Bulk Arrival Two Phase Retrial Queueing System with Impatient Customers, Orbital Search, Active Breakdowns and Delayed Repair

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ABSTRACT

Bulk arrival general service retrial queueing system is analyzed in this paper. Server provides two phases of service-essential and optimal. After each service completion, the server searches for customers in the orbit. Customers may balk or renege at particular times. Accidental and active breakdown of the server is considered. The repair of the failed server starts after a random amount of time known as delay time. After repair the server continues the service of the interrupted customer or waits for the same customer. The necessary and sufficient condition for the system to be stable is presented. By applying supplementary variable technique, the steady state distributions of the server state and the number of customers in the orbit are obtained. Numerical examples are presented to illustrate the influence of the parameters on several performance characteristics

Key Words - Retrial Queue, Server Breakdown, Orbital Search, Delayed Repair, Reserved Time**.**

1. INTRODUCTION

During the last two decades considerable attention has been paid to the analysis of queueing system with repeated calls see for example the book by Falin and Templeton [10], Artalejo and Gomez-Corral [2] and the survey papers of Artalejo [3, 4].

In many waiting line systems, the role of the server is played by mechanical / electronic device, such as computer, ATM, traffic light etc., which is subject to accidental random failure; until the failed server is repaired, it may cause a halt in service. Wang [17], Wang and Li [18], Atencia et. al [6], Geni Gupur [11] and Aissani [1] have studied retrial queueing system with random breakdown. In most of the articles with server breakdown it is assumed that the failed service channel undergoes repair instantaneously. But in several life situations, men have to be employed who may not be available on call to affect the necessary repair. Consequently a delay is caused in starting the repair of the breakdown server. Only few authors, for example Prakash Rani et.al [12, 13] and Rehab et. al [14] have analysed unreliable queueing models with delayed repair.

The next specific feature considered in retrial queueing system is orbital search. Artalejo et al. [5], Dudin et al. [8], Chakravarthy et al. [7] and Udaya Chandrika and Ebenesar Annabagyam [15] have investigated retrial queue with orbital search to reduce the idle time of the server.

In this paper, bulk arrival two phase retrial queueing system with balking, reneging, orbital search and server breakdown is discussed by including the concept of delay time and reserved time. Analytical treatment of this model is obtained by supplimentry variable technique. The main motivation is from the applications to Local Area Networks, client server communication and electronic mail services on internet.

2. MODEL DESCRIPTION

Consider a single server retrial queue in which customers arrive in batches of variable size according to a Poisson process with rate λ. The batch size Y is a random variable with probability mass function $P(Y=k) = C_k$, the probability generating function C(z) and first two moments m_1 and m_2. At the arrival epoch of a batch, if the server is idle, then one of the customers get into the service immediately and others leave the service area and enter the retrial queue. If the server is busy or down, the arriving customers either join a retrial queue with probability p or leave the system with probability $1 - p$. The customer at the head of the retrial queue competes with potential primary customers to decide which customer will enter service next. If a primary customer arrives first, the retrial customer may cancel its attempt for service and either return to its position in the retrial queue with probability q or quit the system with probability $1 - q$. The retrial time of the customer in the retrial queue is generally distributed with distribution function A(x), Laplace Stieltjes transform A*(s) and the hazard rate function $\eta(x)$.

The server provides two phases of service phase 1 and phase 2. The phase 1 service is needed to all arriving customers. As soon as the phase 1 service is completed, the customer may opt to leave the system probability τ or opt for the phase 2 service with probability $1 - \tau$.

The service times of phase i are generally distributed with distribution function $B_i(x)$, Laplace Stieltjes transform $B_i^*(s)$, the hazard rate function $\mu_i(x)$ and the first two moments μ_{i1} and μ_{i2}. If the server becomes free after completing phase i service, then immediately the server searches for customers in the orbit (if any) with probability $1 - \rho_i$ or remains idle with probability ρ_i.

Assume that the server is subject to fail while it is busy in any one of the two phases. The life time of the server in i^{th} phase is exponentially distribution with mean α_i. When the server breaks down, there is some delay to start the repair. The delay

time distributions in both the phases are arbitrarily distributed with probability distribution functions $V_i(x)$, Laplace Stieltjes transform $v_i^*(s)$, the hazard rate function $\gamma_i(x)$ and the first two moments v_{i1}, v_{i2}. The repair time in both service phases are also generally distributed with probability distribution functions $H_i(x)$, Laplace Stieltjes transform $H_i^*(s)$, the hazard rate function $\beta_i(x)$ and the first two moments h_{i1}, h_{i2}.

When the server fails in phase i service, the customer in service either remains in the service position with probability r_i until the server is up or leaves the service area with probability $1 - r_i$ and keeps returning at times exponentially distributed with the rate θ_i. If the customer in service leaves the service area upon server failure, the server after repair completion, waits for the same customer to return. This time is referred as reserved time. The server is not allowed to accept new customers until the interrupted customer leaves the system after completion of service. The server is said to be blocked if the server is busy or down.

At time t, let $N(t)$ be the number of customers in the retrial queue, $W(t)$ the elapsed retrial time of the customer, $X(t)$ the elapsed service time of the customer in service, $U(t)$ the elapsed delay time in attending the failed server, $Y(t)$ the elapsed repair time of the server and $R(t)$ the elapsed reserved time.

Define the following state probabilities:

$I_0(t)$ is the probability that the server is idle at time t and there is no customer in the retrial orbit.

$I_n(t, w)\, dw$ is the joint probability that at time t there are n customers in the orbit, the server is idle and the elapsed retrial time of an orbital customer is between w and $w + dw$, $n \geq 1$.

$P_n^i(t, x)\, dx$ is the joint probability that at time t there are n customers in the retrial orbit, the server is providing phase i service with the elapsed service time between x and $x + dx$, where $n \geq 0$.

$D_{0,n}^i(t, x, u)\, dx\, du$ $(D_{1,n}^i(t, x, u)\, dx\, du)$ is the joint probability that at time t there are n customers in the retrial orbit, the server is down, the elapsed service time is x, the interrupted customer remains in the service position (not in the service position) and the elapsed delay time is between u and $u + du$, where $n \geq 0$.

$F_{0,n}^i(t, x, y)\, dx\, dy$ $(F_{1,n}^i(t, x, y)\, dx\, dy)$ is the joint probability that at time t there are n customers in the retrial orbit, the server is down, the elapsed service time is x, the interrupted customer remains in the service position (not in the service position) and the elapsed repair time is between y and $y + dy$, where $n \geq 0$.

$R_n^i(t, x, r)\, dx\, dr$ is the joint probability that at time t there are n customers in the retrial orbit, the elapsed service time is equal to x and the elapsed reserve time is between r and $r + dr$, where $n \geq 0$.

2.1 Theorem

The necessary and sufficient condition for the system is to be stable is

$$\lambda p m_1 \left[\mu_{11}\left(1+\alpha_1\left(\frac{1-r_1}{\theta_1}+h_{11}+v_{11}\right)\right)+ (1-\tau)\,\mu_{21}\left(1+\alpha_2\left(\frac{1-r_2}{\theta_2}+h_{21}+v_{21}\right)\right)\right] < 1-(1-A^*(\lambda))\,(q+m_1-1)\,(\tau\rho_1+\rho_2(1-\tau))$$

Proof

Let $S^{(k)}$ be the generalized service time of the kth customer in service. Then $\{S^{(k)}\}$ are independently and identically distributed with Laplace transform

$$B^*(s)=B_1^*\left(s+\alpha_1-\alpha_1\left(\frac{r_1 s+\theta_1}{s+\theta_1}\,H_1^*(s)V_1^*(s)\right)\right)$$

$$+(1-\tau)B_2^*\left(s+\alpha_2-\alpha_2\left(\frac{r_2 s+\theta_2}{s+\theta_2}\,H_2^*(s)\,V_2^*(s)\right)\right)$$

and expected value

$$E(S^{(k)}) = \mu_{11}\left(1+\alpha_1\left(\frac{1-r_1}{\theta_1}+h_{11}+v_{11}\right)\right)$$

$$+(1-\tau)\mu_{21}\left(1+\alpha_2\left(\frac{1-r_2}{\theta_2}+h_{21}+v_{21}\right)\right)$$

Let $P(S)$ and $P(I)$ denote respectively the probabilities that the system is blocked and idle. Let $E(S^{(k)})$ be the expected blocked time and $E(I)$ be the expected idle time. Then

$$P(S)= \frac{E(S^{(k)})}{E(S^{(k)})+E(I)} \quad \text{and} \quad P(I) = \frac{E(I)}{E(S^{(k)})+E(I)}$$

The arrival rate at the retrial queue when the system is blocked is $\lambda\, pm_1 P(S)$

The arrival rate at the retrial queue when the server is idle is $(1 - A^*(\lambda))\,(m_1 - 1)\,(\rho_1\,\tau + (1 - \tau)\,\rho_2)\,P(I) / E(I)$.

Total arrival rate at the retrial queue is $\lambda\, pm_1 P(S) + (1- A^*(\lambda))\,(m_1 - 1)\,(\rho_1\,\tau + (1 - \tau)\,\rho_2)\,P(I) / E(I)$.

The exit rate from the retrial queue by entering service when there is no orbital search is $A^*(\lambda))(\rho_1\,\tau + (1 - \tau)\,\rho_2)\,P(I) / E(I)$.

The exit rate from the retrial queue by the orbital search is $(\tau\,(1 - \rho_1) + (1 - \tau)\,(1 - \rho_2))\,P(I) / E(I)$.

The exit rate from the retrial queue by leaving the system when a primary customer arrives first at the server is $(1 - q)\,(1 - A^*(\lambda))\,(\rho_1\,\tau + (1 - \tau)\,\rho_2)\,P(I) / E(I)$.

The total exit rate from the retrial queue is $[1 - q\,(1- A^*(\lambda))\,(\rho_1\,\tau + (1 - \tau)\,\rho_2)]\,P(I) / E(I)$.

For stability, the arrival rate should be less than the exit rate. Hence $\lambda\, pm_1 P(S) + (1-A^*(\lambda))(m_1-1)\,(\rho_1\tau+(1-\tau)\,\rho_2)\,P(I) / E(I) < [1 - q\,(1 - A^*(\lambda))\,(\rho_1\,\tau + (1 - \tau)\,\rho_2)]\,P(I) / E(I)$.

and hence

$$\lambda\, pm_1 \left[\mu_{11}\left(1+\alpha_1\left(\frac{1-r_1}{\theta_1}+h_{11}+v_{11}\right)\right) + (1-\tau)\,\mu_{21}\left(1 + \alpha_2\left(\frac{1-r_2}{\theta_2}+h_{21}+v_{21}\right)\right)\right] < 1 - (1 - A^*(\lambda))\,(q + m_1 - 1)\,(\rho_1\,\tau + (1 - \tau)\,\rho_2).$$

3. STEADY STATE DISTRIBUTIONS

The system of equations that governs the model under steady state, by supplementary variable method are

$$\lambda I_0 = \tau \int_0^\infty p_0^{(1)} \mu_1(x)dx + \int_0^\infty p_0^{(2)} \mu_2(x)dx \tag{1}$$

$$\frac{dI_n(w)}{dw} = -(\lambda + \eta(w))I_n(w), \; n \geq 1 \tag{2}$$

$$\frac{dP_n^{(i)}(x)}{dx} = -(p\lambda + \mu_i(x) + \alpha_i)P_n^{(i)}(x) + \int_0^\infty F_{0,n}^{(i)}(x,y)\beta_i(y)dy$$
$$+ \theta_i \int_0^\infty R_n^{(i)}(x,r)dr + p\lambda(1-\delta_{0n})\sum_{k=1}^n c_k P_{n-k}^{(i)}(x),$$
$$n \geq 0, i = 1, 2 \tag{3}$$

$$\frac{\partial D_{i,n}^{(i)}(x,u)}{du} = -(p\lambda + \gamma_i(u))D_{i,n}^{(i)}(x,u) + p\lambda(1-\delta_{0n})\sum_{k=1}^n c_k D_{i,n}^{(i)}(x,u),$$
$$n \geq 0, i = 1, 2 ; j = 0, 1 \tag{4}$$

$$\frac{\partial F_{j,n}^{(i)}(x,y)}{dy} = -(p\lambda + \beta_i(u))F_{j,n}^{(i)}(x,y) + p\lambda(1-\delta_{0n})\sum_{k=1}^n c_k F_{j,n}^{(i)}(x,y),$$
$$n \geq 0, i = 1, 2 ; j = 0, 1 \tag{5}$$

$$\frac{\partial R_n^{(i)}(x,r)}{dr} = -(p\lambda + \theta_i)R_n^{(i)}(x,r) + p\lambda(1-\delta_{0n})\sum_{k=1}^n c_k R_n^{(i)}(x,r),$$
$$n \geq 0, i = 1, 2 \tag{6}$$

with boundary conditions

$$I_n(0) = \tau \rho_1 \int_0^\infty p_n^{(1)}(x)\mu_1(x)dx + \rho_2 \int_0^\infty p_n^{(2)}(x)\mu_1(x)dx, n \geq 1 \tag{7}$$

$$P_n^{(1)}(0) = \tau(1-\rho_1)\int_0^\infty p_{n+1}^{(1)}(x)\mu_1(x)dx + (1-\rho_2)\int_0^\infty p_{n+1}^{(2)}(x)\mu_2(x)dx$$
$$+ \lambda C_{n+1}I_0 + \lambda(1-q)\sum_{k=1}^{n+1} C_k \int_0^\infty I_{n-k+2}(w)dw$$
$$+ \int_0^\infty I_{n+1}(w)\eta(w)dw + \lambda q(1-\delta_{0n})\sum_{k=1}^n C_k \int_0^\infty I_{n-k+1}(w)dw,$$
$$n \geq 1 \tag{8}$$

$$P_n^{(2)}(0) = (1-\tau)\int_0^\infty p_n^{(1)}(x)\mu_1(x)dx, \; n \geq 0 \tag{9}$$

$$D_{0,n}^{(i)}(x,0) = r_i \alpha_i P_n^{(i)}(x), n \geq 0, i = 1,2 \tag{10}$$

$$D_{1,n}^{(i)}(x,0) = (1-r_i)\alpha_i P_n^{(i)}(x), n \geq 0, i = 1,2 \tag{11}$$

$$F_{o,n}^{(i)}(x,0) = \int_0^\infty D_{o,n}^{(i)}(x,u)\gamma_i(u)du, n \geq 0, i = 1,2 \tag{12}$$

$$F_{1,n}^{(i)}(x,0) = \int_0^\infty D_{1,n}^{(i)}(x,u)\gamma_i(u)du, n \geq 0, i = 1,2 \tag{13}$$

$$R_n^{(i)}(x,0) = \int_0^\infty F_{i,n}^{(i)}(x,y)\beta_i(y)dy, n \geq 0, i = 1,2 \tag{14}$$

Define the following probability generating functions

$$I(z,w) = \sum_{n=1}^\infty I_0(w) z^n, \; P^i(z,x) = \sum_{n=0}^\infty P_n^i(x) z^n,$$

$$D_j^i(z,x,u) = \sum_{n=0}^\infty D_{j,n}^i(x,u) z^n, \; F_j^i(z,x,y) = \sum_{n=0}^\infty F_{j,n}^i(x,y) z^n,$$

$$R^i(z,x,r) = \sum_{n=0}^\infty R_n^i(x,r) z^n, \; i = 1, 2 ; j = 0, 1 \text{ and } |z| \leq |$$

Proceeding in the usual manner with the equations (1)–(14) we can determine the expressions of $I(z,w)$, $P^{(i)}(z,x)$, $D_j^{(i)}(z,x,u)$, $F_j^{(i)}(z,x,y)$ and $R^{(i)}(z,x,r)$, i=1,2 ; j=1,2.

By defining the the partial probability generating function $\psi(z) = \int_0^\infty \psi(z,x)\,dx$ or $\psi(z) = \int_0^\infty \int_0^\infty \psi(z,x,y)\,dx\,dy$, we can find the queue size probability generating functions $I(z)$, $P^{(i)}(z)$, $D_j^{(i)}(z)$, $F_j^{(i)}(z)$ and $R^{(i)}(z)$.

4. PERFORMANCE MEASURES

Taking $z = 1$ in $I(z)$, $P^{(i)}(z)$, $D_j^{(i)}(z)$, $F_j^{(i)}(z)$ and $R^{(i)}(z)$ we obtain the following system state probabilities.

The steady state probability that the system is non-empty and the server is idle is $I = I(1) = I_0 (1-A^*(\lambda)) [K_1'(1) + (1-\tau) K_2'(1) + m_1(\tau\rho_1 + (1-\tau)\rho_2) - 1] / T_1$

The steady state probability that the server is busy is

$$P = P^{(1)}(1) + P^{(2)}(1)$$
$$= I_0 \lambda N_2 [\mu_{11} + (1 - \tau) \mu_{21}] / T_1$$

The steady state probability that the breakdown server waits for its repair to start is given by

$$D = D_0^{(1)}(1) + D_1^{(1)}(1) + D_0^{(2)}(1) + D_0^{(2)}(1)$$
$$= I_0 \lambda N_2 [\alpha_1 \mu_{11} v_{11} + (1 - \tau) \alpha_2 \mu_{21} v_{21}] / T_1$$

The steady state probability that the server is under repair is

$$F = F_0^{(1)}(1) + F_1^{(1)}(1) + F_0^{(2)}(1) + F_0^{(2)}(1)$$
$$= I_0 \lambda N_2 [\alpha_1 \mu_{11} h_{11} + (1 - \tau) \alpha_2 \mu_{21} h_{21}] / T$$

The steady state probability that the server is in reserved time is

$$R = R^{(1)}(1) + R^{(2)}(1)$$
$$= I_0 \lambda N_2[(1-r_1) \alpha_1\mu_{11}/\theta_1 + (1-\tau)(1-r_2) \alpha_2 \mu_{21}/\theta_2] / T_1$$

Using the normalizing condition $I_0 + I + P + D + F + R = 1$ the expression of I_0 is derived as $I_0 = T_1 / N_1$
where
$$T_1 = 1 - K_1'(1) - (1-\tau) K_2'(1) - (1-A^*(\lambda))(q+m_1-1)(\tau \rho_1 + \rho_2(1-\tau))$$
$$N_1 = A^*(\lambda) + (1-q)(1-A^*(\lambda))(\tau\rho_1 + \rho_2(1-\tau)) + (K_1'(1)$$
$$+ (1-\tau) K_2'(1)) (A^*(\lambda)m_1(1-p) + (1-q)(1- A^*(\lambda)))/(pm_1)$$
$$N_2 = m_1 A^*(\lambda) + (1 - q) (1 - A^*(\lambda))$$
$$K_i'(1) = p\lambda \mu_{i1} m_1 (1 + \alpha_i (\frac{1-r_i}{\theta_i} + h_{i1} + v_{i1})), \; i = 1, 2.$$

The probability generating function of the number of customers in the retrial queue is given by

$$P_q(z) = I_0 + I(z) + \sum_{i=1}^2 [P^i(z) + \sum_{j=0}^1 [D_j^i(z) + F_j^i(z)] + R^i(z)]$$

The probability generating function of the number of customers in the system is given by

$$P_s(z) = I_0 + I(z) + z \sum_{i=1}^{2} [P^i(z) + \sum_{j=0}^{1} [D_j^i(z) + F_j^i(z)] + R^i(z)]$$

The mean number of customers in the orbit is

$$L_q = P_q'(1)$$
$$= \frac{T_1 N_4 + T_2 N_3 + T_1 N_3 \overline{C_2} / 2 \overline{C_1}}{p\, T_1\, N_1\, \overline{C_1}}$$

The mean number of customers in the system is

$$L_s = P_s'(1)$$
$$= \frac{T_1 N_5 + T_2 N_3 + T_1 N_3 \overline{C_2} / 2 \overline{C_1}}{p\, T_1\, N_1\, \overline{C_1}}$$

where

$T_2 = 1 - (1 - A^*(\lambda))(\tau\rho_1 + \rho_2(1-\tau))[K_1'(1) (q + m_1 - 1)$
$+ qm_1 + m_2/2] - K_1'(1) - (1-\tau) K_2'(1) [K_1'(1)$
$+ (1-\rho_2) + \rho_2(A^*(\lambda) + (1 - A^*(\lambda))(q + m_1))]$
$- (K_1''(1) + (1-\tau) K_2''(1)) / 2$

$N_3 = pm_1(1 - A^*(\lambda))[1 - (1-q) (\tau\rho_1 + \rho_2(1-\tau))]$
$- [K_1'(1) + (1-\tau) K_2'(1)] [m_1(1-p)A^*(\lambda)$
$+ (1-q) (1 - A^*(\lambda))]$

$N_4 = p[\tau\rho_1 + \rho_2(1-\tau)][(1 - A^*(\lambda))(1-q)(m_1(q + K_1'(1))$
$+ m_2/2) - m_1 A^*(\lambda) K_1'(1)] + pA^*(\lambda)[m_1 + m_2/2]$
$+ pm_1(1-\tau)[\rho_2(1-q)(1 - A^*(\lambda)) K_2'(1)$
$- K_2'(1) (1-\rho_2)/2]$
$+ [K_1'(1) + (1-\tau) K_2'(1)][m_1(1-q)(1 - A^*(\lambda))$
$+ A^*(\lambda)(1-p)(m_1 + m_2/2)] + [K_1''(1) + (1-\tau) K_2''(1)$
$+ 2(1-\tau) K_1''(1) K_2''(1))][N_2 pm_1 A^*(\lambda)]$

$N_5 = N_4 + N_2 [K_1''(1) + (1 - \tau) K_2''(1)]$

$$K_i''(1) = p^2 \lambda^2 m_1^2 \left(\mu_{i2}(1 + \alpha_i(\frac{1-r_i}{\theta_i} + h_{i1} + v_{i1}))^2\right.$$
$$+ 2\alpha_i \mu_{i1}((\frac{1-r_i}{\theta_i})(\frac{1}{\theta_i} + h_{i1} + v_{i1}) + (h_{i2} + 2h_{i1}v_{i1} + v_{i2})/2))$$
$$+ p\lambda m_2 \mu_{i1}(1 + \alpha_i(\frac{1-r_i}{\theta_i} + h_{i1} + v_{i1})), \quad i = 1, 2$$

5. RELIABILITY INDEXES OF THE SERVER

We now consider some reliability quantities of the server. Let A(t) be the system availability at time t.

The steady state availability of the server A is

$$A = \lim_{t \to \infty} A(t).$$
$$= \lim_{t \to \infty} P \{\text{the service station is up at time t}\}$$
$$= I_0 + \sum_{n=1}^{\infty} \int_0^{\infty} I_n(w)\, dw + \sum_{n=0}^{\infty} \sum_{i=1}^{2} [\int_0^{\infty} P_n^i(x)\, dx$$
$$+ \int_0^{\infty} \int_0^{\infty} R_n^i(x, r)\, dx\, dr]$$
$$= \{A^*(\lambda) + (1 - A^*(\lambda))(1-q)(\tau\rho_1 + \rho_2(1-\tau)) - A^*(\lambda) K_1'(1)$$

$$+ (1-\tau) K_2'(1)) + \lambda N_2 [\mu_{11}(1 + \alpha_1(\frac{1-r_1}{\theta_1}))$$
$$+ (1-\tau) \mu_{21}(1 + \alpha_2(\frac{1-r_2}{\theta_2}))]\} / N_1$$

The steady state failure frequency of the server is

$$F = \sum_{n=0}^{\infty} \sum_{i=1}^{2} \int_0^{\infty} \alpha_i\, P_n^i(x)\, dx$$
$$= N_2 [\alpha_1 \mu_{11} + \alpha_2 (1 - \tau) \mu_{21}] / N_1$$

6. SPECIAL CASES

The model under study is a generalization of many queueing models.

Case 1: If C(z) = z, then the model reduces to M/G/1 two phase retrial queueing system with impatient customers, server breakdown, delayed repair and orbital search.

Case 2: If C(z) = z and $\rho_1 = \rho_2 = 1$ the system reduces to two phase retrial queueing system with impatient customers, server breakdown, delayed repair and reserved time [9].

Case 3: If C(z) = z and $p = q = \rho_1 = \rho_2 = 1$ the system reduces to two phase retrial queueing system with server breakdown, delayed repair and reserved time [16].

Case 4: If $\tau = 1$ and $p = q = \rho_1 = \rho_2 = 1$ the system reduces to impatient customer bulk arrival retrial queueing system with server breakdown and delayed repair [12].

Case 5: If $\tau = 1 = r_1 = r_2 = \rho_1 = \rho_2 = 1$ the system coincides with the paper bulk arrival retrial queueing system with server breakdown and delayed repair [13].

7. NUMERICAL RESULTS

Assume that the distributions of service time, retrial time, delay time and repair time follow (i) Erlangian of order 2, (ii) Exponential and (iii) Hyper-exponential distributions. Table 1 provides the availability of the server A and the failure frequency F by varying the values of μ_1, τ and α_1 for the three processes Erlang, exponential and hyper-exponential with the fixed values $(\lambda, \eta, \mu_2, \gamma_1, \gamma_2, \beta_1, \beta_2, r_1, r_2, \rho_1, \rho_2, \theta_1, \theta_2, C_1, C_2, \alpha_2, p, q, a) = (1, 15, 20, 5, 6, 3, 6, 0.5, 0.5, 0.5, 0.5, 6, 3, 0.6, 0.4, 2, 0.8, 0.8, 0.5)$.

For all μ_1 and τ, as α_1 increases the availability of the server decreases and the failure frequency increases. This can be explained intuitively as follows: As failure rate α_1 increases life time decreases and hence availability decreases and failure rate increases.

Table 1. Availability and Failure frequency

μ_1	τ	α_1	Erlang		Exponential		Hyper-exponential	
			A	F	A	F	A	F
25	0.0	1	0.7282	0.3480	0.9268	0.1876	0.9762	0.1006
		2	0.6313	0.4366	0.8989	0.2395	0.9665	0.1289
		3	0.5389	0.5211	0.8714	0.2907	0.6569	0.1569
		4	0.4507	0.6018	0.8442	0.3413	0.9473	0.1849
		5	0.3665	0.6780	0.8175	0.3912	0.9377	0.2127
	0.5	1	0.8041	0.2320	0.9484	0.1223	0.9831	0.0650
		2	0.7017	0.3267	0.9199	0.1754	0.9733	0.0935
		3	0.6044	0.4169	0.8919	0.2278	0.9636	0.1219
		4	0.5116	0.5028	0.8643	0.2795	0.9540	0.1501
		5	0.4231	0.5846	0.8370	0.3305	0.9443	0.1782
	1	1	0.8858	0.1071	0.9706	0.0551	0.9901	0.0289
		2	0.7775	0.2086	0.9416	0.1095	0.9803	0.0577
		3	0.6747	0.3050	0.9130	0.1631	0.9705	0.0863
		4	0.5769	0.3967	0.8848	0.2159	0.9607	0.1148
		5	0.4838	0.4840	0.8570	0.2680	0.9510	0.1432
30	0.0	1	0.7440	0.3340	0.9313	0.1792	0.9779	0.0958
		2	0.6619	0.4090	0.9080	0.2227	0.9698	0.1193
		3	0.5831	0.4810	0.8849	0.2657	0.9618	0.1426
		4	0.5074	0.5503	0.8620	0.3082	0.9538	0.1658
		5	0.4345	0.6169	0.8394	0.3503	0.9459	0.1890
	0.5	1	0.8210	0.2165	0.9531	0.1136	0.9484	0.0601
		2	0.7343	0.2968	0.9292	0.1581	0.9767	0.0838
		3	0.6512	0.3737	0.9057	0.2021	0.9686	0.1073
		4	0.5715	0.4475	0.8824	0.2455	0.9606	0.1308
		5	0.4948	0.5185	0.8594	0.2885	0.9526	0.1542
	1	1	0.9040	0.0900	0.9754	0.0461	0.9918	0.0240
		2	0.8123	0.1760	0.6511	0.0916	0.9836	0.0479
		3	0.7244	0.2584	0.6271	0.1366	0.9755	0.0716
		4	0.6402	0.3373	0.6034	0.1811	0.9674	0.0953
		5	0.5595	0.4130	0.8800	0.2251	0.9593	0.1189
35	0.0	1	0.7553	0.3238	0.9346	0.1731	0.9790	0.0925
		2	0.6842	0.3889	0.9145	0.2106	0.9722	0.1125
		3	0.6155	0.4516	0.8946	0.2476	0.9653	0.1324
		4	0.5492	0.5123	0.8749	0.2844	0.9585	0.1523
		5	0.4850	0.5710	0.8554	0.3207	0.9517	0.1721
	0.5	1	0.8333	0.2053	0.9564	0.1073	0.9860	0.0567
		2	0.7581	0.2749	0.9359	0.1456	0.9790	0.0769
		3	0.6856	0.3420	0.9156	0.1835	0.9721	0.0971
		4	0.3157	0.4067	0.8956	0.2211	0.9653	0.1171
		5	0.5481	0.4693	0.8757	0.2582	0.9584	0.1371
	1	1	0.9173	0.0776	0.9789	0.0396	0.9930	0.0205
		2	0.8376	0.1522	0.9580	0.0788	0.9860	0.0409
		3	0.7610	0.2241	0.9373	0.1176	0.9790	0.0612
		4	0.6841	0.2934	0.9168	0.1560	0.9721	0.0815
		5	0.6158	0.3602	0.8965	0.1940	0.9652	0.1017

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