

Implementation and Contrasting Comparison of Transform Techniques of OFDM

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ABSTRACT

Different type of transform techniques such as Discrete Hartley transform (DHT), Discrete Cosine Transform (DCT), Fast Fourier Transform (FFT) and their inverse are used for the fast computational algorithms in digital signal processing. These transforms are used to perform the modulation and demodulation operations for the OFDM system. In this paper the three transforms are describe, these transforms are used in OFDM and shows the comparison between these different schemes used in OFDM system and also the simulation results based on the BER performances.

General Terms

Orthogonal Frequency Division Multiplexing, Fast Fourier Transform (FFT), Discrete Fourier transform (DFT), Discrete Fourier Transform (DFT).

Keywords

DCT, DHT, FFT, ICI, ISI, MJPEG, MPEG, OFDM.

1. INTRODUCTION

The different transforms used in this paper have their own applications, pros and cons in digital signal processing. These transforms have a wide importance in the orthogonal frequency division multiplexing. These transforms are adopted in OFDM because of their multiplexing and demultiplexing and because of their less computational time. A Fast Fourier Transform (FFT) is an efficient algorithm to compute the discrete Fourier transform (DFT) and its inverse. The Discrete *Fourier Transform* (DFT) is used to produce frequency analysis of discrete non-periodic signals. The FFT is a faster version of the Discrete Fourier Transform (DFT). The FFT utilizes some clever algorithms to do the same thing as the DTF, but in much less time.

The Discrete Hartley Transform is closely related to the Discrete Fourier Transform, but unlike the DFT, the DHT has the advantage of producing real numbers and it is (quasi-)symmetrical. The Hartley transform produces real output for a real input, and is its own inverse. It therefore can have computational advantages over the discrete Fourier transform.

The DCT has found wide applications in image/video processing and other fields. Discrete cosine transforms (DCTs) express a function or a signal in terms of a sum of sinusoids with different frequencies and amplitudes. A DCT operates on a function at a finite number of discrete data points. The obvious distinction between a DCT and a DFT is DCT uses only cosine functions, while DFT uses both cosines and sines. DCT implies different boundary conditions than the DFT or other related transforms. DCT is used in JPEG image compression, MJPEG, MPEG, and DV video compression.

The *modified* discrete cosine transform, or MDCT is used in AAC, Vorbis and MP3 audio compression.

OFDM is the modulation technique used in many new and emerging broadband communication systems including wireless local area networks (WLANs), high definition television (HDTV) and 4G systems. To achieve high data rates OFDM is used in wireless LAN standards like IEEE 802.11a, IEEE 802.11g. Orthogonal Frequency Division Multiplexing (OFDM), which allows the overlapping of the subcarriers but keeps them orthogonal to avoid intercarrier interference (ICI) and inter-symbol interference (ISI) [1], has been adopted for mobile communication standards due to its ability to combat with the frequency-selective multipath fading channel effects and has high spectral efficiency. OFDM is robust against narrowband interference because such interference affects only a small percentage of the subcarriers; also it increases robustness against frequency-selective fading. This paper considers the different sections which are – 2. OFDM system model, 3- Comparison between transforms used in OFDM, 4-simulation results, and 5-Conclusion.

2. OFDM SYSTEM MODEL

2.1 FAST FOURIER TRANSFORM (FFT)

The fast Fourier transform (FFT) is merely a rapid mathematical method for computer applications of DFT. It is the availability of this technique, and the technology that allows it to be implemented on integrated circuits at a reasonable price, that has permitted OFDM to be developed as far as it has. The main reason that the OFDM technique has taken a long time to become a prominence has been practical. It has been difficult to generate such a signal, and even harder to receive and demodulate the signal. The hardware solution which makes use of multiple modulators and demodulators. At the transmitter, the signal is defined in the frequency domain. It is a sampled digital signal, and it is defined such that the discrete Fourier spectrum exists only at discrete frequencies. Each OFDM carrier corresponds to one element of this discrete Fourier spectrum. The amplitudes and phases of the carriers depend on the data to be transmitted. The data transitions are synchronized at the carriers, and can be processed together, symbol by symbol. The FFT is a faster version of the Discrete Fourier Transform (DFT). FFTs are of great importance to a wide variety of applications, from digital signal processing to solving partial differential equations to algorithms for quickly multiplying large integers. Let x_0, \dots, x_{N-1} be complex numbers. The DFT is defined by the formula

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N}nk}$$

k=0, ..., N-1

Evaluating these sums directly would take $O(N^2)$ arithmetical operations. An FFT is an algorithm to compute the same result in only $O(N \log N)$ operations. In general, such algorithms depend upon the factorization of N , but there are FFTs with $O(N \log N)$ complexity for all N , even for prime N . Many FFT

algorithms only depend on the fact that $e^{-\frac{2\pi i}{N}}$ is a primitive root of unity. Since the inverse DFT is the same as the DFT, but with the opposite sign in the exponent and a $1/N$ factor, any FFT algorithm can easily be adapted for it as well.

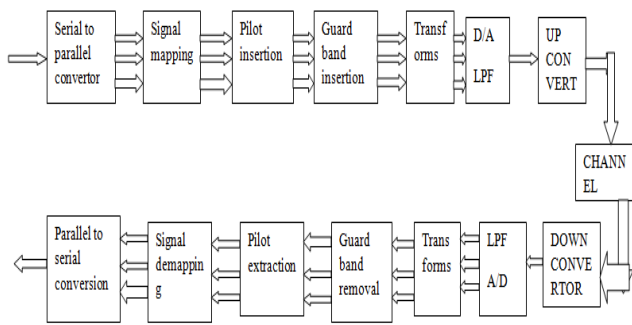


Fig1. Simulation Diagram of OFDM System

2.2. DISCRETE HARTLEY TRANSFORM (DHT)

A discrete Hartley transform (DHT) is a Fourier-related transform of discrete, periodic data similar to the discrete Fourier transform (DFT), with analogous applications in signal processing and related fields. Its main distinction from the DFT is that it transforms real inputs to real outputs, with no intrinsic involvement of complex numbers. Just as the DFT is the discrete analogue of the continuous Fourier transform, the DHT is the discrete analogue of the continuous Hartley transform, introduced by R. V. L. Hartley in 1942.

Because there are fast algorithms for the DHT analogous to the fast Fourier transform (FFT), the DHT was originally proposed as a more efficient computational tool in the common case where the data are purely real. It was subsequently argued, however, that specialized FFT algorithms for real inputs or outputs can ordinarily be found with slightly fewer operations than any corresponding algorithm for the DHT. Formally, the DHT is a linear, invertible function $H: R^n \rightarrow R^n$, where R is the set of real numbers. The N real numbers x_0, \dots, x_{N-1} are transformed into the N real numbers H_0, \dots, H_{N-1} according to the formula

$$H_k = \sum_{n=0}^{N-1} x_n [\cos(\frac{2\pi}{N}nk) + \sin(\frac{2\pi}{N}nk)]$$

k=0, ..., N-1

The IDHT is given by

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} H_k [\cos(\frac{2\pi}{N}nk) + \sin(\frac{2\pi}{N}nk)]$$

n=0, ..., N-1

The overall scale factor in front of the transform and the sign of sine term are different and it does not affect the essential properties and it is a matter of convention.

2.3. DISCRETE COSINE TRANSFORM (DCT)

A discrete cosine transform (DCT) is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. DCTs are equivalent to DFTs of roughly twice the length, operating on real data with even symmetry (since the Fourier transform of a real and even function is real and even), where in some variants the input and/or output data are shifted by half a sample. The Discrete Cosine Transform (DCT) attempts to uncorrelate the image data. After decorrelation each transform coefficient can be encoded independently without losing compression efficiency. The DFT, like the Fourier series, implies a periodic extension of the original function. A DCT, like a cosine transform, implies an even extension of the original function.

However, because DCTs operate on *finite, discrete* sequences, two issues arise that do not for the continuous cosine transform. First, one has to specify whether the function is even or odd at *both* the left and right boundaries of the. Second, one has to specify around *what point* the function is even or odd. In particular, consider a sequence *abcd* of four equally spaced data points, and say that we specify an even *left* boundary. There are two sensible possibilities: either the data is even about the sample *a*, in which case the even extension is *dcbabcd*, or the data is even about the point *halfway* between *a* and the previous point, in which case the even extension is *dcbaabcd* (*a* is repeated). Its DCT is given by

$$x_i = \sqrt{\frac{2}{N_s}} \sum_{n=0}^{N_s-1} d_n \beta_n \cos(\frac{\pi n(2i+1)}{2N_s})$$

$$i = 0, \dots, N_s - 1$$

And IDCT is given by

$$d_n = \sqrt{\frac{2}{N_s}} \beta_n \sum_{i=0}^{N_s-1} x_i \cos(\frac{\pi n(2i+1)}{2N_s})$$

The above two equations are the OFDM system transmitter and receiver sections. The IDCT is performed by $N \times N$ DCT matrix at receiver which is orthogonal to IDCT, it recovers the original transmitted sequence.

3. COMPARISON OF DIFFERENT TRANSFORM SCHEME USED IN OFDM

PARAMETERS	FFT	DHT	DCT
Pulse shaping	Required	Required	Required
Symmetric Extension	Not Required	Not required	Required
Cyclic Prefix	Required	Required	Required
Transform	Complex valued	Real valued	Real valued
Implementation Complexity	Less	Less	Less
Time-Frequency Resolution	Fixed	Dynamic	Fixed

4. SIMULATION RESULTS

In the OFDM system implementation FFT, DCT, DHT were used to perform the modulation and demodulation operations. BPSK is used as digital modulation technique. For OFDM-based transceivers, the modulator needs to compute a long-length inverse transform and the demodulator needs to compute a long length transforms, where the transform length is up to 512 or more. Software used for simulation is MATLAB 7.0. Parameters for simulation the transform based OFDM.

Table1. Parameters for Simulation

PARAMETERS	VALUES
No. of bits transmitted	12000
No. of carriers used	6
Bits per each carrier	2000
Spacing between each carrier	6KHZ
Carrier frequencies	6,12,18,24 and 30 KHZ
Modulation Technique	BPSK

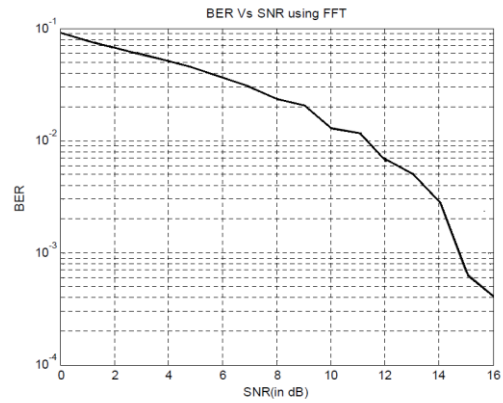


Fig 2. BER versus SNR curve for FFT based OFDM

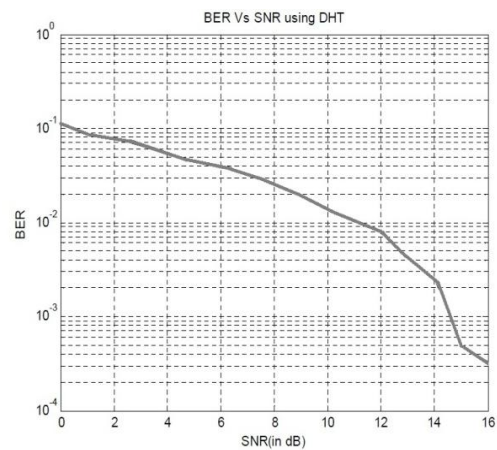


Fig 3. BER versus SNR curve for DHT based OFDM

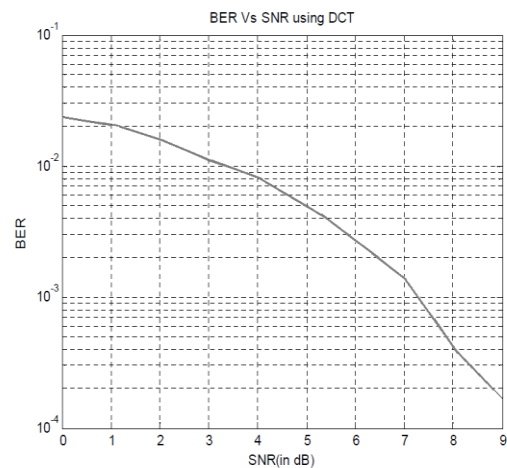


Fig 4. BER versus SNR curve for DCT based OFDM

The following table shows the results of three transform based OFDM. Figure 5 shows comparison of the BER versus SNR curves obtained in the FFT, DHT, and DCT based OFDM systems with BPSK as a modulation technique. The BER for these transform system is inversely proportional to SNR that is if SNR increases then BER reduces. The BER performance of FFT and DHT are almost same but DCT based OFDM curve is below the FFT and DHT curves.

Table 2. BER results for FFT, DHT, DCT based OFDM

SNR	BER USING FFT	BER USING DHT	BER USING DCT
0	0.09042	0.0173	0.0237
1	0.07676	0.0882	0.0210
2	0.06800	0.0775	0.0162
3	0.05730	0.0645	0.0108
4	0.05043	0.0524	0.0085
5	0.04420	0.0448	0.0050
6	0.03685	0.0387	0.0028
7	0.03090	0.0341	0.0014
8	0.02375	0.0246	4.162×10^{-1}

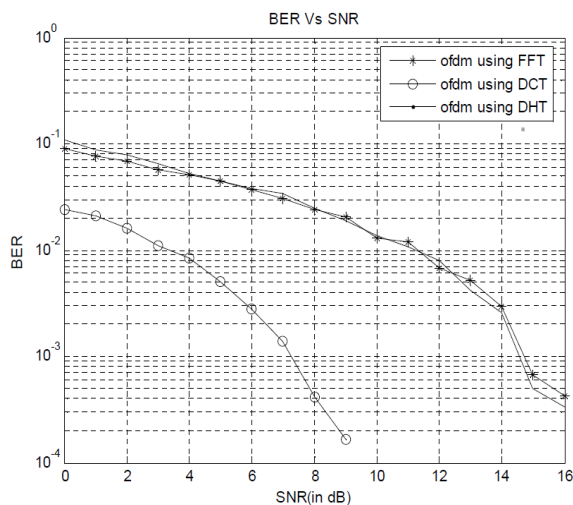


Fig 5. Comparison of BER of FFT, DCT,DHT based OFDM system

5. CONCLUSION

OFDM is a widely used and important modulation technique. It is computationally efficient because of FFT techniques. Using MATLAB software, the performance of OFDM system is taken for three different transform techniques namely FFT, DHT, and DCT. By simulation results, it observed that BER is improved in noisy channel by using BPSK at maximum data transmission capacity. DCT implies different boundary conditions than the DFT or other related transforms because of real valued nature and strong energy compaction property. DHT based OFDM, due to its real valued it require less computation complexity and implementation cost. From the results, it conclude that DHT,DCT transform technique may be alternative of DFT for implementation of OFDM-based communication systems.

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