On L-Fuzzy σ-Generalized Closed Sets and their Properties

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ABSTRACT
The aim of this paper is to introduce the concept of fuzzy σ-generalized closed set and study its properties. Secondly the concept of infra L-fuzzy topological space is introduced. Making use of fuzzy σ-closed sets and fuzzy σ-generalized closed sets the concept of fuzzy σ-generalized continuous mapping is introduced. Lastly the concept of fuzzy σ-generalized closed irresolute map in L-fuzzy topological spaces are given.

General Terms
L-Fuzzy topological space

Keywords
coprime element, fuzzy σ-closed set, fuzzy σ'-open set, fuzzy σ-generalized closed set, fuzzy σ'-generalized open set.

1. INTRODUCTION
The concept of generalized closed set was first introduced by Levine [6] in ordinary topological space. Later on Balasubramaian and Sundaram [1] introduced the concept of generalized fuzzy closed set in fuzzy topological space. In this paper the generalization of L-fuzzy topological spaceis studied. In section 2 some definitions as ready references are given. Section 3 is devoted to study fuzzy σ-generalized closed sets and their properties. In this section a new space called infra L-fuzzy topological space also defined. In section 4 fuzzy σ-generalized continuous mapping is defined and studied its properties. Lastly the concept of fuzzy σ-generalized closed irresolute map in L-fuzzy topological spaces are given in section 5

Throughout this work X and Y will be non empty ordinary sets and $L = L(\leq, \vee, \wedge, ')$ will denote a fuzzy lattice i.e. a complete completely distributive lattice with a smallest element 0 and a largest element 1 ($0 \neq 1$) and with an order-reversing involution $a \to a'$ ($a \in L$). L is therefore a continuous lattice. Also L^X will denote the lattice of L-fuzzy subsets of X.

2. PRELIMINARIES
2.1. Definition Let X be a non empty ordinary set, L be a fuzzy lattice, $\delta \subset L^X$. δ is called an L-fuzzy topology on X, and (L^X, δ) is called an L-fuzzy topological space if δ satisfies the following conditions

i) $\underline{0}, \underline{1} \in \delta$

ii)for any $A, B \in \delta, A \wedge B \in \delta$

iii)$\forall A \subset \delta, \vee A \in \delta$

Particularly, when L=[0,1], call an L-fuzzy topological space (L^X, δ) a fuzzy topological space and denote it by (X, δ).

2.2 Definition [5] An element p of L is called prime iff $p \neq 1$ and whenever $a, b \in L$ with $a \wedge b \leq p$ then $a \leq p$ or $b \leq p$. The set of all prime elements which are not 1 of L will be denoted by $pr(L)$.

2.3 Definition [6] Let L be a fuzzy lattice. $\alpha \in L$ is called a union-irreducible (or a molecule or coprime) element of L, if for arbitrary $a, b \in L$, we have $\alpha \leq a \vee b \Rightarrow \alpha \leq a$ or $\alpha \leq b$. The set of all nonzero union-irreducible elements of L is denoted by $M(L)$. The set of all molecules for a fuzzy lattice L^X is denoted by $M^*(L^X)$. Clearly $p \in pr(L)$ iff $p' \in M(L)$.

2.4 Definition [8] Let (L^X, δ) be an L-fuzzy topological space and $\alpha \in M(L)$ and $A \in L^X$, A is called an α-closed set , if for any $x \in X$, $ClA(x) \geq \alpha \Rightarrow A(x) \geq \alpha$

The set of all α-closed set in (L^X, δ) is denoted by $C_\alpha(\delta)$. Clearly, $\forall \alpha \in M(L)$, $\delta' \subset C_\alpha(\delta)$.

2.5 Theorem Let (L^X, δ)be an L-fts, $\alpha \in M(L), A \in L^X$. Then A is α-closed iff $A'$is α'-open.

2.6 Definition [1] A fuzzy set A in an fts X is called generalized fuzzy closed (in short, gf-closed) if cl(A)$\leq U$ whenever $A \leq U$ and U is fuzzy open.

2.7 Definition [1] A function $f : X \to Y$ *is* called generalized fuzzy continuous (in short gf-continuous)if the inverse image of every fuzzy closed set in Y is gf-closed in X.

2.8 Definition [1] A function $f : X \to Y$ *is* called fuzzy gc-irresolute if the inverse image of every gf-closed set in Y is gf-closed in X.

2.9 Definition [3] Let (L^X, δ)be an L-fts, $\alpha \in M(L), \lambda \in L^X$.λ is called fuzzy generalized α-closed set,if $cl_\alpha(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzyα'-open set.

3. FUZZY σ-GENERALIZED CLOSED SETS IN L-FUZZY TOPOLOGICAL SPACES
In this section, fuzzy σ-closed set, fuzzy σ-generalized closed set is defined and studied its properties.

3.1 Definition
Let (L^X, δ) be an L-fuzzy topological space and $\sigma \in M(L)$ and $A \in L^X$, A is called a fuzzy σ-closed set , if for any $x \in X$, Cl (Int (Cl$A(x)$))$\geq \sigma \Rightarrow A(x) \geq \sigma$

The set of all fuzzy σ -closed set in (L^X,δ) is denoted by $C_\sigma(\delta)$.

3.2 Remark

Fuzzy closed set and fuzzy σ-closed set are independent concept as seen in the following example.

3.3 Example

Let $X = \{x_1, x_2, x_3\}$. Define $f_1, f_2, f_3 : X \longrightarrow [0,1]$ as follows
$f_1 = 0_X$, $f_2 = 1_X$, $f_3 = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.6)\}$
Clearly $\delta = \{f_1, f_2, f_3\}$ is an L-fts on X.
$\{(x_1, 0.5), (x_2, 0.3), (x_3, 0.4)\}$ is fuzzy closed but this is not a fuzzy σ -closed set for $\sigma = 0.3$.

3.4 Definition

Let (L^X,δ) be an L-fuzzy topological space and $\sigma \in M(L)$ and $A \in L^X$, A' is called a fuzzy σ'-open set iff A is called an fuzzy σ-closed set.

3.5 Definition

If λ is an L-fuzzy set in a L-fts L^X and $\sigma \in M(L)$ then $cl_\sigma(\lambda) = \cap\{\mu : \mu \geq \lambda\}$, μ is "fuzzy σ-closed set", is called σ-closure of λ.
An L-fuzzy set λ in a L-fts (L^X,δ) is a fuzzy σ-closed iff $\lambda = cl_\sigma(\lambda)$.

3.6 Definition

Let (L^X,δ) be an L-fts, and $\sigma \in M(L)$, and $\lambda \in L^X$. λ is called an fuzzy σ-generalized closed set(in short Fσ-gclosed set), if $cl_\sigma(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzyσ'-open set.

3.7 Definition

If λ_1 and λ_2 are Fσ-gclosed sets then $\lambda_1 \vee \lambda_2$ is a Fσ-gclosed set.

Proof: Let $\lambda_1 \vee \lambda_2 \leq \mu$ where μ is fuzzy σ'-open set. Since λ_1 and λ_2 are Fσ-gclosed sets therefore $cl_\sigma(\lambda_1) \leq \mu$ whenever $\lambda_1 \leq \mu$ and $cl_\sigma(\lambda_2) \leq \mu$ whenever$\lambda_2 \leq \mu$.
$cl_\sigma(\lambda_1 \vee \lambda_2) = cl_\sigma(\lambda_1) \vee cl_\sigma(\lambda_2)$ whenever $\lambda_1 \vee \lambda_2 \leq \mu$ and μ is fuzzy σ'-open set.

However the intersection of two Fσ-gclosed sets is not Fσ-gclosed set as the following example shows.

3.8 Example

The intersection of two Fσ-gclosed sets is not Fσ-gclosed set.
Let $X = \{x_1, x_2, x_3\}$. Define $f_1, f_2, f_3, f_4, f_5 : X \longrightarrow [0,1]$ as follows
$f_1 = 0_X$, $f_2 = 1_X$, $f_3 = \{(x_1, 0.5), (x_2, 0.5), (x_3, 0.5)\}$,
$f_4 = \{(x_1, 0.4), (x_2, 0.3), (x_3, 0.2)\}$,
$f_5 = \{(x_1, 0.7), (x_2, 0.6), (x_3, 0.8)\}$
Clearly $\delta = \{f_1, f_2, f_3, f_4, f_5\}$ is an L-fts on X.
Define$\lambda_1, \lambda_2 : X \longrightarrow [0,1]$ as follows:
$\lambda_1 = \{x_1, 0.9), (x_2, 0.4), (x_3, 0.3)\}$ and
$\lambda_2 = \{(x_1, 0.5), (x_2, 0.4), (x_3, 0.9)\}$.
Here λ_1 and λ_2 are Fσ-gclosed sets but $\lambda_1 \wedge \lambda_2 = \{(x_1, 0.5), (x_2, 0.4), (x_3, 0.3)\}$ is not Fσ-gclosed set where $= \frac{1}{2}$.

3.9 Theorem

If λ is Fσ-gclosed set and $\lambda \leq \mu \leq cl_\sigma(\lambda)$, then μ is Fσ-gclosed set .

Proof: Let β be a fuzzy σ'-open set such that $\beta \geq \mu$. Since $\mu \geq \lambda$, $\beta \geq \lambda$, and λ is Fσ-gclosed set , $\beta \geq cl_\sigma(\lambda)$. But $cl_\sigma(\lambda) \geq cl_\sigma(\mu)$. since $cl_\sigma(\lambda) \geq \mu$ and so $\beta \geq cl_\sigma(\mu)$. Hence μ is Fσ-gclosed set.

3.10 Definition

A fuzzy set λ is called fuzzy σ'-generalized open (in short Fσ'-gopen) iff 1- λ is Fσ-gclosed.
We now prove some properties of fuzzy σ'-generalized open sets.

3.11 Remarks

(1) The union of two Fσ'-gopen sets is not generally Fσ'-gopen.
Example **3.8** serves the purpose.
(2) The intersection of any two Fσ'-gopen sets is Fσ'-gopen.
Proof: Let λ_1 and λ_2 are Fσ'-gopen sets in L^X then $\lambda_1{}'$ and $\lambda_2{}'$ are Fσ-gclosed set in L^X. By **theorem 3.7** $\lambda_1{}' \vee \lambda_2{}'$is F$\sigma$-gclosed. $\lambda_1{}' \vee \lambda_2{}' = (\lambda_1 \wedge \lambda_2)'$ is Fσ-gclosed set. Therefore $(\lambda_1 \wedge \lambda_2)$ is Fσ'-gopen set.

3.12 Definition

If λ is an L-fuzzy set in a L-fts L^X and $\sigma' \in P(L)$ then $int_{\sigma'}(\lambda) = \cup\{\mu : \lambda \geq \mu\}$, μ is fuzzy σ'-open set, is called fuzzy σ'-interior of λ.
An L-fuzzy set λ in a L-fts (L^X,δ) is a fuzzy σ'-open iff $\lambda = int_{\sigma'}(\lambda)$.

3.13 Theorem

An L- fuzzy set λ is Fσ'-gopen $\Leftrightarrow \mu \leq int_{\sigma'}(\lambda)$ whenever μ is Fσ-closed set and $\mu \leq \lambda$.

Proof: Let λ be a fuzzy σ'-gopen set and μ be a Fσ-closed set such that $\mu \leq \lambda$.Therefore 1- $\lambda \leq$ 1- μ and 1- λ is Fσ-gclosed set. $cl_\sigma(1-\lambda) \leq$ 1- μ.i.e 1-$cl_\sigma(1-\lambda)$ $\geq$1-(1- μ) = μ.
But 1-$cl_\sigma(1-\lambda) = int_{\sigma'}(\lambda)$. Therefore$\mu \leq int_{\sigma'}(\lambda)$.
Conversely suppose that λ is an L-fuzzy set such that $\mu \leq int_{\sigma'}(\lambda)$ whenever μ is Fσ-closed set and $\mu \leq \lambda$. We claim that 1- λ is Fσ-gclosed set. So 1- $\lambda \leq$ μ where μ is fuzzy σ'-open. 1- $\lambda \leq$ $\mu \Rightarrow$1- $\mu \leq \lambda$.
Hence by assumption we must have 1- $\mu \leq int_{\sigma'}(\lambda)$ .i.e. 1-$int_{\sigma'}(\lambda) \leq \mu$. But 1-$int_{\sigma'}(\lambda) = cl_\sigma(1-\lambda) \leq$ μ .This shows that $1-\lambda$ is Fσ-gclosed set. Therefore λ is Fσ'-gopen.

3.14 Theorem

If $int_{\sigma'}(\lambda) \leq \mu \leq \lambda$ and λ is fuzzy σ'-generalized open, then μ is fuzzy σ'-generalized open.

Proof: Given $int_{\sigma'}(\lambda) \leq \mu \leq \lambda$. We have 1- $\lambda \leq$1- $\mu \leq 1 - int_{\sigma'}(\lambda) = cl_\sigma(1-\lambda)$. As λ is fuzzy σ'-generalized open, 1- λ is Fσ-gclosed and so it follows by theorem **3.9** that is 1- μ is fuzzy σ-generalized closed, i.e. μ is fuzzy σ'-generalized open.

3.15 Definition

Let X be a non empty ordinary set, L be a fuzzy lattice, $\delta \subset L^X$. δ is called an L-fuzzy infra topology on X, and (L^X, δ) is called an L-fuzzy infra topological space if δ satisfies the following conditions

i) $\underline{0}, \underline{1} \in \delta$

ii) for any $A,B \in \delta$, $A \wedge B \in \delta$

3.16 Theorem

Let $FGO_{\sigma'}(\delta)$ = {The set of all Fσ'-gopen in (L^X, δ) }. Then $(L^X, FGO_{\sigma'}(\delta))$ forms an infra L-fuzzy topological space.

Proof: It is obvious from remarks **3.11**.

3.17 Definition

A function $f: L^X \longrightarrow L^Y$ is called fuzzyσ-continuous if the inverse image of every fuzzy closed set in L^Y is fuzzyσ-closed set in L^X.

Example. Let $f: L^X \longrightarrow L^Y$ where $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$. We define A: $X \longrightarrow [0,1]$ as follows $A(x_1) = A(x_3) = 1$, $A(x_2) = 0$. Clearly $\tau = \{1_X, 0_X, A\}$ forms an L-fts on X. Let $Y = \{y_1, y_2, y_3\}$, B: $Y \longrightarrow [0,1]$ defined as follows $B(y_1) = B(y_2) = 1$, $B(y_3) = 0$.$\delta = \{1_Y, 0_Y, B\}$ forms an L-fts on Y. Let f is defined as $f(x_1) = y_1, f(x_2) = y_2, f(x_3) = y_3$.

Here f is fuzzyσ-continuous.

3.18 Remarks

L-fuzzy continuous$\leftrightarrow$ fuzzyσ-continuous

Example. (i) $f: L^X \longrightarrow L^Y$ $X = \{x_1, x_2, x_3\}$. We define A: $X \longrightarrow [0,1]$ as follows $A(x_1) = 0.5$, $A(x_2) = 0.7$, $A(x_3) = 0.6$. Clearly $\tau = \{1_X, 0_X, A\}$ forms an L-fts on X. Let $Y = \{y_1, y_2, y_3\}$, B: $Y \longrightarrow [0,1]$ defined as follows $B(y_1) = 0.7$, $B(y_2) = 0.5$, $B(y_3) = 0.6$.$\delta = \{1_Y, 0_Y, B\}$ forms an L-fts on Y. Here f is defined as $f(x_1) = y_2, f(x_2) = y_1, f(x_3) = y_3$ .Here f L-fuzzy continuous but f is not fuzzyσ-continuous. Since the inverse image of fuzzy closed set $\{(y_1, 0.3), (y_2, 0.5), (y_3, 0.4)\}$in L^Y is not fuzzyσ-closed set in L^X.

(ii) $f: L^X \longrightarrow L^Y$ $X = \{x_1, x_2, x_3\}$, Define A: $X \longrightarrow [0,1]$ as follows $A(x_1) = 1$, $A(x_2) = 0$, $A(x_3) = 0$. Clearly $\tau = \{1_X, 0_X, A\}$ forms an L-fts on X. Let $Y = \{y_1, y_2, y_3\}$, B: $Y \longrightarrow [0,1]$ defined as follows $B(y_1) = 1$, $B(y_2) = 1$, $B(y_3) = 0$. $\delta = \{1_Y, 0_Y, B\}$ forms an L-fts on Y. Let f is defined as $f(x_1) = y_1, f(x_2) = y_3, f(x_3) = y_2$. Here f is fuzzyσ-continuous but not L-fuzzy continuous . Since the inverse image of fuzzy closed set$\{(y_1, 0), (y_2, 0), (y_3, 1)\}$in L^Y is not fuzzy closed set in L^Xbut it is fuzzyσ-closed in L^X.

3.19 Definition

A map $f: L^X \longrightarrow L^Y$ is called fuzzy σ-closed (in short Fσ-closed) set if the image of every fuzzy σ-closed set in L^X is fuzzy σ-closed in L^Y.

3.20 Theorem

If λ is Fσ-gclosed set in L^X and if $f: L^X \longrightarrow L^Y$ is fuzzyσ-continuous and Fσ-closed, then $f(\lambda)$ is Fσ-gclosed set in L^Y.

Proof: If $f(\lambda) \leq \mu$ where μ is fuzzy open in L^Y, then $\lambda \leq f^{-1}(\mu)$. Since λ is Fσ-gclosed and $f^{-1}(\mu)$ is fuzzy σ'-open, $cl_\sigma(\lambda) = \bar{\lambda} \leq f^{-1}(\mu)$. i.e. $f(\bar{\lambda}) \leq \mu$. Now by assumption, $f(\bar{\lambda})$ is Fσ-closed $\overline{f(\lambda)} \leq \overline{f(\bar{\lambda})} = f(\bar{\lambda}) \leq \mu$. We know that $\overline{f(\lambda)} = cl_\sigma(f(\lambda))$. Thus$cl_\sigma(f(\lambda)) \leq \mu$. This means $f(\lambda)$ is Fσ-gclosed set in L^Y.

3.21 Example

Under Fσ-closed, fuzzyσ-continuous maps fuzzyσ'-generalized open sets are generally not taken into fuzzy σ'-generalized open sets.

Let $X = \{a\}$, $Y = \{a, b, c\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_Y, 1_Y, B_1\}$, $B_1 : Y \to [0,1]$ is such that $B_1(a) = B_1(c) = 1, B_1(b) = 0$. Clearly τ_1 and $\tau_2$are L-fuzzy topologies on X and Y respectively.

Define $f: L^X \longrightarrow L^Y$ as follows$f(a) = b$.

One can verify f is fσ-continuous and Fσ-closed.Now we shall show that f does not take fuzzy σ'-generalized open sets to fuzzy σ'-generalized open. Clearly 1_X is fuzzy σ'-generalized open in L^X but $f(1_X) = \{(a,0)(b,1),(c,0)\}$ is not fuzzy σ'-generalized open in L^Y.

4. FUZZYσ -GENERALIZED CONTINUOUS MAPPING AND ITS PROPERTIES

4.1 Definition

A map $f: L^X \longrightarrow L^Y$ is called fuzzy σ-generalized continuous (in short F σ-gcontinuous) if the inverse image of every fuzzy closed set in $L^Y$is Fσ-gclosed in L^X.

Some properties of fuzzy σ-generalized continuous function has given.

4.2 Theorem

If $f: L^X \longrightarrow L^Y$ is fuzzy σ-continuous then it is fuzzy σ-generalized continuous but the converse is not true.

4.3 Example

Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, , $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_Y, 1_Y, B_1\}$, $B_1 : Y \to [0,1]$ is such that $B(y_1) = 1$, $B(y_2) = 1$, $B(y_3) = 0$. Clearly τ_1 and $\tau_2$are L-fuzzy topologies on X and Y respectively. Define $f: L^X \longrightarrow L^Y$ as follows $f(x_1) = y_1, f(x_2) = y_3, f(x_3) = y_2$. Here f is fuzzy σ-generalized continuous but not fuzzy σ-continuous.

4.4 Theorem

Let $f: L^X \longrightarrow L^Y$ be a function,

The following statements are equivalent

(i) f is fuzzy σ-generalized continuous

(ii) The inverse image of each fuzzy open set in L^Y is Fσ'-gopen in L^X.

Balasubramanian and Sundaram [1] defined the generalized fuzzy closure operator cl* to obtain some properties of *gf*-continuity. So, in similarway, we define the fuzzyσ-generalized closure operator cl* for any L-fuzzy set A in (L^X, τ) as follows:

cl*(λ) = $\wedge\{\mu: \mu \geq \lambda$ and λ is Fσ-gclosed$\}$.

4.5 Theorem

Let $f: L^X \longrightarrow L^Y$ be a fuzzy σ-generalized continuous, then f(cl*(λ))≤cl($f(\lambda)$) where λ is any fuzzy set in L^X.

Proof: Let λ be any fuzzy set in L^X. Then $\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(cl(f(\lambda)))$. Now cl($f(\lambda)$) is a fuzzy closed in L^Y. As f is fuzzy σ-generalized continuous, f^{-1}(cl($f(\lambda)$) is fuzzy σ-generalized inL^X. cl*(λ) $\leq f^{-1}$(cl($f(\lambda)$) .Hence f(cl*(λ))≤cl($f(\lambda)$).

5. FUZZY σ-gc-IRRESOLUTE FUNCTIONS AND THEIR PROPERTIES

5.1 Definition

A function $f: L^X \longrightarrow L^Y$ is called fuzzy σ-gc irresolute if the inverse image of every Fσ-gclosed set in $L^Y$is Fσ-gclosed in L^X.

Following are the properties of fuzzy σ-gc irresolute maps.

5.2 Theorem

If $f: L^X \longrightarrow L^Y$ is fuzzy σ-gc irresolute if the inverse image of every Fσ'-gopen set in $L^Y$is Fσ'-gopen in L^X.

5.3 Remarks

Fuzzy σ-gc irresolute$\nleftrightarrow$ fuzzy σ-generalized continuous

5.4 Example

(i) Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, , $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_Y, 1_Y\}$, $A: X \to [0,1]$ is such that $A(x_1) = 0$, $A(x_2) = 1$, $A(x_3) = 0$. Clearly τ_1 and $\tau_2$are L-fuzzy topologies on X and Y respectively. Define$f: L^X \longrightarrow L^Y$ as follows $f(x_1) = y_2, f(x_2) = y_1, f(x_3) = y_3$. Here f is fuzzy σ-generalized continuous but not fuzzy σ-gc irresolute. Since $\{(y_1, 1), (y_2, 0), (y_3, 1)\}$ is Fσ-gclosed set in L^Y but its inverse image is not Fσ-gclosed set in L^X.

(ii) Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, , $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_Y, 1_Y, B\}$, $A, B: X \to [0,1]$ is such that $A(x_1) = 0.7$, $A(x_2) = 0.5$, $A(x_3) = 0.6$; $B(x_1) = 0.5$, $B(x_2) = 0.7$, $B(x_3) = 0.6$. Clearly τ_1 and $\tau_2$are L-fuzzy topologies on X and Y respectively. Define $f: L^X \longrightarrow L^Y$ as follows $f(x_1) = y_2, f(x_2) = y_1, f(x_3) = y_3$. Here f is fuzzy σ-gc irresolute but not fuzzy σ-generalized continuous.

5.5 Theorem

Let$f: (L^X, \tau) \longrightarrow (L^Y, \delta)$ and $g: (L^Y, \delta) \longrightarrow (L^Z, \eta)$ be any two functions. Then

(i) $g \circ f$ is fuzzy σ-generalized continuous, if g is fuzzy σ-generalized continuous and f is fuzzy σ-gc irresolute.

(ii) $g \circ f$ is fuzzy σ-gc irresolute, if g is fuzzy σ-gc irresolute and f is fuzzy σ-gc irresolute.

Proof: **(i)** Let F be a fuzzy closed in (L^Z, η). Then $g^{-1}(F)$ is fuzzy σ-generalized closed in (L^Y, δ). Since g is fuzzy σ-generalized continuous.$f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is fuzzy σ-generalized closed in(L^X, τ).Since f is fuzzy σ-gc irresolute. Hence $g \circ f$ is fuzzy σ-generalized continuous.

(ii) Let F be a fuzzy σ-generalized closed in (L^Z, η). Then $g^{-1}(F)$ is fuzzy σ-generalized closed in (L^Y, δ). Since g fuzzy σ-gc irresolute. $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is fuzzy σ-generalized closed in(L^X, τ).Since $f$is fuzzy σ-gc irresolute. Hence $g \circ f$ is fuzzy σ-gc irresolute.

5.6 Remarks

The composition of two fuzzy σ-generalized continuous need not be fuzzy σ-generalized continuous function.

6. CONCLUSION

The concept of infra L-fuzzy topological space is introduced with the help of the properties of Fσ'-gopen sets. There is a scope to study the properties of infra L-fuzzy topological space and its application. There is a future scope to study Compactness, connectedness and separation axioms in infra L-fuzzy topological space.

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