Subtraction in Traditional and Strange number system by r's and r-1's Compliments

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ABSTRACT
As most digital system cannot subtract, they can only add, so we needed a method of compliments to subtract. The knowledge of Compliments ,r's and (r-1)'s in number systems, their representation, limits, is essential for understanding of computers and successful programming for digital devices In this paper we discussed r's and (r-1)'s compliments subtraction through Traditional number system and Strange number system, their properties and subtraction in the light of different prospective. Traditional number system – binary, octal, decimal and hexadecimal, in regular use and Strange number system, with an extra edge of memorized information with greater density, zero redundancy problem, avoiding sign problem and reducing complexity of interconnections, - unodecimal, duodecimal, tridecimal, quadrodecimal, pentadecimal, heptadecimal, octodecimal, nona decimal, vigesimal and further are discussed through compliments.

Keywords
Compliments, r's and r-1's, signed and unsigned numbers.

1. INTRODUCTION
Compliments are used to simplify the subtraction operation and for logical manipulation. The r's and(r-1)'s compliments are generalized representation of compliments. r stands for radix or base of number system, thus r's compliment is referred as radix compliment and (r-1)'s compliment is referred as diminished radix compliment. Examples of r's compliment are 2's, 10's and 18's compliment and examples of (r-1)'s compliment are 1's, 9's and 17's compliment. In a base r system, the r's and (r-1)'s compliment of the number N having n digits, can be defined as

(r-1)'s compliment of N = (r^n-1) - N

and

r's compliment of N = r^n- N= (r-1)'s compliment of N + 1

The (r-1)'s complement can also be obtained by subtracting each digit of N from (r-1).Using the above methodology we can also define the 7's and 8's compliments for octal system and 15's and 16's compliments for hexadecimal system (Traditional number system) and 13's and 14's compliments of pentadecimal system and 17's and 18's compliments of nonadecimal system (Strange number system).

1.1 (r-1)'s Compliment Subtraction [2]
Let difference D = M – S by (r-1)'s compliment be evaluated by two numbers, M (minuend) and S (subtrahend), either or both the numbers may be signed or unsigned.

Unsigned data ---

M_u = 3078,S_u= 32 and D_u= M_u-S_u

Signed data-----

M_s=-176, S_s=2312 and D_s= M_s-S_s
Equating the length
M_u=3078 , S_u=32 →S_u=0032
M_s=-176, S_s=2312→M_s=-0176

If either or both of operands are negative then take the (r-1)'s compliment of the number

M_s=-176

(r-1)'s of M_s=9999-0176=9823

S_s=2312

In order to evaluate difference taking (r-1)'s compliment of the representation obtained for subtrahend

S_u and S_s

S_u=0032

(r-1)'s compliment of 0032

S_u=9999-0032=9967

M_u=3078

S_s=2312

(r-1)'s compliment of 2312

S_s=9999-2312=7687

M_s=9823

Add the two numbers and check whether or not carry generated from MSB due to addition

M_u=3078, S_u=9967→ D_u = M_u-S_u

=13045

↓

CY

M_s=9823, S_s=7687→ D_s = M_s-S_s

=17510

↓

CY

We obtained the result as CY. The CY from MSB contains some useful information especially in some unsigned arithmetic. Processing of carry for (r-1)'s complement is, in this case if a carry is generated from MSB, add this carry to the LSB of the result we got

CY=1 , D_u=3045

and

CY=1, D_s=7510

After adding carry to LSB, we get

D_u=3045+1=3046

and

D_s=7510+1=7511

Result Manipulation for

(a) Unsigned

i. If a carry is generated then the result is positive and the digits in the result shows the correct magnitude of result.

ii. If there is no carry from MSB then result is negative and digits in result are not showing correct magnitude. Post processing is required to determine correct magnitude of result.

(b) Signed

i. If the MSB of result obtained is lesser than half radix(i.e MSB < r/2),then result is positive and representing correct magnitude.

ii. If the MSB of the result is not lesser than half the radix (i.e MSB ≥ r/2) then result is negative and correct magnitude is obtained by post processing.

Post Processing

If the result is positive (+ve) it represents the correct magnitude whether it is signed or unsigned arithmetic. However, the negative results are not showing correct magnitudes so post processing in principle is needed for declaration of negative results.

(a) Declare positive results. As per the rules the result of the unsigned arithmetic is positive. Therefore,

Du = 3046

(b) Process and declare negative results. As per the rules result of signed arithmetic is negative and is in complemented form. Take the (r – 1)'s complement to find the complement and declare the result.

(r – 1)'s of Ds = 9999 – 7511 = –2488

Therefore, Ds = –2488

1.2 r's compliment subtraction [2]

Unsigned data---

$$M_u = 3078, S_u = 32 and D_u = M_u - S_u$$

Signed data-----

$$M_s = -176, S_s = 2312 and D_s = M_s - S_s$$

Equating the length

$$M_u = 3078, S_u = 32 \rightarrow S_u = 0032$$
$$M_s = -176, S_s = 2312 \rightarrow M_s = -0176$$

Taking the r's compliment of negative operands

$$M_s = -176,$$

r's compliment of (-176)

$$M_s = 9999 - 176 + 1 = 9824$$

and

$$S_s = 2312$$

Taking r's compliment of (0032) and (2312)

r's compliment of 0032

$$S_u = 10000 - 0032 = 9968$$
$$M_u = 3078$$

r's compliment of (2312)

$$S_s = 10000 - 2312 = 7688$$
$$M_s = 9824$$

Adding the two numbers and checking whether or not carry generated from MSB due to addition.

$$M_u = 3078, S_u = 9968 \rightarrow D_u = 13046$$

↓

CY

$$M_s = 9824, S_s = 7688 \rightarrow D_s = 17512$$

↓

CY

If there is carry from MSB then simply discard it.

$$CY = 1, D_u = 3046 and$$
$$CY = 1, D_s = 7512$$

After discarding the carry we get

$$D_u = 3046 and D_s = 7512$$

Result Manipulation

(a) Unsigned

i. If carry is generated then the result is positive and the digits in the result show the correct magnitude of the result.

ii. If there is no carry from MSB then the result is negative and the digits in result are not showing correct magnitude. Post processing of result must be done to determine correct magnitude.

(b) Signed

i. If the MSB of result obtained is lesser than the half radix (i.e ,MSB <r/2) then result is +ve and representing the correct magnitude. Thus no post processing is required.

ii. If the MSB of result is not lesser than the half radix (i.e, MSB ≥ r/2) then result is –ve and correct magnitude of which must be obtained by post processing.

Post Processing

i. Declare positive results, positive result shows correct magnitude. As result of unsigned arithmetic is positive, so

$$D_u = 3046$$

ii. Process and declare negative results. As the result obtained of signed arithmetic is negative and is in complimented form.

Take the r's compliment to find the compliment and declare the result

$$r'sofD_s = 10000 - 7512 = -2488$$

Therefore

$$D_s = -2488$$

2. TRADITIONAL NUMBER SYSTEM [3]

Number System	Base	Symbol	Number Representation
Binary	2	(0 ,1)	$(110.11) = 1\times2^{2} + 1\times2^{1} + 0\times2^{0} + 1\times2^{-1} + 1\times2^{-2}$
Octal	8	(0….7)	$(36.43) = 3\times8^{1} + 6\times8^{0} + 4\times8^{-1} + 3\times8^{-2}$
Decimal	10	(0…..9)	$(234.28) = 2\times10^{2} + 3\times10^{1} + 4\times10^{0} + 2\times10^{-1} + 8\times10^{-2}$
Hexadecimal	16	(0…9 , A...E)	$(B4.A3) = B\times16^{1} + 4\times16^{0} + A\times16^{-1} + 3\times16^{-2}$

Table 1. Subtraction in Traditional Number System using r's and (r-1)'s compliments

.Unsigned Number = $(625)_8$-$(76)_8$

Signed Number = $(-76)_8$-$(625)_8$

r's compliment subtraction (8's)

Unsigned

$(625)_8$-$(76)_8$

Taking 8's compliment of (076)

$(1000)_8$-$(076)_8$= $(702)_8$

Adding with $(625)_8$

$(625)_8$+$(702)_8$= $(1527)_8$

As carry is generated result is positive and after discarding carry it is $(527)_8$

So $(625)_8$-$(760_8$=$(527)_8$

Signed

$(-76)_8$-$(625)_8$

Taking 8's compliment of $(076)_8$

$(1000)_8$-$(076)_8$=$(702)_8$

8's compliment of $(625)_8$

$(1000)_8$-$(625)_8$=$(153)_8$

Adding with $(702)_8$

$(153)_8$+$(702)_8$=$(1055)_8$

Discarding carry = $(55)_8$

As MSB ≥ r/2 or 5 ≥ 8/2 so taking 8's compliment of $(55)_8$

So $(1000)_8$-$(055)_8$=$(723)_8$

$(-76)_8$-$(625)_8$= -$(723)_8$

(b) (r-1)'s compliment subtraction (7's)

Unsigned = $(625)_8$-$(76)_8$

Taking 7's compliment of $(076)_8$

$(777)_8$-$(076)_8$ =$(701)_8$

Adding with $(625)_8$

$(701)_8$+$(625)_8$= $(1526)_8$

As it is generating carry, discarding and adding

$(526)_8$+1=$(527)_8$

So $(625)_8$-$(76)_8$=$(527)_8$

Signed

$(-76)_8$-$(625)_8$

7's compliment of $(76)_8$

$(777)_8$-$(076)_8$=$(701)_8$

Taking 7's compliment of $(625)_8$

$(777)_8$-$(625)_8$=$(152)_8$

Adding with $(701)_8$

$(701)_8$+$(152)_8$=$(1053)_8$

As it is generating carry , discarding and adding

$(053)_8$+1= $(054)_8$

As MSB ≥ r/2 or 5 ≥ 8/2
Taking 7's compliment of $(054)_8$
$(777)_8$-$(054)_8$=$(723)_8$
So $(-76)_8$-$(625)_8$=-$(723)_8$

2. STRANGE NUMBER SYSTEM [3]

Number System	Base	Symbol	Number Representation
Unodecimal	11	(0….9, A)	$(1A1.28)_{11} = 1\times11^{2} + A\times11^{1} + 1\times11^{0} + 2\times11^{-1} + 8\times11^{-2}$
Duodecimal	12	(0…9, A,B)	$(2B0.17)_{12} = 2\times12^{2} + B\times12^{1} + 0\times12^{0} + 1\times12^{-1} + 7\times12^{-2}$
Tridecimal	13	(0…9, A...C)	$(C81.23)_{13} = C\times13^{2} + 8\times13^{1} + 1\times13^{0} + 2\times13^{-1} + 3\times13^{-2}$
Quadrodecimal	14	(0…9, A….D)	$(DA1.67)_{14} = D\times14^{2} + A\times14^{1} + 1\times14^{0} + 6\times14^{-1} + 7\times14^{-2}$
Pentadecimal	15	(0….9, A….E)	$(E18.A4)_{15} = E\times15^{2} + 1\times15^{1} + 8\times15^{0} + A\times15^{-1} + 4\times15^{-2}$
Heptadecimal	17	(0….9, A….G)	$(B2G.07) = B\times17^{2} + 2\times17^{1} + G\times17^{0} + 0\times17^{-1} + 7$
Octodecimal	18	(0…9, A….H)	$(90H.1A)_{18} = 9\times18^{2} + 0\times18^{1} + H\times18^{0} + 1\times18^{-1} + A\times18^{-2}$
Nonadecimal	19	(0…9, A…I)	$(DI2.38)_{19} = D\times19^{2} + I\times19^{1} + 2\times19^{0} + 3\times19^{-1} + 8\times19^{-2}$
Vigesimal	20	(0…9, A….J)	$(EJ8.14)_{20} = E\times20^{2} + J\times20^{1} + 8\times20^{0} + 1\times20^{-1} + 4\times20^{-2}$

Table 2. Subtraction in Strange Number System using r's and (r-1)'s compliments

Unsigned Number = $(G2)_{18}$-$(2HA)_{18}$

Signed Number = $(-G2)_{18}$-$(2HA)_{18}$

(a) r's compliment subtraction (18's)

Unsigned

= $(G2)_{18}$-$(2HA)_{18}$

Taking 18's compliment of $(2HA)_{18}$

$(1000)_{18}$-$(2HA)_{18}$= $(F08)_{18}$

Adding with $(0G2)_{18}$

$(0G2)_{18}$+$(F08)_{18}$=$(FGA)_{18}$

As there is no carry generated, result is negative, it is recomplimented and minus sign is attached to it, so

$(1000)_{18}$-$(FGA)_{18}$= $(218)_{18}$

$(0G2)_{18}$-$(2HA)_{18}$ = -$(218)_{18}$

Signed

$(-G2)_{18}$-$(2HA)_{18}$

Taking 18's compliment of $(0G2)_{18}$

$(1000)_{18}$-$(0G2)_{18}$=$(H1G)_{18}$

Taking 18's compliment of $(2HA)_{18}$

$(1000)_{18}$-$(2HA)_{18}$=$(F08)_{18}$

Adding with $(H1G)_{18}$

$(H1G)_{18}$+$(F08)_{18}$= $(1E26)_{18}$

Discarding carry & MSB ≥ r/2

Or 14 ≥ 18/2 so

Recomplimenting $(E26)_{18}$ and attaching minus sign to it

$(1000)_{18}$-$(E26)_{18}$= $(3FC)_{18}$

So $(-G2)_{18}$- $(2HA)_{18}$ = -$(3FC)_{18}$

(b) r-1's compliment subtraction (17's)

Unsigned

$(G2)_{18}$-$(2HA)_{18}$

Taking 17's compliment of $(2HA)_{18}$

$(HHH)_{18}$-$(2HA)_{18}$= $(F07)_{18}$

Adding with $(0G2)_{18}$

$(F07)_{18}$+$(0G2)_{18}$= $(FG9)_{18}$

As no carry is generated, result is negative so recomplimenting it and attaching minus sign to it.

$(HHH)_{18}$-$(FG9)_{18}$= $(218)_{18}$

So $(0G2)_{18}$-$(2HA)_{18}$= -$(218)_{18}$

Signed

$(-G2)_{18}$-$(2HA)_{18}$

Taking 17's compliment of $(0G2)_{18}$

$(HHH)_{18}$-$(0G2)_{18}$= $(H1F)_{18}$

And 17's compliment of $(2HA)_{18}$

$(HHH)_{18}$-$(2HA)_{18}$=$(F07)_{18}$

Adding with $(H1F)_{18}$

$(F07)_{18}$+ $(H1F)_{18}$ = $(1E24)_{18}$

As it generated carry, removing and adding

$(E24)_{18}$+ 1 = $(E25)_{18}$

As MSB ≥ r/2 or 14 ≥ 18/2 so

Recomplimenting $(E25)_{18}$ and attaching minus sign to it

$(HHH)_{18}$-$(E25)_{18}$= $(3FC)_{18}$

$(-G2)_{18}$- $(2HA)_{18}$ =-$(3FC)_{18}$

4. CONCLUSION

In this paper we tried to explain and discuss compliments, particularly their use in subtraction. Traditional and Strange number system has been taken and discussed in the light of compliments. With the help of compliments digital system can subtract two numbers, while actually performing addition. Both unsigned and signed numbers are taken and with the help of compliments, are subtracted. This study will be very helpful for researchers and knowledge seekers to easy understanding and practicing of subtraction of number systems as well as to understand traditional and strange number systems for those who are in the field of computer science and technology.

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