

# **A Novel Approach to Improve Invisibleness and Robustness of a Digital Watermark in Copyright Protection**

**D. Vaishnavi**

Research Scholar (Ph.D.),  
Dept. of Computer Science and Engineering,  
Annamalai University, T.N, India.

**T.S.Subashini, Ph.D**

Associate Professor,  
Dept. of Computer Science and Engineering,  
Annamalai University, T.N, India.

## **ABSTRACT**

Rapid penetration of internet and advancements in communication technology is a paved way to easy access of digital images. Nevertheless, these advancements also create ways for malicious users to pirate and sell the copyrighted content. Digital watermarking techniques have been deployed in combating the piracy issue. The current digital watermarking methods are facing the problems in maintaining invisibleness, robustness, capacity and security. In this article, authors are analyzing the various popular algorithms used in digital watermark of copyright protection and propose a novel approach by combining three such algorithms aiming to increase the invisibleness and robustness of the watermarked image. The experimental results, exemplifies much increase in invisibleness and a nominal increase in robustness.

## **General Terms**

LWT, SVD, DCT, DWT, Watermarking

## **1. INTRODUCTION**

Emergence of the World Wide Web witnesses remarkable growth in digital communication techniques and allows large volumes of multimedia signal distribution. The digital contents such as images, audio and video files are prone to digital piracy as they are easy to copy and distribute. Such dissemination of digital content compels the copyright protection as cases of image stealing and piracy of copyright images are ever increasing. It plays a vital role in authenticating any digital image [1]. Image authentication can be classified into two types such as active watermarking and passive forensics. Active watermarking method which is also called as digital watermarking is majorly applied for copyright protection and owner identification, tamper localization whereas passive approach used in forensic analysis of the digital image which includes variety of techniques for detecting traces of digital tampering in any given image in absence of watermark. Digital watermarking embeds some sought of logo or any kind of information as a digital image into the original image. Even when the human vision system (HVS) is not perfect, Digital watermarking is effective. Digital watermark utilizes the limitation of HVS to make it invisible, thus avoiding degrading original digital products, as well being hard to get identified or destroyed.

The major factor in digital watermarking technique is to find a balance between the aspects such as robustness to various attacks, security and invisibleness. The invisibleness of watermarking technique is based on the intensity of embedding watermark. Better invisibleness could be achieved for less intensity watermark. So we must select the optimum

intensity to embed a watermark. In general, there is a little trade-off between the embedding strength (the watermark robustness) and quality (the watermark invisibleness). Increased robustness requires a stronger embedding, which in turn increases the visual degradation of the images [2]. A watermark to be effective, it should satisfy the following features:

- **Invisibleness:** The watermarked image must be look indistinguishable from the original one even on the highest quality equipment is used.
- **Robustness:** The watermark should be resilient when the falsification is introduced during either unintentional attack or an intentional attempt to disable or remove the watermark present.
- **Unambiguous:** Retrieval of the watermark should unambiguously identify the owner. Furthermore, the accuracy of owner identification should degrade gracefully in the face of attack.

In case of frequency based watermarking schemes, there has to be a trade-off between robustness and invisibleness. When a watermark is embedded in the most significant components, it becomes robust to attacks but the watermark becomes difficult to hide. Whereas, when embed a watermark in the lesser significant components, it is easier to hide it but the system is least resistant to attacks. Recently Wavelet based transforms gained reputation because of the property of multiresolution analysis that it provides. Wavelets can be orthogonal or bi-orthogonal. Most of the wavelets used in watermarking are orthogonal wavelets. [3].

In this paper, the authors propose a combined watermarking approach to improve the invisibleness and increase the robustness against various kinds of attacks. Lifting Wavelet Transform (LWT), Discrete Cosine Transform (DCT) and Singular Value Decomposition (SVD) were employed for watermark embedding and extraction. The rest of this article is organized as follows: Section 2 illustrates the related research work, section 3 gives the theoretical concepts about the proposed techniques, section 4 depicts the proposed method, section 5 expresses the quality metrics, section 6 discusses the experimental results and evaluation and finally section 7 concludes the paper.

## **2. RELATED WORK**

According to research findings, it's evident that, if a watermarked image undergoes any manipulation, then it becomes difficult and at times it's impossible to detect the watermark in its original form. Various research works are

attempting to provide a highly robust, and invisible digital watermarks to the digital images.

Researchers applied the theory of integral transform invariants and explained that, this technique can be used to embed watermarks that are resistant to rotation, scaling, and translation (RST)[4]. In their approach, the Discrete Fourier transform (DFT) of an image is computed and then the Fourier-Mellin transform is performed on the magnitude. The watermark is embedded in the magnitude of the resulting transform. The watermarked image is then constructed by performing the inverse DFT and an inverse Fourier-Mellin transform after considering the original phase [4][5]. Fourier Mellin transform is a log-polar mapping (LPM) followed by a Fourier transform, while an inverse Fourier-Mellin transform is an inverse log-polar mapping (ILPM) followed by an inverse Fourier transform. In this scheme, the embedded watermark is extracted by transforming the watermarked image into RST invariant domain. This approach was effective in theory, but difficult to implement [4].

Cox et.al presented a global DCT domain watermarking approach, in which the image is transformed by DCT and then watermark information is embedded into the first L largest coefficients in DCT domain generally which are the low-frequency coefficients[6]. Watermarking information with Gaussian random sequence is embedded into those L largest coefficients. Cox et.al and Quan et.al embedded the singular values of the watermark to the singular values of the mid band DCT coefficients[6][7]. Combination of Discrete Wavelet Transform (DWT) and DCT techniques is proposed by Jianshenget.al. In this method, DCT is used to transform the watermark image and the host image by DWT[8]. Transformed watermark is embedded into the host image of high frequency band in order to improve the robustness of the image under the image processing attacks and invisibility.

A semi-blind reference watermarking scheme based on DWT-SVD for copyright protection and authenticity is presented by Bhatnagaret.al and Al-Haj et.al generated an imperceptible and robust digital image watermarking algorithm based on DWT and SVD[9][10]. Senthil et.al proposed a work, which is based on wavelet transformation methods that embeds and extracts the watermark in digital images with the considerations of perceptual transparency and robustness against geometric attacks[11]. Arya et.al argues that generating an infinite number of discrete bi-orthogonal wavelets starting from an initial one, singular values (SV) allow us to make changes in an image[12].

### 3. TECHNICAL BACKGROUND

#### 3.1 Singular Value Decomposition

Singular value decomposition is one of the numerical analysis tools used in image coding and other signal processing applications [13]. The main features of SVD under the perspective of image processing are as follows:

- SVD is able to efficiently represent the intrinsic algebraic properties of an image, where singular vectors reflect geometry properties of an image.
- Singular values (SVs) of an image have very good stability, i.e., Even if a small perturbation is added to an image, its SV's do not change significantly.

- The quality of the reconstructed image will not degrade a lot, even on ignoring the small SV's in the reconstruction of an image.

Let  $X$  is a real matrix in the SVD transform and can be decomposed into a product of three matrices:  $X = U * S * V'$ . Where  $U$  and  $V$  are called left and right singular vectors and are orthogonal matrices such that  $U'U = I$  and  $V'V = I$ , and  $S$  is a diagonal matrix. The diagonal elements of  $S$  are called as singular values of  $X$ .

#### 3.2 Lifting Wavelet Transform (LWT)

LWT with standard 4-tap orthonormal filter with two vanishing moments is used for digital image watermarking. LWT is an alternative approach for DWT to transform image into frequency domain for real time applications [14][3]. Lifting wavelet is the second generation fast wavelet transform. In this, translation and dilation are not fundamental to obtain lifting wavelets. In this wavelet transformation up and down sampling is replaced simply by split and merge in each of the level [15]. The poly phase components of the signal are filtered in parallel by the corresponding wavelet filter coefficients, producing better result than up and downsampling which is required in traditional DWT approach [16]. Its use has grown due to low memory consumption, easy implementation and reduced the computation time by half. The following LWT scheme below is adapted from [15].

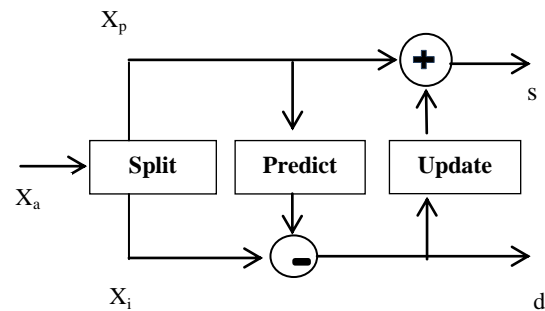


Fig. 1 Lifting Wavelet Transform

In the fig. 1, there are three basic operations: split, predict and update. In split stage, the input  $x_a$  is separated into odd ( $x_i$ ) and even ( $x_p$ ) samples. So that each of these variables contains half the number of samples of  $x_a$ . In the prediction stage, even samples are used to predict the odd samples. The details coefficients or high frequency ( $h$ ) are calculated as prediction errors of the odd samples through the use of the prediction operator  $P$ :

$$d = x_i - P(x_p) \quad (1)$$

To create the low frequency samples  $s$ , the even samples are updated through the update operator  $U$ :

$$s = x_p + U(d) \quad (2)$$

In lifting scheme the signal is divided like a zipper. Then a series of convolution-accumulate operations across the divided signals is applied. In comparison with general wavelets, reconstruction of image by lifting wavelet is impeccable because, it increases smoothness and reduces aliasing effects. It reduces loss in information, increases intactness of embedded watermark in the image and helps to increase the robustness of watermark.

### 3.3 Discrete Cosine Transform

Discrete Cosine Transform (DCT) is a widely accepted method which divides the digital image into independent pieces. These pieces are divided based on its relative important to the quality of the image. The image can also be divided as spectral sub bands. Like as Discrete Fourier Transform (DFT), DCT also converts a signal or image to frequency domain from spatial domain. Since its good capacity of energy compression and de-correlation, it is widely used. And it is faster than DFT because its transform kernel is real cosine function whereas it is complex exponential function in DFT [17]. DCT based watermarking is built on the basic facts that: The signal energy lies at low-frequencies sub-band which contains the most important visual parts of the image and the high frequency components of the image are usually removed through compression and noise attacks.

## 4. PROPOSED METHOD

The proposed method is done in two steps: Embedding, Recovering; which is described as follows:

### 4.1 Watermark Embedding

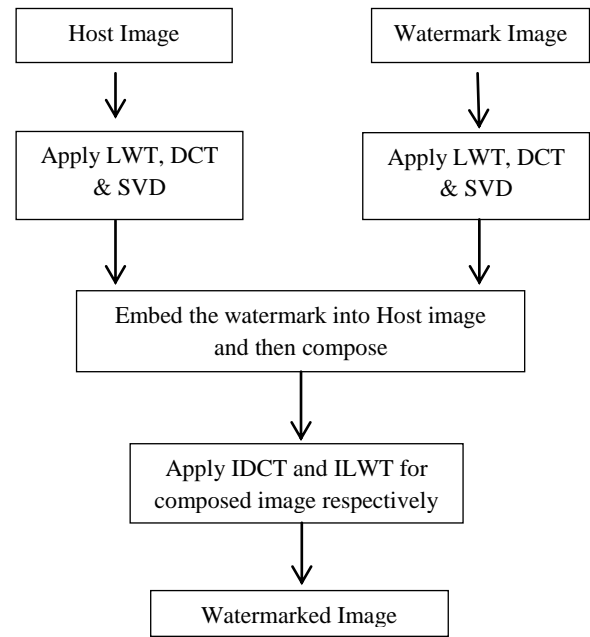
The following steps and fig 2 explains insertion of watermark into the host image.

1. Apply LWT to decompose the host image into sub-bands of approximate coefficients LA, horizontal coefficients LH, vertical coefficients LV and diagonal coefficients LD.
2. Select LD sub band to apply DCT and then apply SVD for all discretized coefficients to obtain U, S and V.
3. Apply LWT to decompose the watermark W into sub-bands of approximate coefficients WLA, horizontal coefficients WLH, vertical coefficients WLV and diagonal coefficients WLD.
4. Select WLD sub band to apply DCT then SVD to get U1, S1 and V1.
5. Convert S as  $S2 = S + k * S1.k$  – scale/gain factor.
6. Obtain B\* using  $B* = U * S2 * V^T$
7. Apply IDCT for B\* to get LD sub band
8. Apply ILWT to LD of modified band with three unmodified bands of LA, LH, LV to get watermarked image.

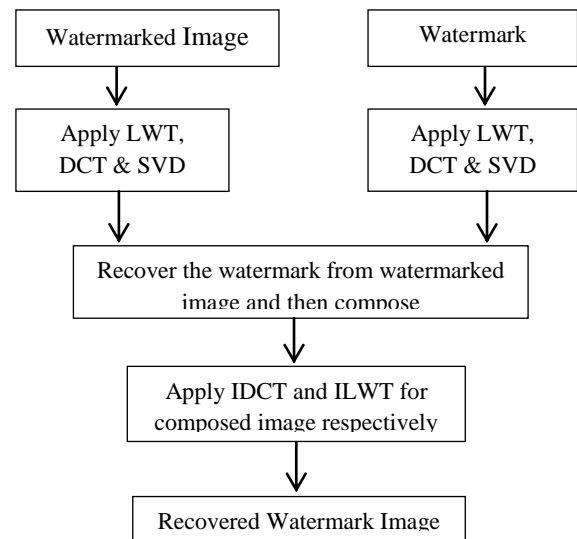
### 4.2 Watermark Recovery

The following steps and fig 3 explains extraction of watermark from Watermarked image.

1. Apply LWT to decompose the watermark image W into four sub-bands WLA1, WLH1, WLV1 and WLD1.
2. Select WLD1 band and apply DCT
3. For all discretized frequency coefficients apply SVD to get U1\*, S1\*, V1\*
4. Convert S as  $S2* = S - S1*/k$ .
5. Obtain B\* with U1, S2\*, V1<sup>T</sup>.
6. Apply inverse DCT for B\* to get LD sub band.
7. Apply ILWT to the LD of modified band with three unmodified bands of watermark image to get a recovered watermark.



**Fig. 2 Watermark Embedding**



**Fig. 3 Watermark Recovery**

## 5. QUALITY METRICS

### 5.1 Peak Signal to Noise Ratio

Peak signal to noise ratio (PSNR) which measures the image quality by computing the Mean-Squared Error (MSE) through the equation given below:

$$MSE = \frac{\sum_{M,N} [I_1(m,n) - I_2(m,n)]^2}{M * N}$$

In the above equation, M and N are the rows and columns of the input images.  $I_1(m,n)$ ,  $I_2(m,n)$  are the pixel values of host image and watermarked image. Then it calculates the PSNR using following equation:

$$PSNR = 10 \log_{10} \left( \frac{R^2}{MSE} \right)$$

Where, R is the maximum fluctuation of an input image. In Case, if the input image has a double-precision floating-point data type, R is taken as 1. If it has 8-bit unsigned integer data type, R is taken as 255.

### 5.2 Structural Similarity

Structural similarity (SSIM) considers that image degradation as an anticipated change in structural information. The SSIM is computed on various windows of an image. The measure between two windows and of common size NxN is:

$$SSIM(x, y) = \frac{(2\mu_x \mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)}$$

where,

- $\mu_x, \mu_y$  the average of  $x, y$  respectively;
- $\sigma_x^2, \sigma_y^2$  the variance of  $x, y$  respectively;
- $\sigma_{x,y}$  the covariance of  $x$  and  $y$ ;
- $c_1 = (k_1 L)^2$ ,  $c_2 = (k_2 L)^2$  variables to stabilize the division with weak denominator;
- L the dynamic range of the pixel-values (typically which is  $2^{\text{#bits per pixel}} - 1$ );
- $k_1 = 0.01$  and  $k_2 = 0.03$  by default.

The SSIM yields a decimal value between -1 and 1. The value 1 is represents two sets of data are identical. Normally it is calculated on window size of  $8 \times 8$ .

## 6. RESULTS AND DISCUSSION

The experimental setup was done using Mathworks Matlab version 9. In the experiment, single level LWT with Haar wavelet is applied to the input images. The watermark is embedded into the wavelet of diagonal sub band (LD). The input of host and watermark images is converted to gray scale and whose resolution is  $512 \times 512$ . Fig. 4 shows the original image is embedded with watermark and watermark is recovered from watermarked image.

To experimentally ascertain the robustness and invisibleness, proposed approach was evaluated with watermarked image against five kinds of attacks: 1) geometrical attack: Rotation (RO), Scaling (SC), Translation (TR); 2) noising attack: Gaussian noise (GN); 3) de-noising attack: adaptive wiener filtering (WF); 4) format-compression attack: JPEG compression (JPEG); 5) image-processing attack: Motion Blur

(MB). Fig. 5 exhibitions that proposed system is resilient to all the attacks.

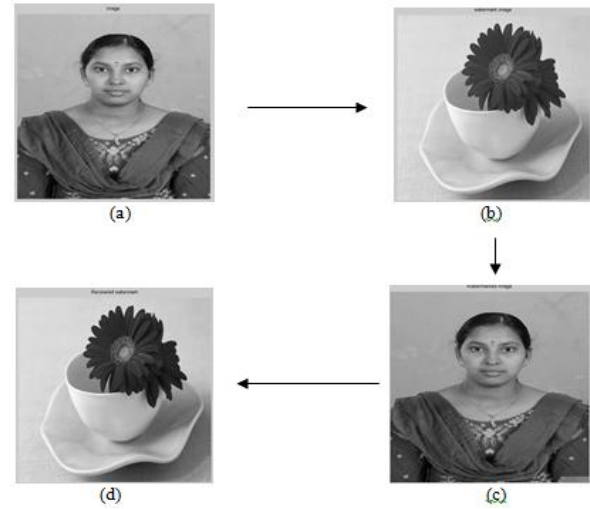


Fig. 4 (a) Host image, (b) Watermark image, (c) Watermarked image, (d) Recovered Watermark image

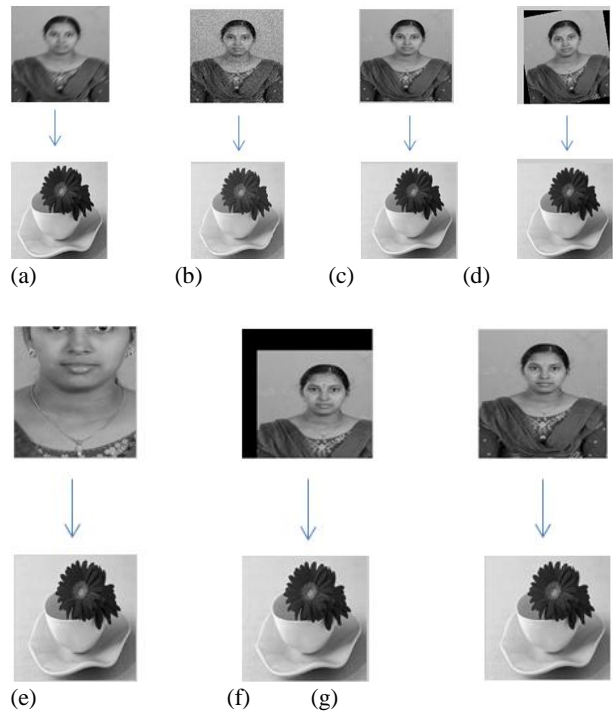


Fig. 5 (a), (b), (c), (d), (e), (f), (g) Watermarked Image and corresponding Recovered Watermark images from the watermarked images with Motion Blur, Gaussian Noise, Wiener Filter, Rotation, Translation, Scaling, JPEG compression attacks

In order to justify the proposed method, authors, also implemented the DWT-SVD based watermarking method [18] and LWT-SVD based approach [12] to do the comparative study. The watermark is embedded into the host image using different gain factors (k). In embedding process, the singular value component are multiplied by a gain factor and then

**Table 1. Comparison of invisibleness (PSNR) of proposed method with DWT-SVD [18], LWT-SVD [12]for different gain factor (k)**

Method	DWT-SVD	LWT-SVD	Proposed Method
Gain factor			
0.1	52.6390	55.7847	74.2897
0.5	48.4516	42.1395	60.6179
1.0	35.4807	36.5194	54.7257
1.5	31.0143	33.3804	51.2872

embedded with the host image coefficients. Therefore, changing the value of gain factor has an obvious effect on both the watermarked image and the recovered watermark.

**Table 2 Parameter values taken for different attacks**

Attacks	Values
Motion Blur	20,40
Gaussian noise	Default
wiener filter	Mask [3, 3]
Rotation and cropped	10°
Translation and cropped	x=60, y=90 shift
Scaling and cropped	2.5 enlarge
JPEG compression	75%

The results in the proposed technique were tested for the k values of 0.1, 0.5, 1.0, and 1.5.

In order to quantitatively evaluate the invisibleness and robustness of algorithm, two guide lines were employed: one is peak signal- noise ratio(PSNR), which is used to measure the invisibleness of the watermark. The other one is SSIM, which is used to measure the structural similarity between the original and recovered watermark images. The experiment was carried out with different gain factor k and the PSNR was calculated. Table 1 offers a comparative analysis of invisibleness. It shows that the proposed approach has better PSNR as compared with other existing methods. It also presentations that smaller gain factor better the image quality. Similarly the experiment was carried out to evaluate the robustness of the proposed system against different attacks. Table 3 gives the SSIM calculated for the seven different attacks for different values of gain factor k. Table 2 depicts the parameter values chosen for the different types of attacks. Table 3 illustrate that, in starting for the k value of 0.1, robustness is low except the wiener filter, scaling and JPEG compression attacks. But, the robustness is gradually increasing as the gain factor increasing from 0.1. The Table 3 also depicts that the gain factor k of 1.5 is high robustness against all attacks.

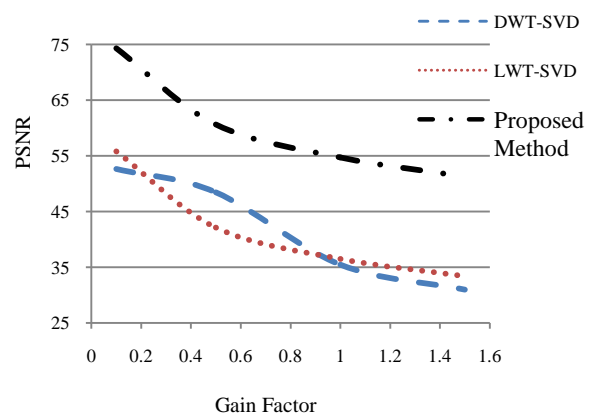
**Table 3. Evaluation of robustness (SSIM values) of proposed approach under various attacks with different gain factor (k)**

Gain factor	0.1	0.5	1.0	1.5
Attacks				
Motion Blur	0.8331	0.9906	0.9989	0.9998
Gaussian noise	0.7547	0.7876	0.8265	0.8903
wiener filter	0.9692	0.9989	0.9994	0.9991
Rotation	0.8331	0.9789	0.9995	0.9997
Translation	0.7447	0.9785	0.9989	0.9989
Scaling	0.9692	0.9896	0.9904	0.9987
JPEG compression	0.9554	0.9899	0.9983	0.9987

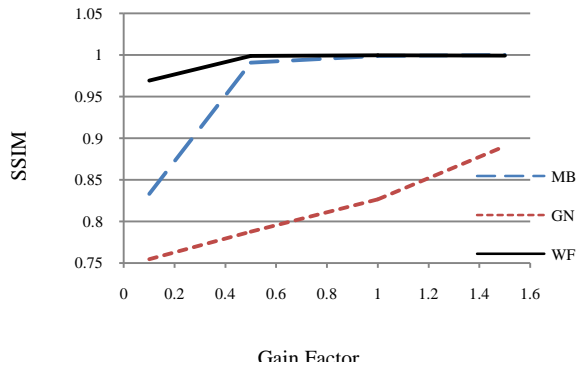
In Table 4, while comparing the DWT-SVD [18] , LWT-SVD [12] the proposed approach shows that the robustness of recovered watermark is better for all attacks except Gaussian Noise, but which is much increased for motion blur, wiener filter, rotation and JPEG compression attacks.

**Table 4. Comparative of robustness (SSIM ) recovered watermark with the methods of DWT-SVD[18], LWT-SVD [12]And our proposed with Gain factor k=1.5**

Method	DWT-SVD	LWT-SVD	Proposed Method
Attacks			
Motion Blur	0.9398	0.9424	0.9998
Gaussian noise	0.9853	0.9850	0.7903
Wiener filter	0.9899	0.9979	0.9991
Rotation	0.9847	0.9884	0.9997
Translation	0.9532	0.9524	0.9989
Scaling	0.9562	0.9524	0.9987
JPEG compression	0.9879	0.9893	0.9997

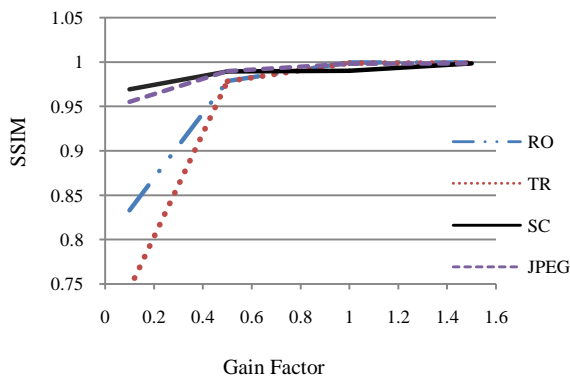


**Fig. 6 Gain factor Vs. PSNR (Invisibleness)**



**Fig. 7(a) Gain factor vs. SSIM (Robustness) for Motion Blur (MB), Gaussian Noise (GN), Wiener Filter (WF) attacks**

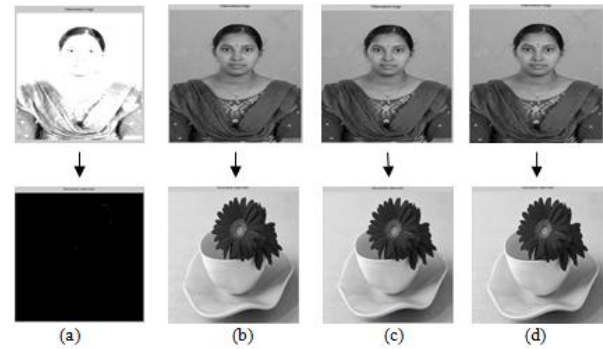
The graph shown in fig. 6 gives the comparison of the proposed system with the existing systems. PSNR of the proposed system is high compared to existing systems for all gain factor values ranging from 0.1 to 1.5. The robustness of the proposed system under various attacks namely Motion Blur, Gaussian noise, Wiener filter is depicted in fig. 7(a) and geometric attacks and the JPEG compression in 7 (b). The fig. 7 shows that the proposed system is highly resilient for gain factor of 1.5. Further from fig. 6 & 7, it also shows that the invisibility of the system decreases with increases in gain factor, whereas the robustness increases with increase in gain factor.



**Fig 7(b) Gain factor vs. SSIM (Robustness) for Rotation Translation, Scaling and JPEG compression attacks**

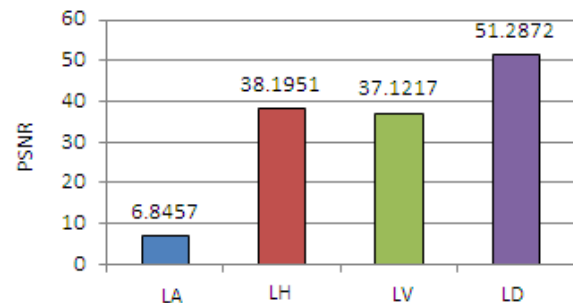
**Table 5 Effect of robustness(SSIM) for different sub bands with k=1.5**

Sub Band Attacks	LA	LH	LV	LD
Motion Blur	-3.3684	0.9994	0.9975	0.9998
Gaussian noise	-3.3662	0.8006	0.7938	0.7903
wiener filter	-3.3685	0.9760	0.9686	0.9991
Rotation	0.0025	0.9835	0.9790	0.9997
Translation	-0.5112	0.9785	0.9712	0.9989
Scaling	-0.7058	0.9949	0.9931	0.9987
JPEG compression	-0.5612	0.9711	0.9630	0.9997



**Fig 8 (a), (b), (c), (d) Watermarked and Recovered Watermark images of LA, LH, LV, and LD sub band respectively**

Table 3 presents the robustness of the proposed system is high with the gain factor of 1.5. Further the table 5, it also shows that the robustness is good when the watermark is embedded on the Diagonal coefficients sub band (LD). Fig 8 and 9 shows that watermark is embedded on LA, LH, LV and LD sub bands. From the fig. 8, it visually exposes that the invisibility of watermark is good when embedding process involved in diagonal sub band (LD) and it is very poor when embedding on Approximate coefficients sub band (LA) and also from the fig. 9, the PSNR calculated for the different sub bandsexpresses that the LD band gives a very good PSNR value, whereas the PSNR value of 6.845 for the LA band depicts that the watermark image cannot be embedded into LA band.



**Fig. 9 PSNR of LA, LH, LV, and LD sub bands**

## 7. CONCLUSION

A novel approach to watermarking for copyright protection using three popular algorithms, such as Lifting Wavelet Transform, Discrete Cosine Transform and Singular Value Decomposition to improve invisibility and robustness was proposed. The system was implemented and the various quality metrics namely PSNR and SSIM were calculated. PSNR is measured to evaluate invisibility and SSIM is measured to evaluate robustness. The system was tested for robustness under different attacks and different gain factors. The study shows that the invisibility of the system decreases with increases in gain factor, whereas the robustness increases with increase in gain factor. The robustness of the proposed system is high with the gain factor of 1.5. It also depicts that the robustness is good when the watermark is embedded on the Diagonal sub band (LD). A comparison was made with other schemes namely LWT-SVD [12] and DWT-SVD[18].



The study depicts that proposed system produces high PSNR and SSIM effectively for a watermarking technique for copyright protection.

## 8. ACKNOWLEDGMENT

Authors express their sincere gratitude to the researchers those who developed the various algorithms used in this article.

## 9. REFERENCES

- [1] C. Fei, D. Kundur, and R. H. Kwong, "Analysis and design of watermarking algorithms for improved resistance to compression," *Trans. Img. Proc.*, vol. 13, pp. 126–144, Feb. 2004.
- [2] M. M. Sathik and S. Sujatha, "An Improved Invisible Watermarking Technique for Image Authentication," *International Journal of Advanced Science and Technology (IJAST)*, vol. 24, 2010.
- [3] W. Sweldens, "The Lifting Scheme: A New Philosophy in Biorthogonal Wavelet Constructions," in *Wavelet Applications in Signal and Image Processing III*, 1995, pp. 68–79.
- [4] J. J. O'Ruanidh and T. Pun, "Rotation, scale and translation invariant digital image watermarking," in *Image Processing, 1997. Proceedings., International Conference on*, 1997, vol. 1, pp. 536–539.
- [5] M. Alghoniemy and A. H. Tewfik, "Progressive quantized projection watermarking scheme," in *Proceedings of the seventh ACM international conference on Multimedia (Part 1)*, 1999, pp. 295–298.
- [6] I. J. Cox, J. Kilian, F. T. Leighton, and T. Shamoon, "Secure spread spectrum watermarking for multimedia," *IEEE TRANSACTIONS ON IMAGE PROCESSING*, vol. 6, pp. 1673–1687, 1997.
- [7] L. Quan and A. Qingsong, "A combination of DCT-based and SVD-based watermarking scheme," in *Signal Processing, 2004. Proceedings. ICSP '04 7th International Conference on*, 2004, vol. 1, pp. 873–876 vol.1.
- [8] M. Jiansheng, L. Sukang, and T. Xiaomei, "A digital watermarking algorithm based on DCT and DWT," in *Proceedings of the 2009 International Symposium on Web Information Systems and Applications*, 2009, pp. 104–107.
- [9] G. Bhatnagar and B. Raman, "A new robust reference watermarking scheme based on DWT-SVD," *Computer Standards & Interfaces*, vol. 31, pp. 1002–1013, 2009.
- [10] A. M. Al-Haj and T. Manasrah, "Non-invertible copyright protection of digital images using DWT and SVD," in *ICDIM, 2007*, pp. 448–453.
- [11] V. Senthil and R. Bhaskaran, "Wavelet Based Digital Image Watermarking with Robustness against Geometric Attacks," in *Conference on Computational Intelligence and Multimedia Applications, 2007. International Conference on*, 2007, vol. 4, pp. 89–93.
- [12] M. S. Arya, R. Siddavatam, and S. Ghrera, "A hybrid semi-blind digital image watermarking technique using lifting wavelet transform and Singular value decomposition," in *Electro/Information Technology (EIT), 2011 IEEE International Conference on*, 2011, pp. 1–6.
- [13] K. He, J. Gao, L. Hu, and H. Gao, "Watermarking for images using the HVS and SVD in the wavelet domain," in *Mechatronics and Automation, Proceedings of the 2006 IEEE International Conference on*, 2006, pp. 2352–2356.
- [14] A. Calderbank, I. Daubechies, W. Sweldens, and B. Yeo, "Wavelet Transforms That Map Integers to Integers," *Applied and Computational Harmonic Analysis*, vol. 5, pp. 332–369, 1998.
- [15] I. Daubechies and W. Sweldens, "Factoring wavelet transforms into lifting steps," *Journal of Fourier analysis and applications*, vol. 4, pp. 247–269, 1998.
- [16] W.-H. Chang, Y.-S. Lee, W.-S. Peng, and C.-Y. Lee, "A line-based, memory efficient and programmable architecture for 2D DWT using lifting scheme," in *Circuits and Systems, 2001. ISCAS 2001. The 2001 IEEE International Symposium on*, 2001, vol. 4, pp. 330–333.
- [17] M. Kansal, G. Singh, and B. V. Kranthi, "DWT, DCT and SVD Based Digital Image Watermarking," in *Computing Sciences (ICCS), 2012 International Conference on*, 2012, pp. 77–81.
- [18] C.-C. Lai and C.-C. Tsai, "Digital Image Watermarking Using Discrete Wavelet Transform and Singular Value Decomposition," *IEEE T. Instrumentation and Measurement*, vol. 59, pp. 3060–3063, 2010.