Performance Evaluation of Non-linear Equalizer using Different Modulation Techniques

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ABSTRACT
An equalization technique based on nonlinear Hammerstein type filters to combat the inter symbol interference (ISI) effect is proposed. This technique is nothing but nonlinear generalization of the linear equalizer. Linear frequency selective fading channels in presence of additive white Gaussian noise is considered using DPSK and QAM modulation techniques in this work. Simulation results shows that the proposed technique is found superior compared to when linear equalizer is used. Better BER performance at moderate and higher SNRs is achieved for M-QAM modulation. Results also show better MSE performance than the linear structure.

Keywords
Channel Equalization, Frequency Selective Fading Channels, Hammerstein Filter, QAM, DPSK.

1. INTRODUCTION
Generally in frequency selective channels, the input signal after being transmitted suffer from inter-symbol interference (ISI) and noise. In order to reduce these effects, an optimum receiver based on maximum likelihood sequence estimation (MLSE) is designed. MLSE which is a non-linear method has high computational complexity that increases exponentially with the channel memory length. Therefore in frequency selective channels, MLSE is replaced by suboptimum receivers. Linear and decision feedback equalizers (DFE) are the most common techniques [1]. Linear equalizer (LE) is simply a linear transversal filter with a limited number of taps. Linear transversal filters are used in DFE as feed-forward and feedback blocks. Many other equalization techniques are presented in practice to reduce ISI effect[1-3].

In this paper, discussion is on generalized nonlinear structure for channel equalization that is based on Hammerstein type filters. Normally this technique is performed on frequency selective fading channels. Hammerstein filter is a nonlinear polynomial filter which is used in many applications. Some of them are system identification [4],[5], modeling[6],[7], echo cancellation[8],[9] and noise cancellation[10]. Hammerstein decision feedback equalization (HDFE) is used in fiber-wireless channel for compensation of nonlinear distortion in the electrical-to optical converter [11], [12].
In the next section a system model is presented. Section III introduces the nonlinear Hammerstein equalization technique.. Simulation results and discussions are presented in section IV, before concluding the paper in section V.

2. SYSTEM MODEL
In this section the equivalent low-pass discrete time model of the system is considered. DPSK and M-QAM modulation techniques are employed in this system.. A frequency selective fading channel modeled by a tapped delay line with *L* taps is considered to be:

$$H = [h_1 h_2 .... h_{L]}] \quad (1)$$

Where h_i is the random gain of the *i*th tap. These components are assumed to be real valued zero-mean Gaussian random variables with variance σ_{hi}^2 . Further, they are assumed uncorrelated and normalized to unity, i.e.:

$$\sum_{i=1}^{L} E\{|h_i|^2\} = 1 \quad (2)$$

The channel fading is assumed to be slow, such that the tap gains do not vary during one data frame. It is also assumed that the frequency selective fading channel has a specific power delay profile (PDP), which is the profile of the mean square values of the tap gains. The received signal which is corrupted by ISI and noise is expressed as

$$y(n) = \sum_{i=1}^{L} h_i x(n-i+1) w(n) \quad (3)$$

Where $w(n)$ is a real-valued zero-mean white Gaussian noise with variance σ_w^2 . Eq. (3) can be expressed in matrix form:

$$y(n) = HX(n) + W(n) \quad (4)$$

Where **H** is the channel vector and **X**(n) is the received data vector, defined as:

$$X(n) = [x(n)\, x(n-1) .... x(n-L+1)^T \quad (5)$$

In sub-optimum receivers, the detected signal is obtained by passing *y(n)* through an equalizer and a hard detector.

3. GENERALISED HAMMERESTEIN EQUALIZATION TECHNIQUE

3.1 Equalizer Model

The block diagram of Generalized Hammerstein Equalization technique (GHE) is shown in below Fig 1. Here the received signal is passed through a delay line with L_{eq} taps. Then, the signal at every tap is applied to a Hammerstein filter of order *D*. The output polynomial of the i^th filter is then

$$Z_i(n) = \sum_{k=1(k\,odd)}^{D} g_{ik}\tilde{y}_i^{\,k}(n)$$
$$for\ i = 1,2\ \ldots.\ \ldots.L_{eq} \qquad (6)$$

Where g_{ik} is the k^{th} coefficient of the output polynomial of the i^{th} filter, and $\tilde{y}_i(n)$ is defined as the signal at i^{th} tap, i.e.:

$$\tilde{y}_i(n) = y\left(n - i + \frac{L_{eq}+1}{2}\right) \qquad for\,i = 1,2,\ldots..L_{eq} \quad (7)$$

The summation of Eq. (6) contains only the odd powers. It can be shown that the terms containing the even powers are equal to zero. The filters outputs are summed to produce the equalizer output $z(n)$

$$z(n) = \sum_{i=1}^{L_{eq}}\sum_{k=1(odd)}^{d} g_{ik}\tilde{y}_i^{\,k}(n) \qquad (8)$$

Eq. (8) when expressed in matrix form:

$$z(n) = G_H^T Y_H(n) \qquad (9)$$

Where G_H is a $^{L}eq\,(D+1)/2\times 1$ vector that consists of coefficients g_{ik} and $Y_H(n)$ is a $^{L}eq\,(D+1)/2\times 1$ vector defined as:

$$Y_H(n) = [\tilde{y}_1^{\,T}(n)\tilde{y}_3^{\,T}(n)\tilde{y}_5^{\,T}(n)\ldots.\tilde{y}_D^{\,T}(n)]^T, D odd \qquad (10)$$

where $\tilde{Y}_P(n)$ is a L_{eq} vector defined by using Eq. (7):

$$\tilde{Y}_P = [\tilde{y}_1^{\,P}(n)\tilde{y}_2^{\,P}(n)\ldots.\tilde{y}_{L_{eq}}^{\,P}(n)]^T \qquad (11)$$

Z(n) is an estimate of the transmitted symbol x(n) To minimize the mean square error we have to find out the coefficients g_{ik} . The output decision $\hat{x}(n)$ is obtained by passing *Z(n)* through a hard detector

3.2 Calculation of the coefficients

The MSE criterion is used for calculating the coefficients of Hammerstein filters from the training mode. The training mode having the transmitter sends a training sequence, let us assume that it is known to receiver as the desired signal d(n).The difference between the desired and estimated values gives the error signal which is given by the eq.

$$e(n) = d(n) - z(n) = x(n) - z(n) \qquad (12)$$

The cost function is defined as below:

$$\zeta = E\{e^2(n) \qquad (13)$$

The coefficients are computed so as to minimize ζ .Using Eqs. (9) And (12) in (13), we get:

$$\zeta = E\{[x(n) - G_H^T Y_H(n)][x(n) - Y_H^T G_H]\} =$$
$$E[X^2(n)] - G_H^T E[Y_H(n)x(n)\} - \qquad (14)$$
$$E\{x(n)Y_H^T(n)\}G_H + G_H^T E[Y_H(n)Y_H^T(n)\}G_H$$

If we define the $^{L}eq\,(D+1)/2\times 1$ cross correlation vector:

$$P_H = E\{Y_H(n)x(n)\} \qquad (15)$$

And the $^{L}eq\,(D+1)/2\times{}^{L}eq\,(D+1)/2$ autocorrelation matrix:

$$R_H = E\{Y_H(n)Y_H^T(n)\} \qquad (16)$$

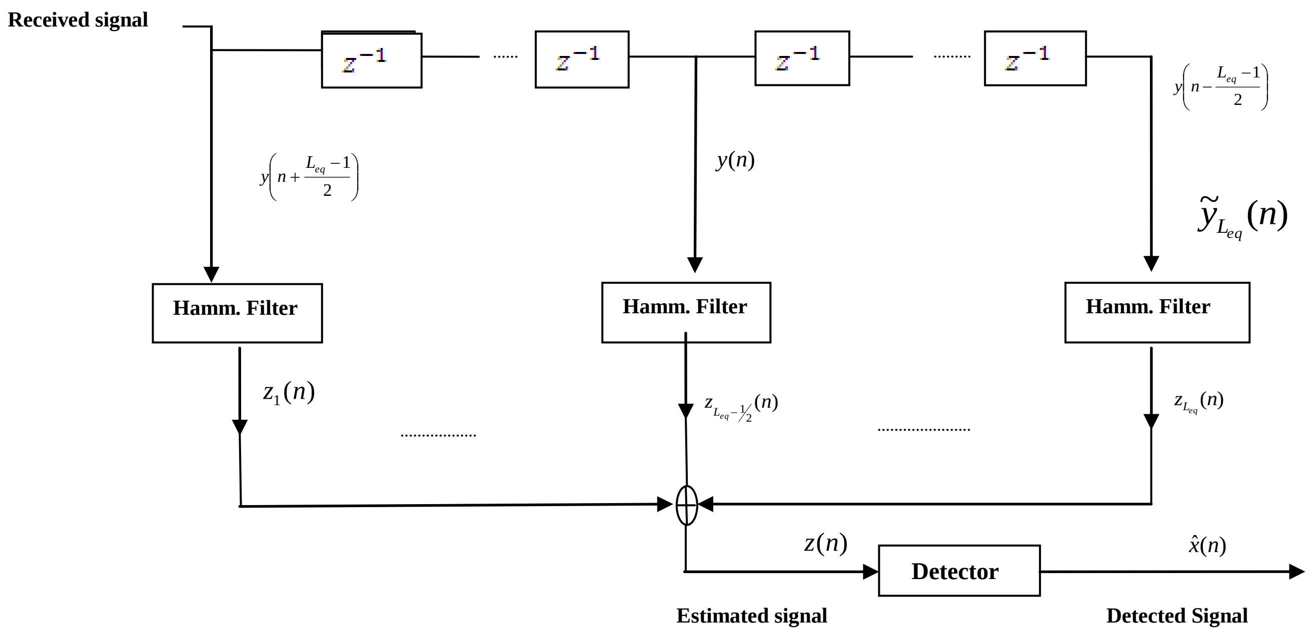

Fig. 1Generalized Hammerstein Equalizer

And note that $E\{x(n)Y_H^T(n)\} = P_H^T$

$G_H^T P_H = P_H^T G_H$,and $E\{x^2(n)\} = 1$, we obtain:

$$\zeta = 1 - 2G_H^T P_H + G_H^T R_H G_H \quad (17)$$

This is a quadratic function of vector G_H with a single global minimum. To minimize ζ , we need to have:

$$\nabla\zeta = 0 \quad (18)$$

Where ∇ is the gradient operator From Eqs. (17) and (18) and using the gradient properties we can write:

$$\nabla\zeta = 2R_H G_H - 2P_H = 0 \quad (19)$$

Finally, the coefficients of Hammerstein filters are obtained by solving Eq. (19):

$$G_H = R_H^{-1} P_H \quad (20)$$

Assuming that R_H^{-1} is invertible.

4. SIMULATION RESULTS

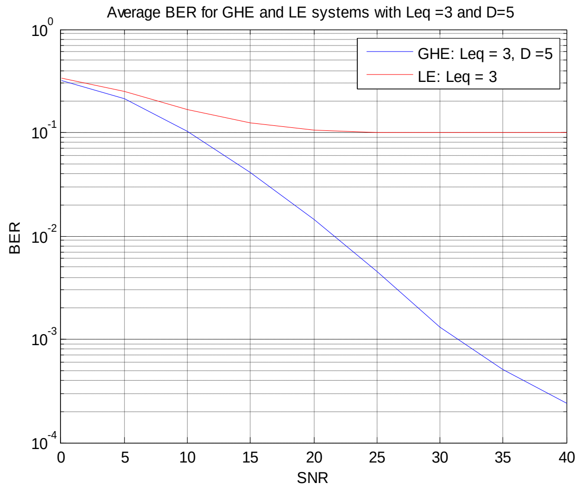

Fig. 2 Average BER for GHE & LE systems for Leq= 3 and D=5

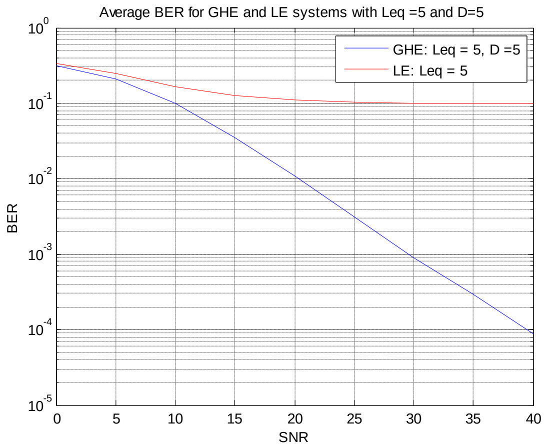

Fig.3Average BER for GHE & LE systems for L_{eq}=5, D = 5

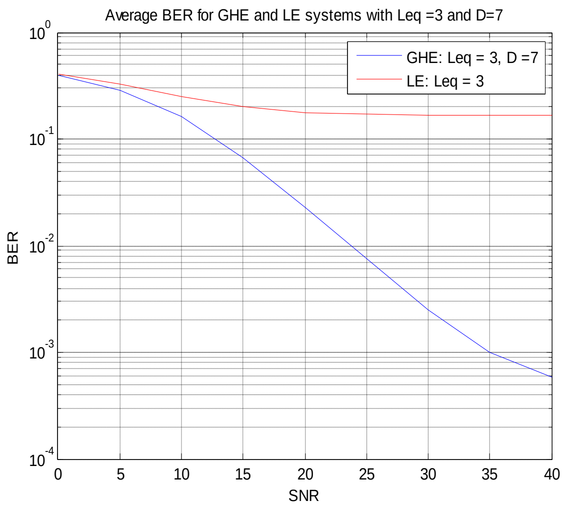

Fig. 4 Average BER for GHE & LE systems for Leq=3, D =7

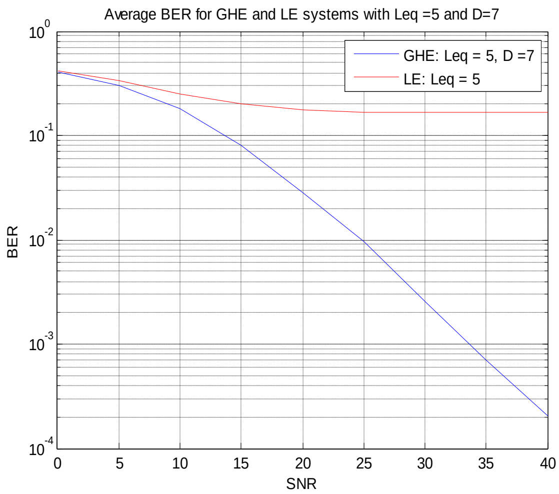

Fig 5 Average BER for GHE & LE systems for Leq=5 and D = 7

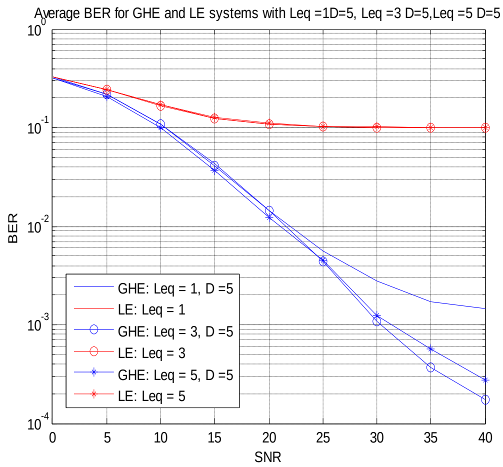

Fig. 6 Average BER for GHE & LE systems for L_{eq}={1,3,5} D = 5

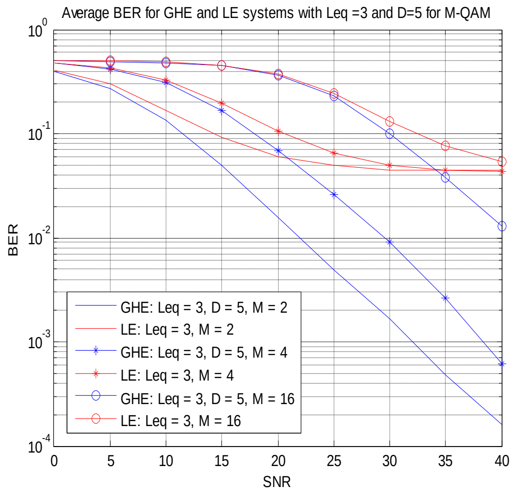

Fig 7 Average BER for GHE & LE systems for L_{eq}=3, D = 5 for M= (2, 4, 16)

4.1 Conclusions:

The average Bit Error Rate (BER) for different SNR values are plotted for both linear equalizer and Generalized Hammerstein Equalizer techniques. Fig. 2&3 shows that BER decreases as the number of taps of the equalizer increases from 3 to 5 with the order of the filter taken as 5 i.e. D = 5. In other words taking the order of the filter fixed the BER rate decreases considerably for moderate to high SNR values as the number of taps increases from 3 to 5 i.e. L_{eq}= {3,5}. Results were also obtained by increasing the order of the filter to 7 i.e. D = 7 for different number of taps L_{eq} =3,5 from fig. 4&5.

The results obtained reveal that considerable improvement in BER, but when compared with the previous figures the performance of GHE with the order of the filter D=5 out performs than the filter with the order D = 7 for moderate to high SNR values. The conclusion that is drawn from the above results is the proposed GHE to perform well the order of the filter has to be limited to a value less than or equal to 5 i.e. D = 5. Fig. 6 shows how the proposed GHE performs if the taps takes different values i.e. L_{eq}= {1, 3, 5} for the order of the filter D = 5. The conclusion that is drawn is as the number of taps increases for fixed order the BER decreases gradually.

The BER of GHE decreases drastically for moderate to high SNR values. The above results are obtained provided the modulation technique being used is DPSK. Performance of the proposed GHE and LE is also carried out for fixed order and fixed number of taps for different values of the M-QAM modulation scheme and found that as the value of M increases BER also increases. Excellent performance for the proposed GHE is obtained for M = 2 as shown in fig

4.2 Results: MSE

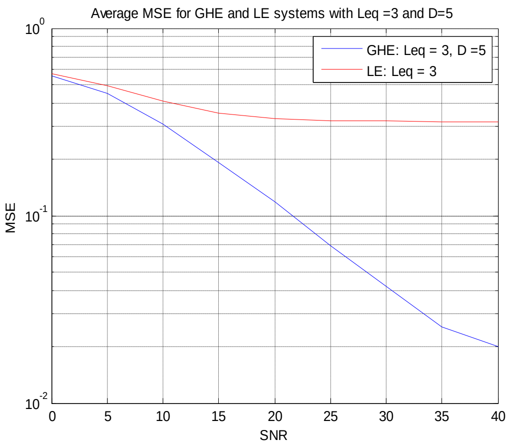

Fig. 8 Average MSE for GHE & LE systems for Leq =3 and D = 5

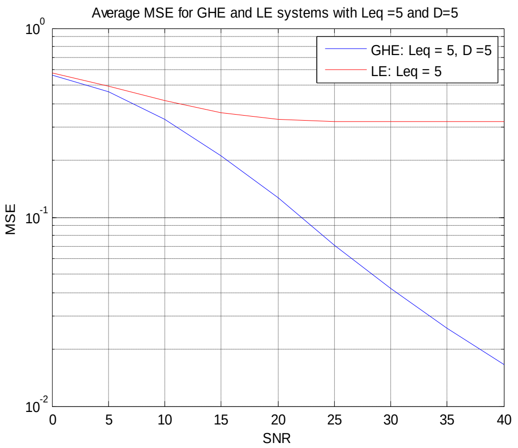

Fig. 9 Average MSE for GHE & LE systems for Leq =5 and D = 5

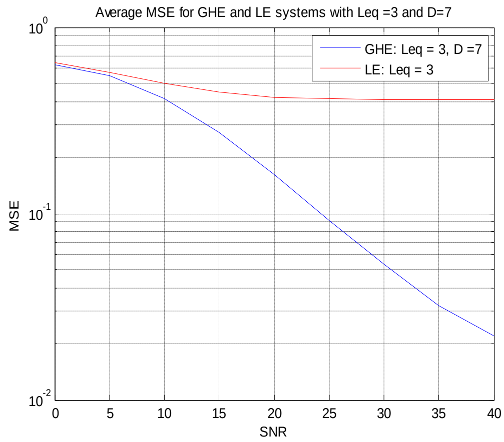

Fig. 10 Average MSE for GHE & LE systems for Leq =3 and D =7

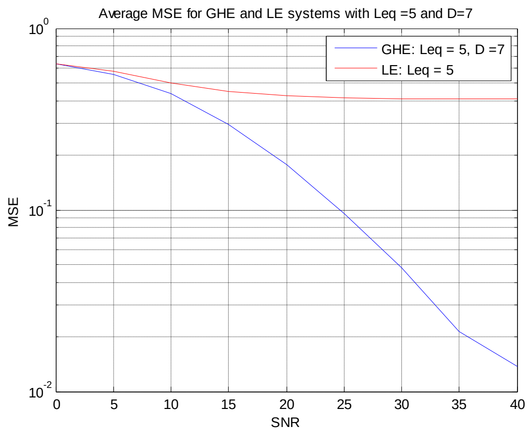

Fig. 11 Average MSE for GHE & LE systems for Leq =5 and D =7

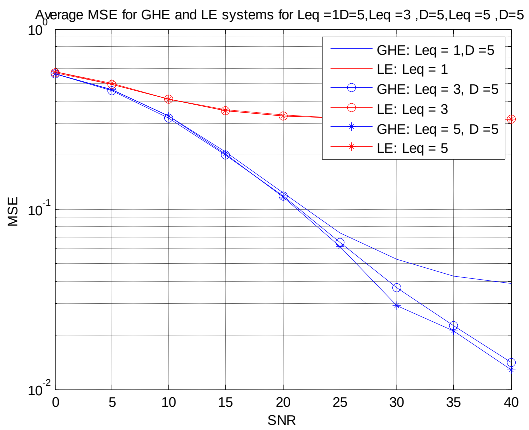

Fig. 12 Average MSE for GHE & LE systems for Leq =1,3,5& D = 5

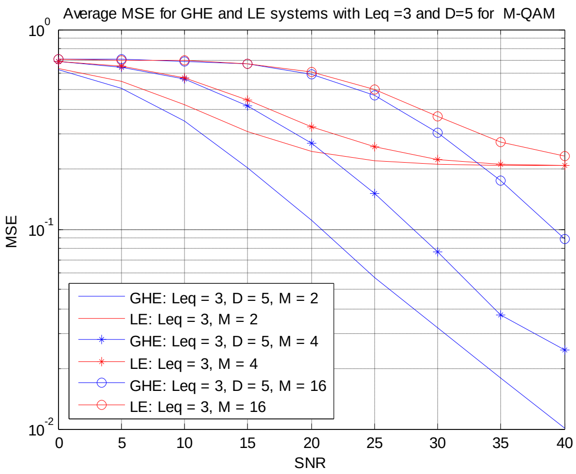

Fig. 13 Average MSE for GHE & LE systems for Leq=3, D = 5 & M= (2, 4, 16)

4.3 MSE Conclusions:

The performance of the proposed GHE is found to be superior to the linear equalizer in terms of its average Mean Square Error (MSE) for different SNR values. Significant out performance is obtained for GHE systems at moderate and high values of SNR. Fig. 8 and 9 reveals that as the number of taps increases for fixed order of the filter the MSE decreases considerably. Observing fig. 10 and 11 it can be concluded that as the order of the filter is increased for two different values of the taps the performance of the proposed GHE is almost same as the previous results but significant improvement is seen at high SNR values ie when the SNR values are in the range of 30 to 40.

Comparing the results obtained from the fig. 12 considerable improvement in the performance of the GHE is seen at moderate to high SNR values. The Modulation technique used for the above results is DPSK. Performance of the proposed GHE and LE is also carried out for fixed order and fixed number of taps for different values of the M-QAM modulation scheme and found that as the value of M increases MSE also increases. Superior performance for the proposed GHE is obtained for M = 2. The decrease in MSE is considerable at moderate to high SNR values as shown in fig 13.

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