A Diffusion-Augmented Level Set Method with Efficient Two Step Implementation

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ABSTRACT

The level set method was devised by Osher and Sethian [2] in as a simple and versatile method for computing and analyzing the motion of an interface Γ in two or three dimensions. Γ bounds a region Ω. The goal is to compute and analyze the subsequent motion of Γ under a velocity field v [1]. This velocity can depend on position, time, the geometry of the interface and the external physics. The interface is captured for later time as the zero level set of a smooth function $\phi(x, t)$, i.e., $\Gamma(t) = \{x|\phi(x, t) = 0\}$. ϕ is positive inside Ω, negative outside Ω and is zero on $\Gamma(t)$ [1]. This paper presents a reaction-diffusion method used to describe a physico-chemical phenomenon that comprises two elements, namely chemical reactions and diffusion for implicit active contours[21][37][39][40], which is completely free of the costly re-initialization procedure in level set evolution (LSE). A diffusion term is introduced into LSE, resulting in a diffusion-augmented level set method with efficient two step implementation. First we iteratively solve the diffusion term and then iteratively solve the level set equation. By solving equation in two steps we can stabilize the level set function without re-initialization. This is also called two step splitting method for image segmentation.

General Terms

Image segmentation, two step splitting method, partial differential equation, active contour

Keywords

Level set method, image segmentation, diffusion; level set evolution, re-initialization, signed distance function

1. INTRODUCTION

Level set methods have seen tremendously applications in many areas over the past decade [13]. This has been made possible by the exibility of the level set formulation in dealing with dynamic evolutions and topological changes of curves and surfaces, and by the mathematical theory and numerical tools developed in studying these methods[15][18][23]. The level set methods (LSM) can be categorized into partial differential equation (PDE) based ones and variational ones.

Level set method was first introduced by Osher and Sethian, and has become a more and more popular theoretical and numerical framework within image processing, fluid mechanics, graphics, computer vision, etc.[2] The level set method is basically used for tracking moving fronts [1-2] by considering the front as the zero level set of an embedded function, called the level set function. In image processing, it is used for propagating curves in 2D or surfaces in 3D. The applications of the level set method cover most fields in image processing, such as noise removal, image inpainting, image

segmentation and reconstruction [13]. In image segmentation, the level set method has some advantages compared to the active contour model. The level set method conquers the difficulties of topological transformations. The level set approach is able to handle complex topological changes automatically [2].To stop evolution of level set function traditionally we are usually find gradient of given image [26], which is less efficient. Later in advancement of level set method, we used different approach to stop evolution of level set function. Chan and Vese , the authors propose a different active contour model that based on the Mumford-Shah functional for segmentation [14], which does not use gradient to stop the evolution. In addition, by using this model and its level set formulation, interior contours are automatically detected, and the initial curve can be anywhere in the image. The liberty of formulation of these level set methods gives us countless possibilities.In recent years, some variational level set [6] formulations have been proposed to regularize the LSF during evolution, and hence the re-initialization procedure can be eliminated. These variational LSMs without re-initialization have many advantages over the traditional methods, including higher efficiency and easier implementation. Chunming Lia proposed new variational formulation for geometric active contours that forces the level set function to be close to a signed distance function [8], and therefore completely eliminates the need of the costly re-initialization procedure. Other problems of Intensity inhomogeneities occur in real-world images and may cause considerable difficulties in image segmentation. Chunming Li a new variational level set formulation in which the regularity of the level set function such that the derived level set evolution has a unique forward-and-backward (FAB) diffusion effect, which is able to maintain a desired shape of the level set function, particularly a signed distance profile near the zero level set. This yields a new type of level set evolution called distance regularized level set evolution (DRLSE) [41].This paper presents a reaction-diffusion method used to describe a physico-chemical phenomenon that comprises two elements, namely chemical reactions and diffusion for implicit active contours, which is completely free of the costly re-initialization procedure in level set evolution (LSE). A diffusion term is introduced into LSE, resulting in a diffusion-augmented level set method with efficient two step implementation, thus re-initialization procedure is completely eliminated from LSE. The rest of the paper is organized as follows. Section 2 introduces the background and related works. Section 3 presents the Reaction-diffusion systems on moving surfaces. Section 4 implements Diffusion based LSM, and analyzes the consistency between theory and implementation. Section 5 presents experimental results and Section 6 concludes the paper.

2. BACKGROUND AND RELATED WORKS

The original idea behind the level set method was a simple one. Given an interface Γ in R^n of codimension one, bounding a open region Ω [1], we wish to analyze and compute its subsequent motion under a velocity field $\vec{v}$. This velocity can depend on position, time, the geometry of the interface e.g. its normal or its mean curvature and the external physics. The idea, as devised in 1987 by S. Osher and J.A. Sethian [2] is merely to define a smooth function $\phi(x, t)$, that represents the interface as the set where $\phi(x, t) = 0$. Here x = x $(x_1 . . . x_n)$ ϵ R^n. The level set function ϕ has the following properties

$$\phi(x, t) > 0 \text{ for x } \epsilon \ \Omega$$

$$\phi(x, t) < 0 \text{ for x } \notin \bar{\Omega}$$

$$\phi(x, t) = 0 \text{ for x } \epsilon \ \partial\Omega = \Gamma(t) \qquad (1)$$
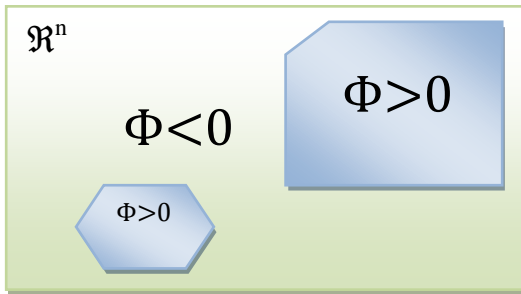

Fig 1: implicit and explicit regions

Hence we can identify the interface by locating the set for which $\Gamma(t)$ for which ϕ vanishes. These phenomena can be useful for numerical computation, primarily for topological changes such as breaking and merging [1][2][3]. The motion is analyzed by convecting the ϕ values (levels) with the velocity field $\vec{v}$. This elementary equation is

$$\frac{d\phi}{dt} + \vec{v} . \nabla\varphi = 0. \qquad (2)$$

Here $\vec{v}$ is the desired velocity on the interface, and is arbitrary elsewhere. Actually, only the normal component of v is needed: $V_N = \vec{v} \cdot \nabla\varphi$, so (1) becomes

$$\frac{d\phi}{dt} + V_N . \nabla\varphi = 0. \qquad (3)$$

Since F is defined as the speed in the outward normal direction, then $\bar{x}\cdot$ n = F, where n $=\frac{\Delta\varphi}{|\Delta\varphi|}$. This yields an evolution equation for φ:

$$\frac{d\phi}{dt} + F. |\nabla\varphi| = 0. \qquad (4)$$

This equation is defined by the authors "osher and sethian"[2]. Using this equation we can find the topological changes using iteration methods. One disadvantage of this level set method is that level set function (LSF) becomes to flat or too steep near the zero level set, causing numerical errors [15][23]. This problem can be solved by re-initializing the level set function and repeat the same procesedure until find the desired level set.

2.1 Re-initialization:

Using re-initialization method we can find topological changes in the image like merging and splitting. a procedure called re-initialization is periodically employed to reshape it to be an SDF [2][8][41]. This method is used extensively with

level set method for stable numerical solution. The standard re-initialization method is to solve the following re-initialization equation,

$$\varphi_t + Sign(\varphi_0)(1-|\nabla\varphi|) = 0 \qquad (5)$$

Where φ_0 is the function to be re-initialized, and sign(φ) is the sign function. If φ_0 is not smooth or is too steeper at one side of the interface than the other, the resulting zero level set function may be incorrect from that of the original function. If signed distance function is far away from the level set function then it becomes difficult to re-initialize the level set function to be signed distance function [9]. In practice, the evolving level set function can deviate greatly from its value as signed distance in a small number of iteration steps, especially when the time step is not chosen small enough. Re-initialization has been extensively used as a numerical remedy for maintaining stable curve evolution and having desirable results. The re-initialization process can be quite complicated, expensive and have subtle side effects. Most of the level set methods are fight with their own problems, such as when and how to re-initialize the level set function to a signed distance function.

2.2 Signed distance function:

As to get stable and accurate evolution of topological changes we are initializing the level set function as signed distance function. Osher and Sethian proposed to initialize the LSF as $\varphi(x) = 1\pm dist^2(x)$, where $dist(\cdot)$ is a distance function and "$\pm$" denotes the signs inside and outside the contour [9]. Later, Mulder *et al.* initialized the LSF as $\varphi(x) = \pm dist(x)$, which is an SDF that can result in accurate numerical solutions.re-initialization methods do not directly compute the SDF since the solution of $|\nabla\varphi| = 1$ is itself an SDF [28]. In, the following re-initialization equation was proposed

$$\varphi_t + S(\varphi_0)(|\nabla\varphi| - 1) = 0 \qquad (6)$$

Where $S(\varphi_0) \triangleq \frac{\varphi}{\sqrt{\varphi^2 + |\nabla\varphi|2.(\Delta x)2}}$, φ_0 is the initial LSF and Δx is the spatial step. Unfortunately, if the initial LSF φ_0 deviates much from an SDF, Eq. will fail to yield a desirable final SDF.

As re-initialization has many drawbacks such as the expensive computational cost, blocking the emerging of new contours, failures when the LSF deviates much from an SDF, and inconsistency between theory and implementation. Solution to this problem is proposed to regularize the variational LSF to eliminate the re-initialization procedure. Li proposed the distance regularized level set evolution (DRLSE) method for accurate evolution of LSF [41]. Li et al. proposed a signed distance penalizing energy functional:

$$P(\varphi) = \int_\Omega (|\nabla\varphi| - 1) 2 \, dx \qquad (7)$$

This equation measures the closeness between an LSF φ and an SDF in the domain $\Omega \subset R^n$ .Although DRLSE methods have many advantages over re-initialization methods, such as higher efficiency and easier implementation, they still have

Table 1: Comparison of different level set method

the following drawbacks as limited application to PDE-based LSMs, limited anti-leakage capability for weak boundaries, and sensitivity to noise. By adding the diffusion term into level set method equation and evaluate it in two steps we can get better accuracy and stability in image segmentation and it becomes free from costly re-initialization procedure.

Comparison of different level set method		
SR NO.	TITTLE	Proposed Method
1	An Efficient Algorithm for Level Set Method Preserving Distance Function	Fast algorithm to preserve distance functions in level set methods. algorithm is inspired by recent efficient $\ell 1$optimization techniques
2	Re-initialization Free Level Set Evolution via Reaction Diffusion	This paper presents a novel reaction diffusion (RD) method for implicit active contours, which is completely free of the costly re-initialization procedure in level set evolution (LSE). A diffusion term is introduced into LSE, resulting in a RD-LSE equation.
3	Distance Regularized Level Set Evolution and Its Application to Image Segmentation	The DRLSE defined with a potential function such that the derived LSE has a unique forward-and-backward (FAB) diffusion effect, which is able to maintain a desired shape of the LSF, particularly a signed distance profile near the zero level set.
4	A Level Set Method for Image Segmentation in the Presence of Intensity Inhomogeneities With Application to MRI	This paper proposes a novel region-based method for image segmentation, which is able to deal with intensity in homogeneities in the segmentation.
5	Modified Gradient Search for Level Set Based Image Segmentation	Active contours, gradient methods: (1) resilient propagation (Rprop) (2) using a momentum term image segmentation, level set method, machine learning, optimization, variational problems.

3. REACTION-DIFFUSION SYSTEM ON MOVING SURFACES

The term reaction-diffusion system is basically related to physico-chemical phenomenon that comprises two elements, namely chemical reactions and diffusion [16][21]. A chemical reaction system consists of N species $(x_1 . . . x_n)$ (e.g. molecules) together with M reaction channels, $(r_1 . . . r_m)$. Each reaction channel defines the stoichiometry of a reaction

$$r_m : \sum_i \propto_{im} X_i \xrightarrow{km} \sum_j \beta_{jm} X_j \qquad (8)$$
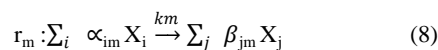
This describes the idea that whenever the species i come together with molar concentrations $\propto_i$, they are interconverted

to the species j with molar concentrations β_j with a specific reaction rate k_m. Using the law of mass action , one can derive a system

$$\frac{dc}{dt} = \sum_1^M k_m \ (\beta_{im} - \propto_{im}) \qquad (9)$$

or in matrix notation

$$\frac{dc}{dt} = M_S \cdot r(t) \qquad (10)$$

Where M_S is the stoichiometric matrix and r the rate vector describing the speed of each reaction.

The second phenomenon, diffusion, refers to the process of thermal motion of molecules [38][39][40]. It is the process by which for example warm and cold water intermingles until the water has a uniform temperature (at thermal equilibrium) or by which a fragrant smell spreads in a room. The general macroscopic diffusion equation for species i = 1, 2,....N is

$$\frac{dc}{dt} = \nabla.(D_i \nabla c_i) \qquad (11)$$

where Di denotes the diffusion tensor of species i, a matrix that defines how well the molecule i diffuses into the different spatial directions.

Combining equations and , we get the so called reaction-diffusion equation. The equations for a reaction-diffusion system on a surface $\Gamma \subset \Omega \subseteq R^3$ are then

$$\frac{dc}{dt} = R_i(C) + D_i \Delta_\Gamma c_i \qquad i = 1,2……N;$$

Since the zero level is used to represent the object contour, we only need to consider the zero level set of the LSF. We can use a function with different phase fields as the LSF. Motivated by the phase transition theory, by adding a diffusion term into the conventional LSE equation. Such an introduction of diffusion to LSE will make LSE stable without re-initialization. By adding a diffusion term "ε∆φ" into the LSE equation, we have the following RD equation for LSM:

$$\varphi_{t} = \varepsilon\Delta\varphi - \frac{1}{\varepsilon} L(\varphi), x \in \Omega \subset R^n$$

4. TWO STEP DIFFUSION BASED LSM

A Two step algorithm to implement Diffusion has been proposed in to generate the curvature-dependent motion. In the reaction function is first forced to generate a binary function with values 0 and 1, and then the diffusion function is applied to the binary function to generate curvature-dependent motion. Different from, where the diffusion function is used to generate curvature-dependent motion, in our proposed LSM, the LSE is driven by the reaction function, i.e., the LSE equation. Therefore, we propose to use the diffusion function to regularize the LSF generated by the reaction function. To this end, we propose the following two step method to solve the equation.

Step 1: Solve the reaction term *till some time T_r* to obtain the intermediate solution, denoted by φn+1/2 =φⁿ;

Step 2: Solve the diffusion term φt =ε∆φ, φ(x,t=0) = φn+1/2 till some time T_d , and then the final level set is φn+1 =φ(x,Td).

4.1 PROPOSED ALGORITHM:

1. Read image

2. Apply Gaussian kernel for smoothing

3. Implementation of Dirac function $\delta(x)$

4. Selection of Time Step value

5. Implementation of penalizing energy term P(\phi) For implementation of penalizing) energy term P(\phi) Set following parameter

- u0: level set function to be updated

- g: edge indicator function

- lambda: coefficient of the weighted length term L(\phi)

- mu: coefficient of the internal (penalizing) energy term P(\phi)

- alf: coefficient of the weighted area term A(\phi), choose smaller alf

- epsilon: the papramater in the definition of smooth Dirac function, default value 1.5

- delt: time step of iteration, see the paper for the selection of time step and mu

- numIter: number of iterations.

6. Define initial level set function

The initial LSF

$$\Phi_0(x_1, x_2) = \sqrt{(x1-50)2 + (x2-50)2} - 30$$

7. Start level set evolution

8. Optimization of results (final contour detection.)
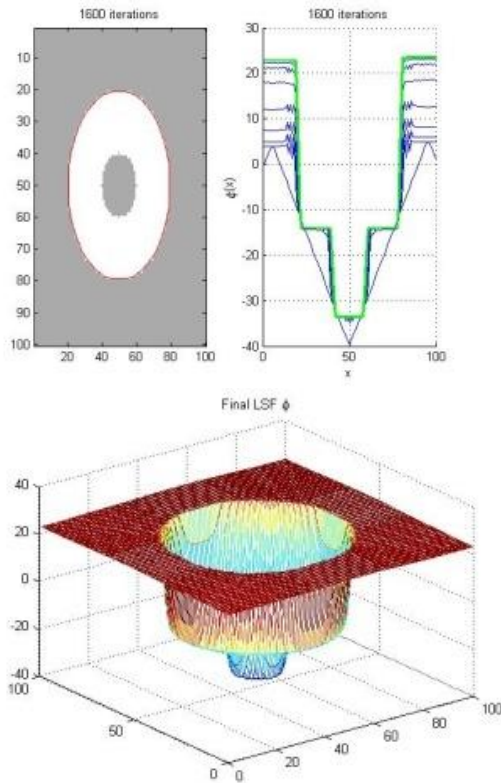
5. EXPERIMENTAL RESULTS

Fig 2: (a) Testing image. red circle represents the initial contour and Middle slices of level set function during LSE. (b) The final LSFs .The initial LSF is $\Phi_0(x_1, x2) =$

$\sqrt{(x1-50)2 + (x2-50)2}$ - 30 , $\Delta t1 = \Delta t2 = 0.1$, $\Delta x1 = \Delta x2 = 1$. The number of iterations is 100.
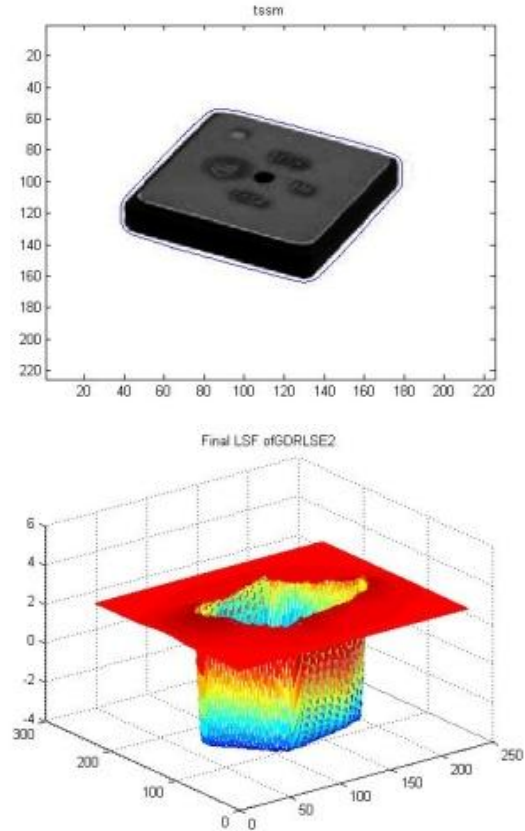

Fig 3: (a) Testing image. (b) The final LSFs. image size = 225 x 225 , type: bmp ,Time step(ΔT)= 1, Mu (μ) =0.2/ΔT , Lambda (γ)= 5, Alpha ($\propto$)= -0.3, Epsilon ($\in$)=1.5, Sigma (σ)= 0.8, iteration = 1500, segment time= 2.03.04 second
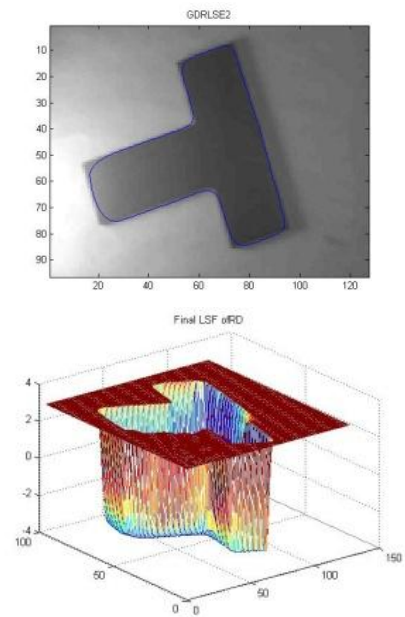

Fig 4: (a) Testing image. (b) The final LSFs. Image size = 127 x 96, type: bmp, Time step (ΔT) = 1, Mu (μ) =0.2/ΔT , Lamda (γ)= 5, Alpha ($\propto$)= -0.3, Epsilon ($\in$)=1.5, Sigma (σ)= 0.8, iteration = 1500, segment time= 1.36.08 second

	ΔT	Mu (μ)	(γ)	$(\propto)$	$(\in)$	(σ)	time
DIFFU-LSM	0.1	$0.2/\Delta T$	5	-0.3	1.5	0.8	4.8
DRLSE	0.1	$0.2/\Delta T$	5	-0.3	1.5	0.8	6.2

Table 2: Comparison of experimental results of Diffusion based LSM and DRLSE method.

6. CONCLUSION

The Two step diffusion based LSM is general, which can be applied to the PDE-based level set methods and variational ones. Second, method has much better performance on weak boundary anti-leakage. Third, the implementation of the method is very simple and it does not need the upwind scheme at all. Fourth, this method is robust to noise. The experiments on synthetic and real images demonstrated the promising performance of our approach.

7. ACKNOWLEDGMENTS

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Fig 5: Two step diffusion based LSM method (a) middle slices of the LSFs in iterations (b) the final LSFs. We set $\Delta t1 = \Delta t2 = 0.1$.

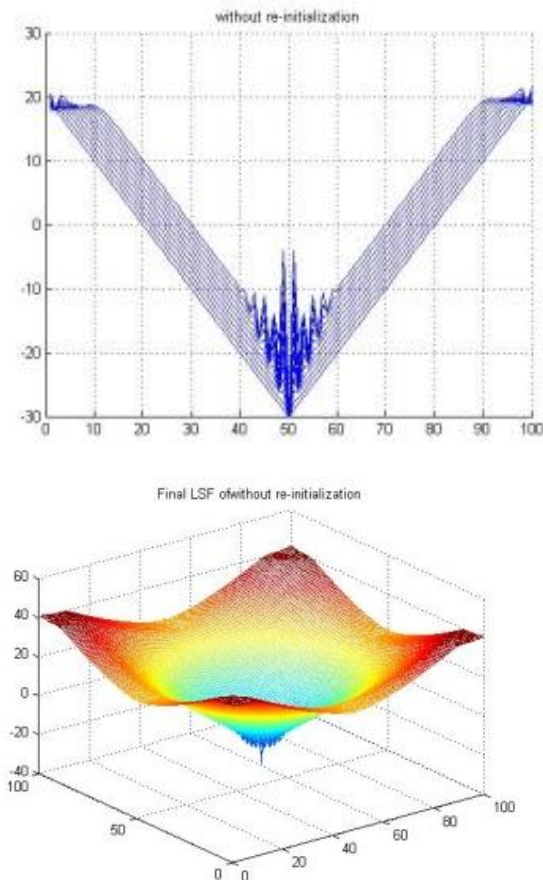**Fig 6: DRLSE method (a) middle slices of the LSFs in iterations (b) the final LSFs. We set $\Delta t1 = \Delta t2 = 0.1$.**

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