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Performance Comparison of Discrete Hartley Transform (DHT) and Fast Fourier Transform (FFT) OFDM System in AWGN Channel

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ABSTRACT

In the current scenario so many advanced techniques are finding a way as a substitute for complex DFT - based OFDM system. One of such type of technique is Discrete Hartley Transform. The requirement of only real arithmetic computations for the proposed technique makes it more advantageous in terms of simplicity and computational speed than conventional one. This technique is very closely related to Discrete Fourier Transform. The performance of DHT based OFDM system is carried out using raw data as well as with some images as the sources and after processing at the transmitter end, the signals are then transmitted through channel. Additive White Gaussian Noise (AWGN) has been considered for channel modeling. For accuracy of this simulation, the measurement of parameters has been repeated multiple times. The simulated resulting graph between Bit Error rate and SNR shows the improvement of performance. The system performance was analyzed for M- PSK mapping schemes with various values of M, where M is order of modulation technique used. Simulation has been performed on MATLAB 7.0.

Keywords:

DHT- OFDM, FFT- OFDM, AWGN, BER

1. INTRODUCTION

OFDM is a multicarrier modulation technique that serves the requirement of today's wireless communication viz. high speed data rate with good spectral efficiency. It is based on the concept of frequency division multiplexing where each of its subcarrier are said to be orthogonal to the other carriers in the given spectrum. At the transmitter end the modulation is carried out by means of Inverse Fast Fourier transform (IFFT). The modulated carriers are summed for transmission and must be separated in the receiver before demodulation using FFT [1].For such long-length DFT/IDFT computations, a great number of complex multiplications are required and each of them basically involves four real multiplications and two real additions. Clearly, the complexity of OFDM-based transceiver would be reduced if the corresponding modulator/demodulator could be implemented using purely real transforms while fast algorithms similar to the Fast Fourier transform (FFT) algorithm can still be applied [10].

In this paper the concept of using a non-conventional transform-DHT (Discrete Hartley Transform) and its inverse, which can be used to replace its pre-existing counterpart-FFT/IFFT, in OFDM over an AWGN channel is presented. DHT and its inverse have similar equation so that there can be same blocks at the transmitter and at the receiver. Moreover FFT/IFFT coefficients can be easily derived from DHT/IDHT coefficients.

The Discrete Hartley transform (DHT) and the Discrete Fourier transform (DFT) are similar but differ from it in two characteristics of the DFT that are sometimes computationally undesirable. Since the inverse DHT is identical with the direct transform, and so it is not necessary to keep track of the $+i$ and $-i$ versions as in the DFT. Also, the DHT has real rather than complex values and thus does not require provision for complex arithmetic operations and separately managed storage for real and imaginary parts [3] [4]. DHT has been established a potential tool for signal processing applications. It has a real valued and symmetric transform kernel [7].

In this particular work the simulation time for the DFT/IDFT and DHT/IDHT for a given set of data have been obtained. It shows that the computing speed of the DHT/IDHT is faster than DFT/IDFT. The faster transform, the FHT algorithm, in essence, is a generalization of the Cooley-Tukey FFT algorithm, but the FHT requires only real arithmetic computations as compared to complex arithmetic operations in any standard FFT. Therefore, the speed of performing a FHT should be about twice as fast as the Cooley- Tukey FFT [9]. By exploiting the complex- to-real property of one of the special cases of this class, it is proposed in [2] a new one-parameter involuntary DHT that is also valid for any size N$\geq$16. The idea of replacing a twiddle factor by another scalar in the kernel of the DFT or DHT can be similarly applied at the algorithm level.

Various algorithms are still available that elaborate more on the comparisons of these two transforms and optimize the simulation timings depending upon the size of data taken. However the work here focuses more on the BER vs. SNR for the two transforms in both 1D and 2D. The result from the 2D comparison are more favorable and shows that DHT/IDHT have a better performance over DFT/IDFT.

In this work the terminology DFT/IDFT - FFT/IFFT has been used interchangeably which should not affect the sense of transform as they have been used here either by means of implementation of the mathematical equations for the transform or the inbuilt functions of the transform. The results however are just the same in both cases.

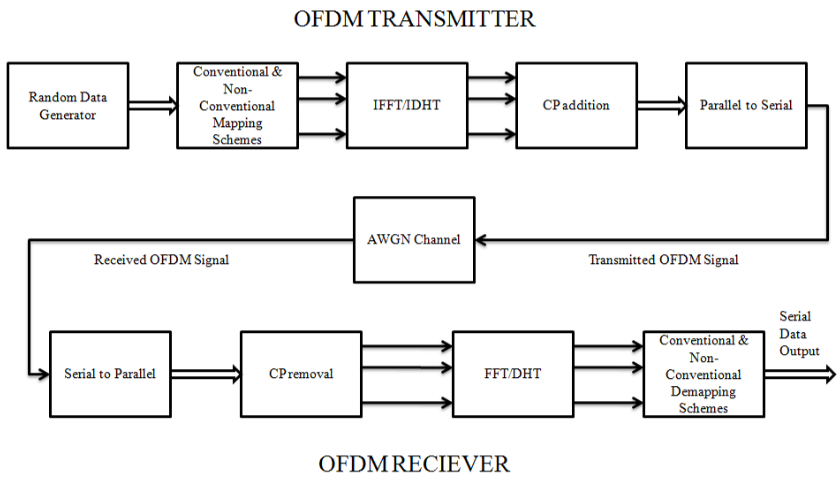

Fig. 1. Simulation Block diagram of OFDM system using an AWGN channel

Table 1. Comparison of DFT and DHT

DFT /IDFT	DHT/IDHT
Has complex values in the transform equation	Has only real values in the transform equation
The forward and inverse transforms are non-identical	The forward and inverse transforms are identical
Need to keep track of $+i$ and i terms	No such requirement
Computational complexity is higher due to imaginary terms	Computational complexity is lower as compared to its counterpart
Distinct hardware required at the transmitter and receiver	Same hardware could be used at both the channel link ends

2. SYSTEM MODEL

2.1 FFT based OFDM

OFDM is a multi carrier system which allows parallel processing and transmission of data on closely spaced sub-carriers. These sub-carriers are orthogonal to each other. The IFFT is used at the transmission end to multiplex the data and encode it before transmission. The FFT is used at the receiving end to decode and demultiplex the received data.

The N-point FFT is given by:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk} \quad where\ \ k = 0, 1...N-1$$

The N-point IFFT for a signal $X(k)$ is given by:

$$x(n) = \tfrac{1}{N}\sum_{n=0}^{N-1} X(k)e^{j2\pi nk} \quad where\ \ k = 0, 1...N-1$$

The DFT for a 2-D data $f(m, n)$ is given by:

$$F(k,l) = \tfrac{1}{\sqrt{MN}}\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} f(m,n)e^{-j2\pi(\frac{mk}{M}+\frac{nl}{N})}$$

$$where\ \ 0 \le m, k \le M-1\ \ and\ \ 0 \le n, l \le M-1$$

The IDFT for a 2-D data $F(k, l)$ is given by:

$$f(m,n) = \tfrac{1}{\sqrt{MN}}\sum_{l=0}^{N-1}\sum_{k=0}^{M-1} F(k,l)e^{j2\pi(\frac{mk}{M}+\frac{nl}{N})}$$

$$where\ \ 0 \le m, k \le M-1\ \ and\ \ 0 \le n, l \le M-1$$

2.2 DHT based OFDM

Just like the IFFT/FFT, OFDM can be implemented using real valued arithmetics by means of DHT/IDHT. The latter reduces computational complexity since it doesnt involve the $+i$ and i terms. It is a real, orthogonal separable transform [3] [4].

The N-point DHT is given by:

$$X(k) = \tfrac{1}{N}\sum_{n=0}^{N-1} x(n)cas(\frac{2\pi nk}{N})$$

$$where\ \ k = 0, 1...N-1$$

The IDHT for a sequence $X(k)$ is given by:

$$x(n) = \tfrac{1}{N}\sum_{n=0}^{N-1} X(k)cas(\frac{2\pi nk}{N})$$

$$where\ \ k = 0, 1...N-1$$

The DHT for a 2-D data $f(\tau_1, \tau_2)$ is given by:

$$H(\nu_1,\nu_2) = M^{-1}N^{-1}\sum_{\tau_1=0}^{M-1}\sum_{\tau_2=0}^{N-1} f(\tau_1,\tau_2) * cas(2\pi\nu_1\tau_1)cas(2\pi\nu_2\tau_2)$$

The IDHT for a 2-D data $H(\nu_1, \nu_2)$ is given by:

$$f(\tau_1, \tau_2) = MN \sum_{\nu_1=0}^{M-1} \sum_{\nu_2=0}^{N-1} H(\nu_1, \nu_2) * cas(2\pi\nu_1\tau_1)cas(2\pi\nu_2\tau_2)$$

where $where \ \ cas\theta = cos\theta + sin\theta$

Table 1 shows a comparison summary of DFT and DHT.

3. SIMULATION RESULTS

Figure 2 shows the BER performance of the 1-D FFT/ IFFT and DHT/ IDHT. This result is in agreement with the work [5] [6]. Both the transforms yield similar performance.

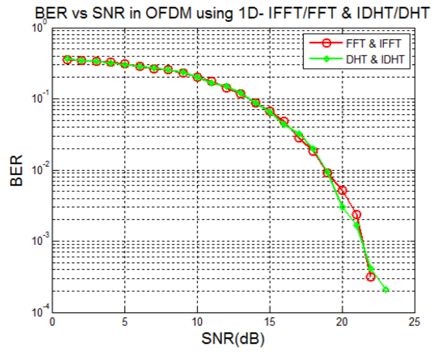

Fig. 2. BER performance of 1D- FFT DHT- OFDM

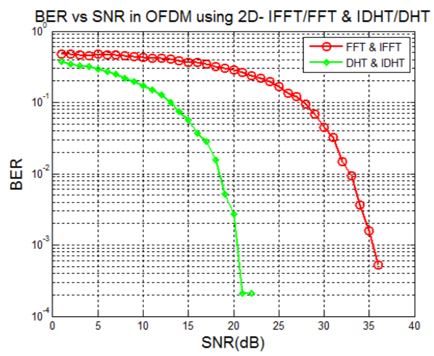

Fig. 3. BER performance of 2D- FFT DHT- OFDM

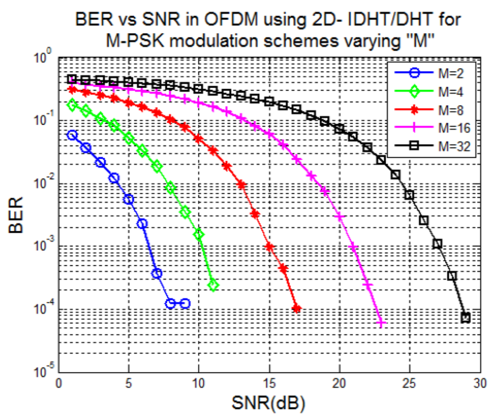

Fig. 4. BER Performance of 2D- DHT OFDM for M-PSK mapping schemes

Figure 3 shows the BER performance of the 2-D FFT/ IFFT and DHT/ IDHT. It can be observed that DHT OFDM gives much better performance than FFT - OFDM.

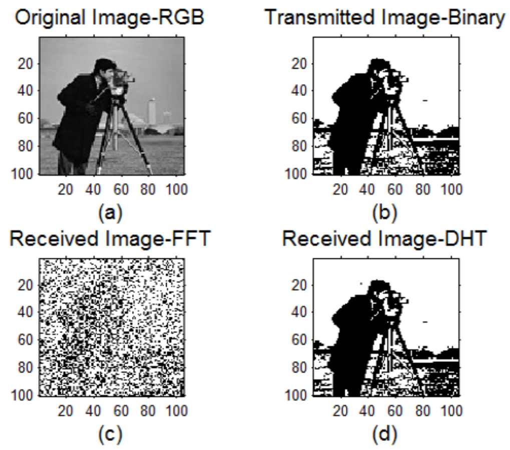

Fig. 5. (a) Original image RGB, (b) Transmitted image Binary, (c) Received image using FFT (d) Received image using DHT

Figure 4 presents the comparison of the BER performances with 2-D DHT/IDHT transforms in OFDM using different mapping schemes by varying the value of M in the M-PSK modulation scheme.

Figure 5 shows the comparative results of using FFT/IFFT and DHT/IDHT on images being transmitted in OFDM over an AWGN channel. This work is comparable to [8] where the 2-D DHT is used on image to transform it and is later retrieved back by its inverse. The work here emphasizes on receiving the image after using it as an input to an OFDM system in an AWGN channel.

4. CONCLUSION

From the above result it can be concluded that 1- dimensional DHT in OFDM has a similar response to FFT on an AWGN channel. While considering the 2- dimensional FFT and DHT OFDM in AWGN channel it can be observed that at a BER of 10^{-3} DHT requires an SNR of 21 dB and FFT requires an SNR of 36 dB. An improvement of 15 dB can be achieved by using DHT in OFDM.

Also by increasing the order of M in DHT OFDM over an AWGN channel, the BER performance becomes gradually poorer. The QPSK can be a reasonably better technique among them.

Additionally one more observation can be drawn by evaluating the time elapsed in implementing the DHT/IDHT and FFT/ IFFT in the OFDM system. On an average DHT/ IDHT consumes less time as compared to FFT/IFFT. Here it has been found that there is a difference of 0.2-1 seconds between the two transforms using the function "$tic - toc$" in MATLAB.

5. REFERENCES

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