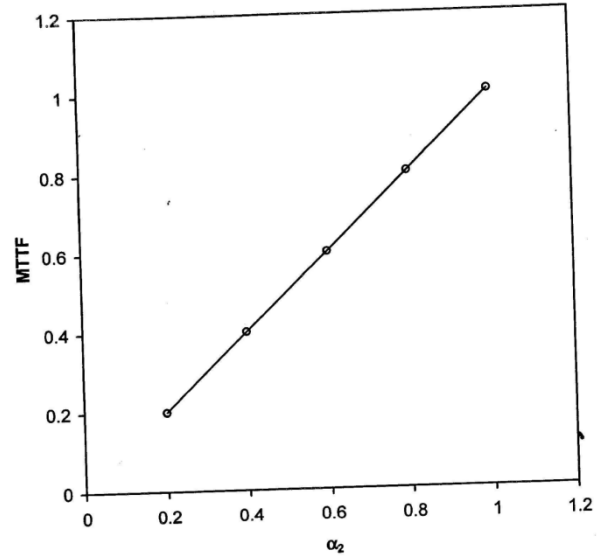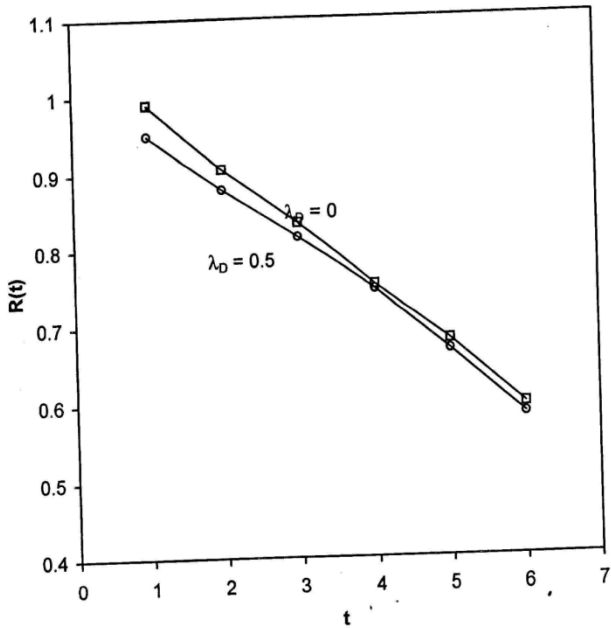
$$\bar{P}_{TT}(s) = \frac{H(s)}{E(s)},$$

$$\bar{P}_{TT}(s) = (\alpha_4 + \lambda_{hD}) . \frac{1}{s+\phi_2} . \frac{H(s)}{E(s)},$$

$$\bar{P}_{C}(s) = \frac{\lambda_C}{s+w_C} \frac{1}{E(s)}.$$

$$MTTF = \int_0^{\infty} R(t)dt$$

$$= \left[\frac{A}{\alpha_1 + y_c} + \frac{B}{\alpha_2} + \frac{C}{\alpha_3} + \frac{D}{\alpha_4 + \lambda_{hD}}\right].$$

NUMERICAL COMPUTATIONS :

$$\alpha_1 = 0.1, \alpha_2 = 0.2, \alpha_3 = 0.3, \alpha_4 = 0.4, \lambda_c = 0.05, \lambda_D = 0.05 / 0.00,$$

$$\phi_1 = \psi_1 = \phi_2 = \beta_1 = w_c = 1.$$

$$A = 8.32 / 8.9,$$

$$B = -10.8 / 12,$$

$$C = 4 / 5.33,$$

$$D = -0.55 / -1.2 \quad .$$

INTERPRETATION :

(a)   Fig. (1) shows that reliability decreases with increase in time   and   also   effect   of λ_{hD} also causes a decrease in   reliability.

(b)   Fig. (2) shows that MTTF decrease for different values of   error.

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