New Discrete Predictive Sliding Mode Control for Non Minimum Phase Systems

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ABSTRACT

This paper shows the development of a Predictive Sliding Mode Controller (PSMC). The proposed controller blends the design of Sliding Mode Control (SMC) with Model Predictive Control (MPC). The combination of SMC and MPC improves the performance of these two control laws. The designed control strategy has stronger robustness and chattering reduction property to conquer within the system uncertainties. The Predictive Sliding Mode Control strategy was improved by giving a New Predictive Sliding Mode Controller (NPSMC). Finally, the performances of the NPSMC, in terms of strong robustness to external disturbance and parameters variation, chattering elimination, fast convergence were judged, in comparaison with PSMC, SMC and MPC, using a non-minimum phase system.

Keywords:

Non minimum phase systems, Sliding mode control, Model predictive control, Predictive sliding mode control, Chattering phenomenon.

1. INTRODUCTION

The classical Bode integral theorems provide unique relationships between the amplitude and the phase of the frequency response for a minimum phase system. A plant with dead-time or with a right-half plane zero is called non-minimum phase system as these relationships are then no longer valid [1]. It is well known that such plants are not easy to control because they have an initial inverse response to step input in the opposite direction to the steady state [2]. In some sense all practical discrete time systems are non-minimum phase. The presence of unstable zero in a process transfer function is thus identified as being responsible for its difficult dynamic behavior; it is also the source of a considerable amount of difficulty in controller design. An other aspect of controlling a process with unstable zero is the problem of instability, which arises in order to achieve high performance when the controller contains an inverse of the process model [3, 4].
Model Based Predictive Control (MPC) has become one of the most popular control methodologies both in industry and academia [7]. Furthermore, the instability problems applying Model Predictive Control to non minimum phase systems have been reported in literature [4, 8], showing that the non-minimum phase systems produce instability when the control horizon and the maximum predictive horizon are equal to 1, while [9, 10] demonstrated that this can be tuned to have stable behavior with unstable zero in the Single Input Single Output (SISO) case. MPC has instability problems because, for non-minimum phase plants, the controller achieves the optimal output by canceling the plant zeros, including the unstable zero, which leads to loss of internal stability of the feedback system [11, 12].

On the other hand, Sliding Mode Control (SMC) is a technique derived from Variable Structure Control (VSC) which was studied originally by Utkin [13]. For a broad class of systems, this kind of control is particulary appealing due to its ability to deal with nonlinearities, parameter uncertainties and disturbances. However, undesired chattering produced by the high frequency switching of the control may be considered as a problem for implementing the sliding mode control methods to some real applications [20].

This paper shows how a combination of MPC and SMC results in a control structure that has the main advantages of both SMC and MPC. Some works based on mixing these two control techniques can be found. An algorithm based on discrete time VSC was proposed in [14, 15]. The transient response shaping allowed by the multistage optimization approach is enhanced by the connection of a generalized predictive controller with a variable structure algorithm. A dual mode control scheme combining MPC and SMC was presented in [16]. In the last work MPC was used to force the state into terminal region within a finite horizon while it is outside the terminal region then an SMC was used when the state got inside the terminal region. The main idea of the controller proposed in this paper is to introduce the prediction of the sliding surface into the control objective. To ameliorate this controller, we introduced a New Predictive Sliding Mode Controller, aiming at eliminating constant disturbances and parameters variations. These two controllers are applied to a non-minimum phase system with parameter uncertainties and external disturbances. The paper is organized as follows: Section 2 describes the basic concepts by giving the system description and preliminaries, the Sliding Mode control and the Model Predictive control theories. Section 3, describes the synthesis of the Predictive Sliding Mode Controller. In the same section, a simulation example is illus-

trated. Section 4 describes the synthesis of the new Predictive Sliding Mode Controller, with a simulation example. In section 5, the advantages of the presented controllers are verified by a non minimum phase system with disturbances and parameter variations. These results are compared with Sliding mode Controller and Model Predictive Controller. Finally, section 6 draws conclusions of the paper.

2. BASIC CONCEPTS

2.1 System description and preliminaries

Consider the following uncertain discrete time system [15]:

$$
\begin{aligned}
x(k+1) &= (A + \Delta A)\, x(k) + (B + \Delta B)\,(u(k) + v(k)) \\
y(k) &= Hx(k) + Du(k)
\end{aligned}
\tag{1}
$$

Where $x(k) \in R^n$ is the state vector and $u(k) \in R$ is the scalar input control. The matrices A, B, H and D are the nominal model matrices with adequate dimensions. The parameter uncertainties are given by ΔA and ΔB. $v(k) \in R$ is the disturbance input.
The system (1) can be written in the following form:

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) + w(k) \\
y(k) &= Hx(k) + Du(k)
\end{aligned}
\tag{2}
$$

with

$$
w(k) = \Delta Ax(k) + \Delta Bu(k) + (B + \Delta B)\,v(k)
\tag{3}
$$

2.2 Sliding mode control

In this section, we present the control law based on the sliding mode. The sliding function is defined as:

$$
s(k) = Cx(k)
\tag{4}
$$

where $C \in R^{1 \times n}$.
A necessary and sufficient condition assuring sliding motion and convergence to the sliding surface is:

$$
|s(k+1)| - |s(k)| < 0
\tag{5}
$$

According to sliding mode function (4), the sliding mode value at the instant $(k+1)$ can be obtained as:

$$
\begin{aligned}
s(k+1) &= Cx(k+1) \\
s(k+1) &= C\,[Ax(k) + Bu(k) + w(k)] \\
s(k+1) &= C\,[Ax(k) + Bu(k)] + Cw(k)
\end{aligned}
\tag{6}
$$

If there are no disturbances, the sliding function (4) value at time $(k+p)$ [17] is:

$$
\begin{aligned}
s(k+p) &= Cx(k+p) \\
s(k+p) &= CA^p x(k) + \sum_{j=1}^{p} CA^{j-1} Bu(k+p-j)
\end{aligned}
\tag{7}
$$

where $k \in Z$ and $p \in N$.
To obtain the desiral performances, such as strong-robustness, fast convergence and chattering elimination, we introduced a reaching law to ensure the convergence of the sliding function $s(k)$ to zero.
To ensure a quasi-sliding mode, the sliding mode function must verify the reaching law [15]:

$$
s(k+1) = s(k) - m\mathrm{sgn}(s(k))
\tag{8}
$$

where sgn is the signum function defined as:

$$
\mathrm{sgn}\,(s\,(k)) := \begin{cases} -1 & \text{if } s\,(k) < 0 \\ 1 & \text{if } s\,(k) > 0 \end{cases}
$$

and m is the discontinuous term magnitude.
Using the last equation and equation(6), we obtain:

$$
\begin{aligned}
s(k+1) - s(k) &= Cx(k+1) - s(k) \\
s(k+1) - s(k) &= CAx(k) + CBu(k) - s(k) \\
s(k+1) - s(k) &= s(k) - m\mathrm{sgn}(s(k)) - s(k) \\
s(k+1) - s(k) &= m\mathrm{sgn}(s(k))
\end{aligned}
$$

Then, the control law can be calculated as:

$$
u(k) = (CB)^{-1}[s(k) - CAx(k) - m\mathrm{sgn}(s(k))]
\tag{9}
$$

2.3 Model Predictive control

Model Based Predictive Control (MPC) has been successfully implemented in many industrial applications, showing a good performance [4].
MPC does not designate a specific control strategy but a very ample range of control methods, which make an explicit use of a model of the process to obtain the control signal by minimizing an objective function. The synthesis of predictive control law consist of three steps. The first step is to predict the process output at future time instants. The second step is to calculate control sequence minimizing an objective function. The third step is to consider that the control law is the first element of control sequence. The basic idea of MPC is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function [4, 22]:

$$
J(N', N, M) = \sum_{j=N'}^{N} q(j)\,[y_r(k+j) - y(k+j/k)]^2
$$
$$
+ \sum_{j=1}^{M} \lambda(j)\,[u(k+j-1)]^2
$$

where $y(k + j/k)$ is an optimum j-step ahead prediction of the system output on data up to time k, N' and N are the minimum and maximum costing horizon, M is the control horizon, $q(j)$ and $\lambda(j)$ are the weighting sequences, and $y_r(k+j)$ is the future reference trajectory.
The objective of MPC is to compute the future control sequence $u(k)$, $u(k+1)$,...in such a way that the future plant output $y(k+j)$ is driven close to $y_r(k+j)$ by minimizing the quadratic cost function $J(N', N, M)$ [4, 22].

3. PREDICTIVE SLIDING MODE CONTROLLER

3.1 Synthesis of Predictive Sliding Mode controller

In this section, we consider the sliding mode control problem for system (1), taking reaching law (8). The reference sliding mode trajectory is chosen as [17, 18]:

$$
\begin{cases} s_r(k+p) = s_r(k+p-1) - m\mathrm{sgn}(s_r(k+p-1)) \\ s_r(k) = s(k) \end{cases}
\tag{10}
$$

The objective is to design a sliding mode predictive control, using equation (6) and consider that $w(k)$ is equal to zero, the sliding function at instant $(k+p)$ can be written as:

$$
s(k+p) = CA^p x(k) + \sum_{j=1}^{p} CA^{j-1} Bu(k+p-j)
$$

where $k \in N$ and $p \in N$.
Therefore, the predictive sliding mode value of time k on time $(k-p)$ can be deduced:

$$
s(k/k-p) = CA^p x(k-p) + \sum_{j=1}^{p} CA^{j-1} Bu(k-j)
\tag{11}
$$

Equation (11) can be described in vector form as follows:

$$
S_p(k+1) = \Gamma x(k) + \Omega U(k)
\tag{12}
$$

where

$$S_p(k+1) = [s(k+1), s(k+2), ..., s(k+N)]^T$$
$$U(k) = [u(k), u(k+1), ..., u(k+M-1)]^T$$
$$\Gamma = \left[(CA)^T \ (CA^2)^T ... \left(CA^N\right)^T \right]^T$$

$$\Omega = \begin{bmatrix} CB & 0 & ... & ... & 0 \\ CAB & CB & ... & ... & 0 \\ . & . & ... & ... & . \\ . & . & ... & ... & . \\ CA^{M-1}B & CA^{M-2}B & ... & CAB & CB \\ . & . & ... & . & . \\ CA^{N-2}B & CA^{N-3}B & ... & CA^{N-M}B & CA^{N-M-1}B \\ CA^{N-1}B & CA^{N-2}B & ... & CA^{N-M+1}B & CA^{N-M}B \end{bmatrix}$$

With N is the prediction horizon, M is control horizon and the minimum costing horizon N' is chosen equal to 1.

In practice, to make correction to the future predictive sliding mode value $s(k+p)$, we introduce the error between practical sliding mode value $s(k)$ and predictive sliding mode value $s(k/k-p)$. Therefore, the output of sliding mode prediction $\tilde{s}_p(k+p)$ is given as follows:

$$\tilde{s}_p(k+p) = s(k+p) + h_p e(k)$$
$$\tilde{s}_p(k+p) = CA^p x(k) + \sum_{j=1}^{p} CA^{j-1} Bu(k+p-j) \quad (13)$$
$$+ h_p e(k)$$

Where $e(k) = s(k) - s(k/k-p)$ and h_p is a correct coefficient. Rewrite Equation (13) in vector form:

$$\tilde{S}_p(k+1) = S_p(k+1) + H_p E(k) \quad (14)$$

where

$$\tilde{S}_p(k+1) = [\tilde{s}_p(k+1), \tilde{s}_p(k+2), ..., \tilde{s}_p(k+N)]^T$$

$$H_p = diag[h_1, h_2, ..., h_N]$$

$$E(k) = S(k) - S_{mp}(k)$$

$$S(k) = [s(k), s(k), ..., s(k)]_{1 \times N}$$

$$S_{mp}(k) = [s(k/k-1), s(k/k-2), ..., s(k/k-N)]^T$$

The following corresponding optimization cost function is defined [19]:

$$j_p = \sum_{j=1}^{N} q_j [\tilde{s}_p(k+j) - s_r(k+j)]^2 + \sum_{l=1}^{M} g_l [u(k+l-1)]^2 \quad (15)$$

where $s_r(k+j)$ is the sliding mode reference trajectory, q_j and g_l are weight coefficients.

In order to simplify the synthesis of the controller, we consider $(q_j = 1)$ and $g_l = g$. So, The following corresponding optimization cost function is written as:

$$j_p = \sum_{j=1}^{N} [\tilde{s}_p(k+j) - s_r(k+j)]^2 + \sum_{l=1}^{M} g [u(k+l-1)]^2 \quad (16)$$

Rewrite Equation(16) in vector form:

$$J_p = \left\| \tilde{S}_p(k+1) - S_r(k+1) \right\| + \|U(k)\|_G^2 =$$
$$[\Gamma x(k) + \Omega U(k) + H_p E(k) - S_r(k+1)]^T [\Gamma x(k) \quad (17)$$
$$+ \Omega U(k) + H_p E(k) - S_r(k+1)] + U(k)^T GU(k)$$

where

$$S_r(k+1) = [s_r(k+1), s_r(k+2), ..., s_r(k+N)]^T$$

$$G = [g, g, ..., g]$$

The optimal sequence of control is obtained by minimizing the cost function J_p:

$$\frac{\partial J_p}{\partial U(k)} = 0$$

Then, this sequence can be calculated as:

$$U(k) = -(\Omega^T \Omega + G)^{-1} \Omega^T [\Gamma x(k) + H_p E(k) - S_r(k+1)] \quad (18)$$

Because of rolling optimization, only the present control input signal is implemented, the next time control signal $u(k+1)$ will be calculated recursively by the control law.

3.2 Simulation example

The behavior, of the PSMC is illustrated by a simulation example. Let's consider the following system [17]:

$$x(k+1) = Ax(k) + Bu(k) + w(k)$$

where:

$$A = \begin{bmatrix} 1 & 0.01 \\ -0.651 & 0.797 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0.731 \end{bmatrix}$$

The sampling period T is chosen, according to the system's dynamics, equal to $0.01s$.
For the Predictive Sliding Mode controller, choosing $G = 0.001 * I_{M \times M}$. C is designed as $C = \begin{bmatrix} 3 & 1 \end{bmatrix}$ and setting the initial period values $x(0) = \begin{bmatrix} 1 & 0.5 \end{bmatrix}$.
Select the predictive horizon $N = 10$, the control horizon $M = 5$ and the correct coefficient matrix:
$H_p = diag \begin{bmatrix} 1 & 0.8 & 0.6 & 0.5 & 0.3 & 0.2 & 0.2 & 0.4 & 0.1 & 0.5 \end{bmatrix}$
Satisfying the robustness condition, the discontinuous term magnitude m is chosen as follow:

$$\begin{cases} m = 3 \quad \text{if} \quad 48 < k < 92 \\ m = 0.02 \quad \text{else} \end{cases}$$

The results presented in this section are obtained with the presence of disturbances, whose evolution is given in figure 1, and parameters variation are given by:

$$\Delta A = 0.5 \begin{bmatrix} 0 & 0 \\ 3\sin(-\frac{2k\pi}{10}) & 3\sin(-\frac{2k\pi}{10}) \end{bmatrix}$$

$$\Delta B = 0.5 \begin{bmatrix} 0 \\ \sin(-\frac{2k\pi}{10}) \end{bmatrix}$$
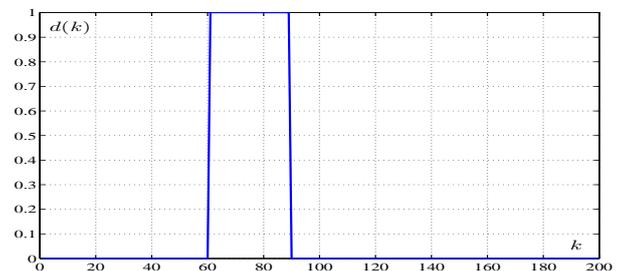
Fig. 1. Evolutions of disturbances.

The simulation results of PSMC are illustrated in figure 2 to figure 5. These figures are a comparaison between the results given by PSMC, SMC and MPC. In fact, figure 2 and 3 show the evolution of the state $x_1(k)$ and $x_2(k)$. Figure 4 presents the evolution of the sliding surface. The evolution of the input $u(k)$ is presented in figure 5.
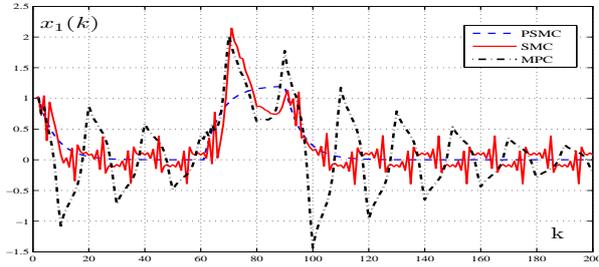
Fig. 2. Evolutions of the state $x_1(k)$.
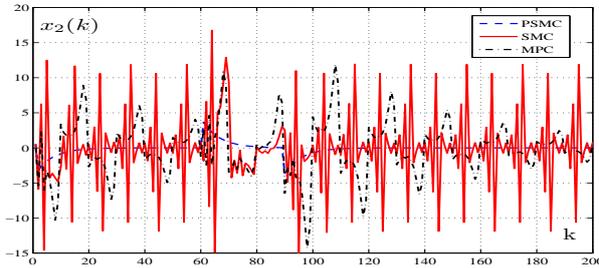
Fig. 3. Evolutions of the state $x_2(k)$.
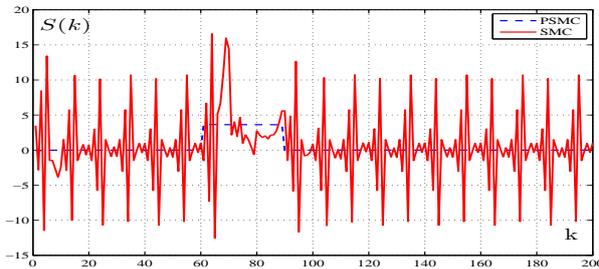
Fig. 4. Evolutions of the sliding function.

These figures prove that the control law, given by Predictive Sliding Mode Controller, can eliminate the chattering and force the states to converge to zero better than Sliding Mode Controller and Model Predictive Controller. But, it is still not able to reject the constant disturbances perfectly.

4. NEW PREDICTIVE SLIDING MODE CONTROLLER

4.1 Synthesis of New Predictive Sliding Mode controller

To ameliorate the performances in term of rejecting constant and periodic disturbances, we propose in this section a New Predictive Sliding Mode Controller (NPSMC).
The optimal control problem is calculated, now, using a cost
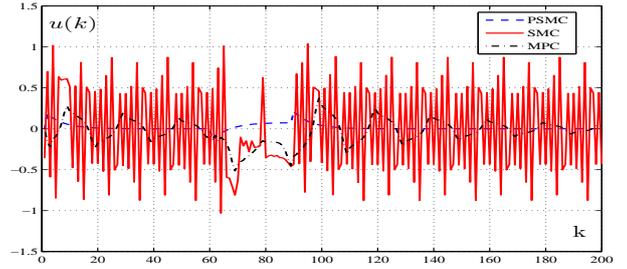
Fig. 5. Evolutions of the control input $u(k)$.

function penalizing deviation of the controlled variables as well as variations in the control signal. So the NPSMC block diagram can be represented as follow: The objective is to design the
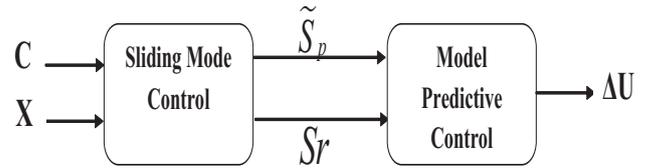
Fig. 6. NPSM Controller block diagram.

sliding mode predictive control, or we have :

$$s(k+p) = CA^p x(k) + \sum_{j=1}^{p} CA^{j-1}Bu(k+p-j)$$

where $k \in Z$ and $p \in N$.
The sliding function at the instance $k+1$, $k+2$ and $k+3$ can be written as:

$$s(k+1) = Cx(k+1)$$
$$= \text{CAx(k)} + \text{CB(u(k)} - \text{u(k}-1)) + \text{CBu(k}-1)$$
$$= \text{CAx(k)} + \text{CB}\delta\text{u(k)} + \text{CBu(k}-1)$$

$$s(k+2) = Cx(k+2)$$
$$= \text{CAx(k}+1) + \text{CBu(k}+1)$$
$$= \text{CA}^2 x(k) + CB\delta u(k+1) + CAB\delta u(k) + \text{CB}\delta\text{u(k)}$$
$$+ \text{CBu(k}-1) + \text{CABu(k}-1)$$
$$= \text{CA}^2 x(k) + CB\delta u(k+1) + C(A+1)B\delta u(k)$$
$$+ C(A+1)Bu(k-1)$$

$$s(k+3) = Cx(k+3)$$
$$= \text{CA}\left[A\left[Ax(k) + Bu(k)\right]\right] + CABu(k+1) + CBu(k+2)$$
$$= \text{CA}^3 x(k) + CB\delta u(k+2) + C(A+1)B\delta u(k+1)$$
$$+ C(A^2 + A + 1)B\delta\text{u(k)} + C(A^2 + A + 1)Bu(k-1)$$

Then, $s(k+p)$ can be calculated as:

$$s(k+p) = CA^p x(k) + CB\delta u(k+p-1)$$
$$+ C(A+1)\delta u(k+p-2) + \cdots + C\left[\sum_{j=0}^{p-1}A^j\right]B\delta u(k)$$
$$+ C\left[\sum_{j=0}^{p-1}A^j\right]Bu(k-1)$$

$$(19)$$

where $\delta u(k) = u(k) - u(k-1)$ Equation (19) can be described in vector form as follows:

$$S_p(k+1) = \Gamma x(k) + \Omega^F \Delta U(k) + \Omega^P u(k-1) \quad (20)$$

where

$$S_p(k+1) = [s(k+1), s(k+2), ..., s(k+N)]^T$$
$$S_p(k) = [s(k), s(k+1), ..., s(k+N-1)]^T$$
$$\Delta U(k) = [\delta u(k), \delta u(k+1), ..., \delta u(k+M-1), 0, ..., 0]^T$$
$$\Gamma = \left[(CA)^T \; (CA^2)^T \; ... \; (CA^N)^T \right]^T$$

$$\Omega^F = \begin{bmatrix} CB & 0 & ... & ... & 0 \\ C(A+I)B & CB & ... & ... & 0 \\ . & . & ... & ... & . \\ . & . & ... & ... & . \\ C\left[\sum_{j=0}^{M-1} A^j\right]B & ... & ... & C(A+I)B & CB \\ . & . & ... & & \\ C\left[\sum_{j=0}^{N-1} A^j\right]B & ... & ... & C\left[\sum_{j=0}^{N-M-1} A^j\right]B & C\left[\sum_{j=0}^{N-M} A^j\right]B \end{bmatrix}$$

With N is prediction horizon, M is control horizon and the minimum costing horizon N' is chosen equal to 1.
and

$$\Omega^P = \begin{bmatrix} CB \\ \vdots \\ C\left[\sum_{j=0}^{M-1} A^j\right]B \\ \vdots \\ C\left[\sum_{j=0}^{N-1} A^j\right]B \end{bmatrix}$$

In practice, to make correction to the future predictive sliding mode value $s(k+p)$, we introduce the error between the practical sliding mode value $s(k)$ and the predictive sliding mode value $s(k/k-p)$. Therefore, the output of sliding mode prediction $\tilde{s}_p(k+p)$ is given as follows:

$$\tilde{s}_p(k+p) = s(k+p) + h_p e(k)$$
$$\tilde{s}_p(k+p) = CA^p x(k) + CB\delta u(k+p-1)$$
$$+C(A+1)\delta u(k+p-2) + \cdots + C\left[\sum_{j=0}^{p-1} A^j\right]B\delta u(k)$$
$$+C\left[\sum_{j=0}^{p-1} A^j\right]Bu(k-1) + h_p e(k)$$

(21)

Where $e(k) = s(k) - s(k/k-p)$ and h_p is a correct coefficient.
Rewrite Equation (21) in vector form:

$$\tilde{S}_p(k+1) = S_p(k+1) + H_p E(k) \tag{22}$$

where

$$\tilde{S}_p(k+1) = [\tilde{s}_p(k+1), \tilde{s}_p(k+2), ..., \tilde{s}_p(k+N)]^T$$

$$H_p = diag[h_1, h_2, ..., h_N]$$

$$E(k) = S(k) - S_{mp}(k)$$

$$S(k) = [s(k), s(k), ..., s(k)]_{1 \times N}$$

$$S_{mp}(k) = [s(k/k-1), s(k/k-2), ..., s(k/k-N)]^T$$

The following corresponding optimization cost function is defined:

$$j_p = \sum_{j=1}^{N} q_j \left[\tilde{s}_p(k+j) - s_r(k+j)\right]^2 + \sum_{l=1}^{M} g_l \left[\delta u(k+l-1)\right]^2$$

(23)

where $s_r(k+1)$ is the sliding mode reference trajectory, q_j and g_l are weight coefficients.
In order to simplify the synthesis of the controller, we consider $(q_j = 1)$ and $g_l = g$. So, The following corresponding optimization cost function is written by:

$$j_p = \sum_{j=1}^{N} \left[\tilde{s}_p(k+j) - s_r(k+j)\right]^2 + \sum_{l=1}^{M} g \left[\delta u(k+l-1)\right]^2$$

(24)

Rewrite Equation(24) in vector form:

$$\begin{aligned} J_p &= \left\| \tilde{S}_p(k+1) - S_r(k+1) \right\|^2 + \|\Delta U(k)\|_G^2 \\ &= [\Gamma x(k) + \Omega^F \Delta U(k) + \Omega^P u(k-1) + H_p E(k) \\ &\quad -S_r(k+1)]^T [\Gamma x(k) + \Omega^F \Delta U(k) + \Omega^p u(k-1) \\ &\quad +H_p E(k) - S_r(k+1)] + \Delta U(k)^T G \Delta U(k) \end{aligned}$$

(25)

where

$$S_r(k+1) = [s_r(k+1), s_r(k+2), ..., s_r(k+N)]^T$$

$$G = [g, g, ..., g]$$

Let $\frac{\partial J_p}{\partial \Delta U(k)} = 0$, the optimal law can be obtained:

$$\Delta U(k) = -((\Omega^F)^T \Omega^F + G)^{-1} (\Omega^F)^T [\Gamma x(k) + H_p E(k) - S_r(k+1)]$$

(26)

Because of rolling optimization, only the present increment of control input signal is implemented, the next time increment of control signal $u(k+1)$ will be calculated recursively by the control law.

$$\delta u(k) = [1, 0, ...0]^T \Delta U(k) \tag{27}$$

So, we have:

$$u(k) = u(k-1) + \delta u(k) \tag{28}$$

4.2 Simulation example

To improve the behavior of the NPSMC, we kept the same simulation example of the last section. Compared to SMC and MPC , the simulation results of NPSMC are illustrated in figure 7 to figure 10.
In fact, figure 7 and 8 show, respectively, the evolution of the states $x_1(k)$ and $x_2(k)$. Figure 9 presents the evolution of the sliding surface. The evolution of the input $u(k)$ is presented in figure 10.
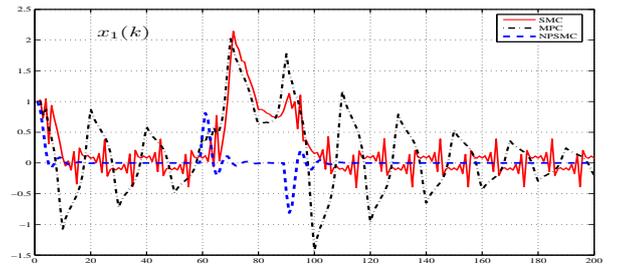
Fig. 7. Evolutions of the state $x_1(k)$.

Then, the application of the new strategy gives a very satisfactory performances in terms of convergence, eliminating chattering, and rejecting constants disturbances.
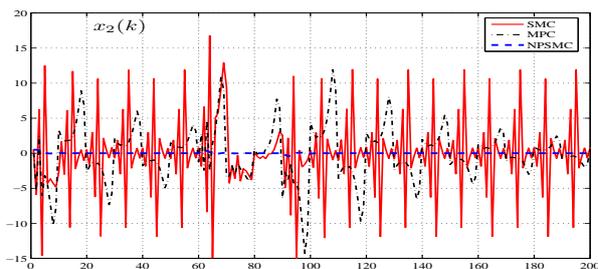
Fig. 8. Evolutions of the state $x_2(k)$.
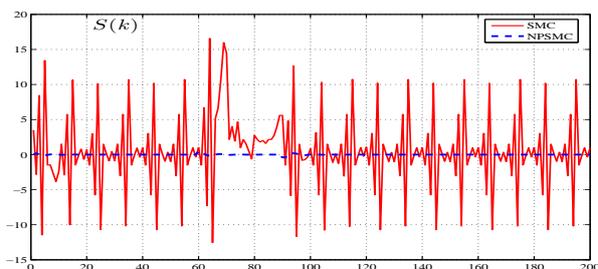
Fig. 9. Evolutions of the sliding function.
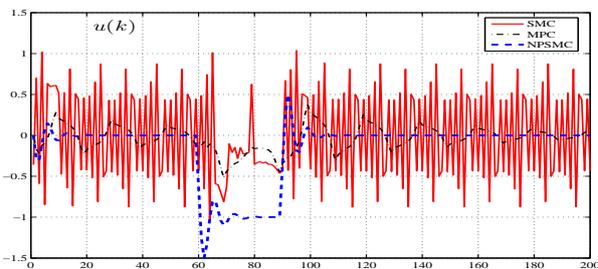
Fig. 10. Evolutions of the control input $u(k)$.

5. SIMULATION RESULTS

To evaluate the robustness of the two control laws equations (18) and equation (26), we consider the isothermal Van de Vussen systems [11]. this system involves series and parallel reactions and is governed by the following equations:

$$\frac{C_a}{dt} = -k_1 C_a - k_3 C_a^2 + (C_{ain} - C_a)\frac{F}{V}$$
$$\frac{C_b}{dt} = k_1 C_a - k_2 C_b \frac{F}{V} \qquad (29)$$

The desired output is the concentration of B, C_b [mol/l], C_a and $C_a in$ are the concentrations of A [mol/l] in the reactor and in the feed respectively, the manipulate input, F is the dilution rate [l/min], V is the volume [l], and the rate constants are given by $k_1 = 5/6[min^{-1}]$, $k_2 = 5/3[min^{-1}]$, $k_3 = 1/6[mol/litermin]$ [11].

Above no linear, non minimum phase process has been used to show the controller performance. After linearizing model (29) about the operating point the physical model gives the following transfer function (30). The discretization has been made with ampling rate $T_e = 0.2$

$$y(k) = \frac{-0.0939 + 0.1745z^{-1}}{1 - 1.2573z^{-1} + 0.3951z^{-2}} u(k-1) \qquad (30)$$

The state space representation of the isothermal Van de Vussen systems is given as follows:

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Hx(k) + Du(k)$$

where:

$$A = \begin{bmatrix} 1.2573 & -0.7902 \\ 0.5 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

$$H = \begin{bmatrix} -0.1878 \\ 0.698 \end{bmatrix} \quad D = [-0.0939]$$

The retained synthesis parameters are: $m = 1.5$, $N = 10$, $M = 5$, $C = \begin{bmatrix} 1 & -1 \end{bmatrix}$, $G = 0.01 I_{N \times N}$, $x(0) = \begin{bmatrix} 3 & 1 \end{bmatrix}$,

$$H_p = diag \begin{bmatrix} 1 & 0.8 & 0.6 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.4 & 0.1 & 0.05 \end{bmatrix}$$

5.1 Case of constant disturbances

The results presented in this section are obtained with the presence of disturbances, whose evolution is given in figure 11. The parameters variation are given by:

$$\Delta A = 0.1 \begin{bmatrix} 5\sin(-\frac{2k\pi}{10}) & 6\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 3\sin(-\frac{2k\pi}{10}) \end{bmatrix}$$

$$\Delta B = 0.1 \begin{bmatrix} 2\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) \end{bmatrix} \quad ; \quad k \geq 300$$
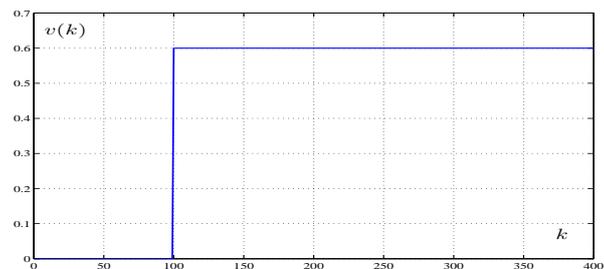
Fig. 11. Evolutions of disturbances.

The comparaison between the NPSMC, PSMC, SMC and MPC are given in figures 12 to 15. Figure 12 and figure 13 illustrate respectively the evolution of states $x_1(k)$ and $x_2(k)$. The evolution of the sliding function is shown in figure 14. Finally, figure 15 shows the evolution of the control input $u(k)$.
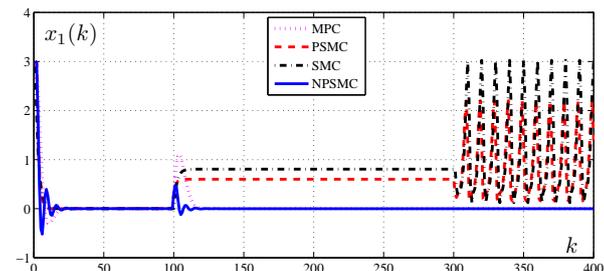
Fig. 12. Evolutions of the state $x_1(k)$.

It can be seen that the performances of NPSMC are better than all the other studied controller, not only, in rejecting constant
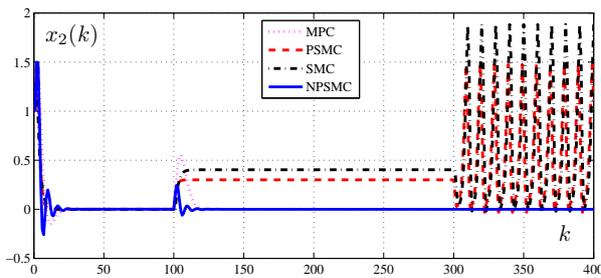
Fig. 13. Evolutions of the state $x_2(k)$.

Fig. 14. Evolutions of the sliding function.

Fig. 15. Evolutions of the control input $u(k)$.
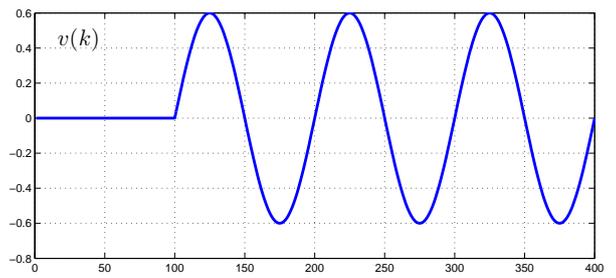
Fig. 16. Evolutions of disturbances.
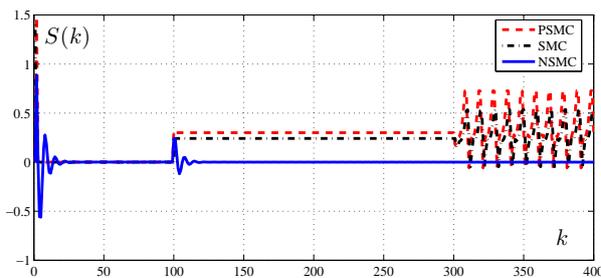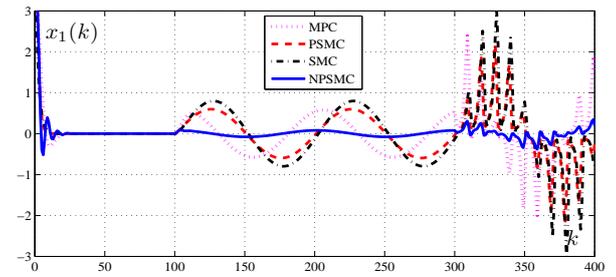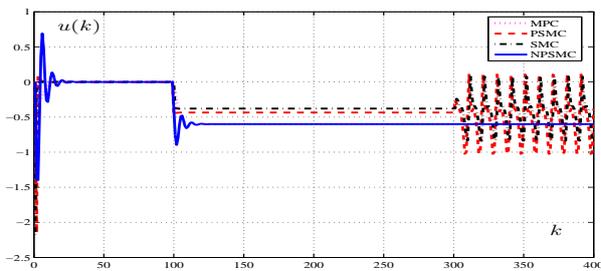
Fig. 17. Evolutions of the state $x_1(k)$.
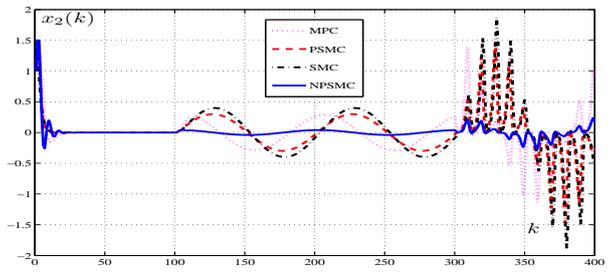
Fig. 18. Evolutions of the state $x_2(k)$.

disturbances, hard parameters variation, but also, in eliminating chattering.

5.2 Case of periodic disturbances

The results presented in this section are obtained with the presence of disturbances, whose evolution is given in figure 16, and parameters variation are given by:

$$\Delta A = 0.1 \begin{bmatrix} 5\sin(-\frac{2k\pi}{10}) & 6\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) & 3\sin(-\frac{2k\pi}{10}) \end{bmatrix}$$

$$\Delta B = 0.1 \begin{bmatrix} 2\sin(-\frac{2k\pi}{10}) \\ 3\sin(-\frac{2k\pi}{10}) \end{bmatrix} \quad ; \quad k \geq 300$$

The states response, the sliding mode function and the control input, with PSMC, NPSMC, SMC, and MPC are given, respectively, in Figures 17 to 20.
.
A comparaison between the NPSMC, PSMC, SMC and MPC reveals that the use of the new control strategy (NPSMC) reduce the chattering problem effectively ($k \geq 300$).
Furthermore, the results obtained prove the capability of the proposed control law to reject periodic disturbances.

6. CONCLUSION

The predictive sliding mode controller, presented in this paper, combines the design technique of SMC and MPC. A predictive sliding mode control strategy is proposed and a discrete-time reaching law is improved by applying a predictive sliding surface and a reference trajectory. It is shown that mixing both control techniques gives a new controller with a better robustness properties. To ameliorate the performances of the PSMC, we introduce the NPSMC whose performance was judged using a non minimum phase system. In fact, The New Predictive Sliding Mode controller can guarantee desired performance, such as chattering elimination, fast convergence, strong robustness to constant and periodic disturbances and parameters variation.

7. ACKNOWLEDGEMENT

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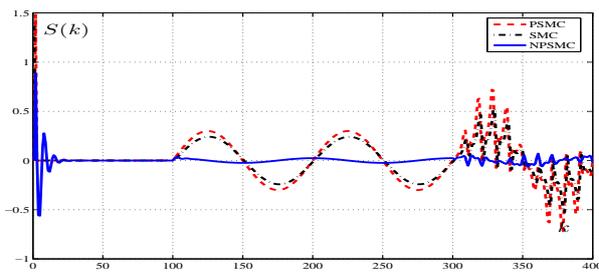
Fig. 19.   Evolutions of the sliding function.
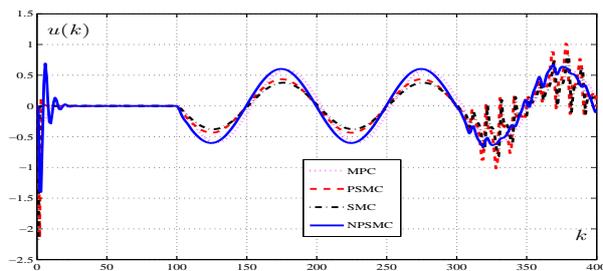
Fig. 20.   Evolutions of the control input $u(k)$**.**

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