EOQ Model with Volume Agility, Variable Demand Rate, Weibull Deterioration Rate and Inflation

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ABSTRACT
The main objective of this paper is to develop a Supply chain model of a volume agile manufacturing process for the deteriorating items. It is assumed that an EOQ model in which inventory is depleted not only by demand also by deterioration. In this study, a model for the producer by assuming stock dependent demand rate is developed. It is assumed that the deterioration rate follows the Weibull distribution. The unit production cost which is treated to be a function of the finite production rate which is treated to be a decision variable. This whole study is studied in the inflationary environment. The mathematical expression for the total cost is derived and it is minimized. The solution procedure is illustrated with the help of numerical example.

Keywords
Volume agility, Stock dependent demand, inflation and Weibull deterioration rate.

1. INTRODUCTION
In the past, inventory models with stock dependent demand had received little attention from researchers. However, stock dependent demand plays an important role in the inventory models. In the supermarkets it is observed that the demand rate is usually influenced by the amount in the stock. Levin et al. (1972) pointed out "at times, the presence of inventory has a motivational effect on the people around it". It is commonly believed that large piles of goods displayed in a supermarket will lead the customers to buy more. This requires consideration of the demand to be a function of the on-hand inventory. As a result, many papers appeared in literature to deal with inventory models using some forms of functional dependencies between the demand rate and on-hand inventory. Gupta and Vrat (1986) had assumed demand rate to be a function of the initial stock. Mondal and Phaujder (1989) considered demand rate to be linear and non linear functions of on hand inventory. Dutta and Pal (1991) extended the above models by allowing deterioration effects and shortages, which are completely backlogged, for finite as well as for infinite time horizon. Chang and Lin (2010) discussed a partial backlogging inventory model for non-instantaneous deteriorating items with stock-dependent consumption rate under inflation. Singh et al. (2011) described a deterministic two warehouse inventory model for deteriorating items with stock dependent demand and shortages under the conditions of permissible delay. Singh et al. (2013) discussed a three stage supply chain model with two warehouse, imperfect production, variable demand rate and inflation.

In the existing literature, most of the inventory models deals with the constant production rate. With a constant production rate, average cost of producing units is minimized but during slumps in demand, the already produced stock needs to be stored and this results the increase in holding cost. But varying production rate may decrease the production cost at discrete points in time and it will decrease the holding cost. Therefore the best production scheme depends on the agility of the production system and on the holding cost. Yet there is a wide scope as well as need of volume agility in the corresponding models to make it more realistic due to present business situation. To improve production efficiency, manufacturing companies have mainly flexible manufacturing systems (FMS). Its offers the hope of eliminating many of the weaknesses of the other manufacturing approaches. Misra (1975) considered the inventory model with optimum production rate. Deb and Chaudhari (1983) developed a model with finite rate of production which was greater than demand rate. Hong et al. (1990) considered the production model with finite uniform production rate. These production rates are not realistic due to market situation. Moon et al. (1991) discussed the effect of slowing down production in the context of a manufacturing equipment of a family of items, assuming a common cycle for all the items. Khouja and Mehrez (1994) and Khouja (1995) and Khouja (1996) extended the EPLS (economic lot size production) model to an imperfect production process with a flexible production rate. Sana and Chaudhuri (2003) considered volume flexibility for deteriorating item with an inventory-level dependent demand rate. Sana and Chaudhuri (2004) developed inventory model with volume flexible production for deteriorating item with time dependent demand and shortage which are completely backlogged. Sana (2004) extended EPLS model in which the rate of production depends upon technology of manufacturing system, capital investment for MRP and powerful with time dependent demand rate**.** Khouja (2005) extended the EPLS model to allow revisions to the demand forecast and multiple production rates for the linear penalty case. Sana et al. (2007) extended the EPLS model which accounts for a production system producing items of perfect as well as imperfect quality with volume FMS. Singh and Urvashi (2010) considered supply chain models with imperfect production process and volume flexibility under inflation. The best production scheme depends on the agility of the production system. Yet there is a wide scope as well as need of volume agility in the corresponding models to make it more realistic due to present business situation.

It is observed that the prices of everything going up over the years. Inflation is a rise in the general level of prices of goods and services in an economy over a period of time. Inflation can also be described as a decline in the real value of money, a loss of purchasing power in the medium of exchange. So it is more realistic to consider the inflation in our model. In this paper most of the factors which effect the inventory management in reality are considered. Deterioration rate is not always constant therefore the Weibull deterioration rate is very realistic concept. In this paper most of the factors which effect the inventory management are considered.

In all of the above mentioned models, the influence of the slump on the demand was not discussed. In this article, an attempt has been made to develop an EOQ model with volume agility, Weibull deterioration rate, and variable demand rate under inflation. The rest of this paper is organized as follows. Section 2 describes the assumptions and notations used throughout this paper. The mathematical model and the minimum total relevant cost are given in Section 3. Illustrative examples, which explain the applications of the theoretical results as well as their numerical verifications, are given in Section 4. Sensitivity analyses are carried out with respect to the different parameters of the system in Section 5, while concluding remarks and suggestions for future research are provided in Section 6.

2. ASSUMPTIONS AND NOTATIONS

The mathematical modeling in this study is developed on the basis of the following assumptions and notations:

2.1 Assumptions

- Replenishment rate is infinite and lead time is zero.
- Deteriorating rate follows the Weibull distribution.
- The production cost per unit item is a function of the production rate.
- The production rate is considered to be a decision variable.
- Demand rate is stock dependent.
- Inflation is also considered.

2.2 Notations

$I_{p1}(t)$	the inventory level at time t during the time interval $[0, t_p]$.
$I_{p2}(t)$	the inventory level at time t during the time interval $[t_p, T]$.
H	the planning horizon.
T	the replenishment cycle
m	the replenishment number in the planning horizon H
C_{1p}	the ordering cost per order
C_{2p}	the purchasing cost per unit
C_p	the holding cost per unit per unit time
P	the production rate per unit time
$\eta(P)$	the production cost per unit item
$a+bI(t)$	the demand rate at time t, where $a > 0$, and b is the stock dependent consumption rate parameter
$\alpha\beta\, t^{\beta-1}$	the two parameter Weibull distribution deterioration rate(unit/unit time). where $0 < \alpha << 1$ is called the scale parameter , $\beta > 0$ is the shape parameter.
R	the net discount rate of inflation
I_m	the maximum inventory level for each cycle
TC (m,P)	the present value of the total relevant inventory cost in the planning horizon H

3. MATHEMATICAL FORMULATION AND SOLUTION

The planning horizon H is divided into m equal parts of length T= H/m. The jth replenishment is made at time jT (j = 0,1,2,3….m). The maximum inventory level for each cycle is I_m. During the time interval $[jT , jT+t_p]$ (j= 0,1,2,3….m-1) the inventory level increases due to production and decreases due to deterioration and demand. During the time interval $[jT+t_p, (j+1)T]$ (j= 0,1,2,3….m-1), the inventory level gradually reduces due to demand and deterioration.

The inventory level at time t during the time interval $[0, t_p]$ is governed by the following differential equation:

$$\frac{dI_{p1}(t)}{dt}+\alpha\beta t^{\beta-1}I_{p1}(t)=P-[a+bI_{p1}(t)], \quad 0\le t\le t_p \qquad (1)$$

With the boundary condition $I_{p1}(0)=0$. The solution of eq. (1) can be represented by

$$I_{p1}(t)=e^{-\alpha t^{\beta}-bt}(p-a)\left(t+\frac{\alpha t^{\beta+1}}{\beta+1}+\frac{bt^2}{2}\right),\ 0\le t\le t_p \qquad (2)$$

Owing to stock dependent demand and deterioration, the inventory level at time t during the time interval $[t_p, T]$ is governed by the following differential equation:

$$\frac{dI_{p2}(t)}{dt}+\alpha\beta t^{\beta-1}I_{p2}(t)=-[a+bI_{p2}(t)], \quad t_p\le t\le T \qquad (3)$$

With the boundary condition $I_{p2}(T)=0$. The solution of eq.

(3) can be represented by

$$I_{p2}(t)=ae^{-\alpha t^{\beta}-bt}\left[\begin{array}{l}T-t+\dfrac{\alpha(T^{\beta+1}-t^{\beta+1})}{\beta+1}\\ +\dfrac{b(T^2-t^2)}{2}\end{array}\right], t_p\le t\le T \qquad (4)$$

The amount of maximum quantity per cycle

$$I_{p2}(0) = I_m = a\left[T + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{bT^2}{2}\right] \qquad (5)$$

The total relevant inventory cost involves following four factors.

3.1 Ordering cost

The present value of the ordering cost in the entire time horizon H is

$$OC_p = C_{1p}\sum_{j=0}^{m} e^{-RjT} = C_{1p}\frac{e^{RH/m} - e^{-RH}}{e^{RH/m} - 1} \qquad (6)$$

3.2 Purchasing cost

The present value of the Purchasing cost in the entire time horizon H is

$$PC_p = C_{2p}\sum_{j=0}^{m-1} I_m\, e^{-RjT}$$
$$= aC_{2p}\left[T + \frac{\alpha T^{\beta+1}}{\beta+1} + \frac{bT^2}{2}\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right) \qquad (7)$$

3.3 Holding cost

The present value of the holding cost in the entire time horizon H is

$$HC_p = \sum_{j=0}^{m-1} C_{3p}\left[\int_0^{t_p} I_{p1}(t)e^{-Rt}dt + \int_{t_p}^{T} I_{p2}(t)e^{-Rt}dt\right]e^{-RjT}$$

$$= C_{3p}\left\{\begin{array}{l} p\left\{\begin{array}{l}\frac{t_p^2}{2} - \frac{t_p^3}{3}\left(\frac{b}{2}+R\right) - \frac{bt_p^4}{8}(b+R) \\ -\frac{\alpha\beta t_p^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha t_p^{\beta+3}}{(\beta+1)}\left(\frac{b}{2}+\frac{R}{\beta+3}\right)\end{array}\right\} + \\ a\left\{\begin{array}{l}\left((b+R)\frac{t_p^2}{2} - t_p + \frac{\alpha t_p^{\beta+1}}{(\beta+1)}\right)T + \left(\begin{array}{l}\frac{1}{2} - \frac{bt_p}{2} \\ + \frac{\alpha b t_p^{\beta+1}}{2(\beta+1)}\end{array}\right)T^2 \\ + \frac{(b-R)T^3}{6} + \frac{(b+R)bT^4}{8} + \left(\begin{array}{l}\frac{\alpha(b+R)t_p^2}{2(\beta+1)} \\ -\frac{\alpha t_p}{(\beta+1)}\end{array}\right)T^{\beta+1} \\ + \left(\frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right) + \frac{\alpha T^{\beta+3}}{(\beta+3)}\left(\frac{b+R}{2} + \frac{b}{\beta+1}\right)\end{array}\right\}\end{array}\right\}$$
$$\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right)$$

(8)

3.4 Production cost

We consider a self-manufacturing system in which the items are manufactured in a machine and the market demand is filled by these manufactured items. Demand is less than the production and Production cost per unit is

$$\eta(P) = \left(\mu + \frac{g}{P} + sP\right)$$

The production cost is based on the following factors:

1. The material cost μ per unit item is fixed.

2. As the production rate increases, some costs like labor and energy costs are equally distributed over a large number of units. Hence the per-unit production cost $\left(\frac{g}{P}\right)$ decreases as the production rate (P) increases.

3. The third term (sP), associated with tool/die costs, is proportional to the production rate.

Therefore present value of the production cost in the entire time horizon H is

$$\eta(P) = \left(\mu + \frac{g}{P} + sP\right)\sum_{j=0}^{m-1}\left[\int_0^{t_p} Pe^{-Rt}dt\right]e^{-RjT}$$
$$= P\left(\mu + \frac{g}{P} + sP\right)\left(t_p - \frac{Rt_p^2}{2}\right)\left(\frac{1-e^{-RH}}{1-e^{-RH/m}}\right) \qquad (9)$$

Hence, the Present value of the average total cost of the producer in the entire time horizon H is

$$TC(P,m) = OC_p + PC_p + HC_p + \eta(P)$$
$$= equ(3.6) + equ(3.7) + equ(3.8) + equ(3.9) \qquad (10)$$

Now equation (3.10) can be minimized but as it is difficult to solve the problem by deriving a closed equation of the solution of equation (3.10), Matlab Software has been used to determine optimal value of m and P and hence the optimal total cost can be evaluated.

4. NUMERICAL EXAMPLE

The preceding theory can be illustrated by the following numerical example:

If C_{1p}= \$600/order , C_{2p} = \$6 /unit , C_{3p} = \$3 /unit/month , R= 0.2, α = 0.05 , β = 2, a = 60 units/month , b = 0.04 , μ = 0.01 , g = 4000 , s = 0.001 , H = 48 month , t_p = 4 month.

The software *Mathematica5.2* is used to derive the optimal solution. The optimal value of total cost is obtained as \$22654.2, production rate (p^*) is as 187.765 units/month and no of cycles (m*) is as 10.753.

From the figure 2., convexity of the total cost can be analyzed, which shows that the total cost is minimum for the above numerical setup for an optimal value of the p and m.

5. SENSITIVITY ANALYSIS

The sensitivity of the optimal solution has been analyzed for various system parameters through Table 1 and Table 2.

Table 1. Effect of deterioration rate and shape parameter on decision policy

β α		2.0	2.5	3.0
0.05	M	10.5327*	12.0169	13.5448
	P	187.333*	3381.33	9254
	TC	22654.2*	25027.3	28176.7
0.10	M	11.9594	13.6373	15.3424
	P	4134	10522	22267.3
	TC	25608.5	29093.8	33983.4
0.15	M	12.9488	14.7489	16.5771
	P	8080.67	17662.7	35280.7
	TC	28059.1	32536.8	39074.5

Table 2. Effect of initial demand and stock dependent consumption rate on decision policy

a b		60	100	140
0.04	M	10.5327	11.9243	13.0061
	P	187.333	187.333	187.333
	TC	22654.2	25963	28768.8
0.06	M	10.799	12.2317	13.344
	P	907.333	907.333	907.333
	TC	23158	26628.9	29570.9
0.08	M	11.0536	12.5258	13.6673
	P	1659.33	1659.33	1659.33
	TC	23656.9	27278.4	30355.2

From the above sensitivity analysis, the relative effects of the demand parameters and deterioration rate, on the total cost of the model can be analyzed.

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6. FIGURES

Inventory modal and the convexity of the total cost is represented by figure 1. and figure 2

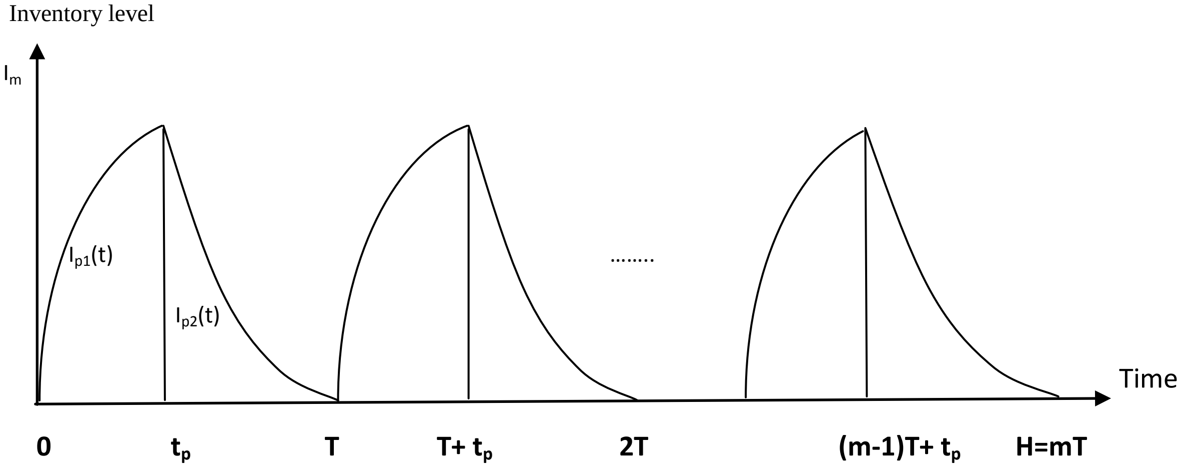

Fig 1: The graphic representation of inventory model

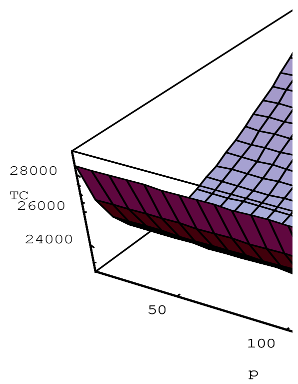

Fig 2: The graphical representation of total cost w.r.t production rate and no of cycles

7. CONCLUSION

In this model, an inventory model has been proposed with stock dependent demand, Weibull deterioration rate in the environment of inflation and volume agility. Volume agility has become an important factor because customers require increasingly customized products that answer their unique needs. The greater the uncertainties in supply and demand, it is essential to emphasized the importance of a long term strategic relationship between the producer and the customer. The purpose of the proposed study is to give a dimension to the production system of the producer. Here production rate is assumed as a decision variable and it is found that, what should be the production rate according to a model. Numerical example has been solved which gives the optimal values of total cost, Production rate and no of cycles. With the help of sensitivity analysis, the effect of deterioration rate and demand rate on the production rate, no of cycles and total cost can be analyzed.

Finally, the proposed model can be extended in several ways. For example, this model can be extended with fuzzy parameters.

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