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On Reliability of $(n+1)$-unit Warm Standby System based on Imperfect Repair Facility and Two Types of Failures

M. A. W. Mahmoud
Mathematics Department,
Faculty of Science, Al-Azhar University,
Nasr City (11884), Cairo, Egypt.

M. A. Attallah
Department of Statistics,
Faculty of Science,
King Abdul Aziz University, Jeddah, SA.

A. Alharby
Department of Statistics,
Faculty of Science,
King Abdul Aziz University, Jeddah, SA.

ABSTRACT

This paper deals with an $(n + 1)$-unit warm standby system based on imperfect repair facility and two types of failures. These types of failure are hardware and human error failures. Various measures of the system reliability are obtained using the regenerative point technique. Finally a numerical example is presented to illustrate the theoretical results.

General Terms:

Reliability and Economic Measures

Keywords:

reliability, warmstandby, humanerror failure, hardware failure, availability, cost benefit.

1. INTRODUCTION

The theory of reliability is one of the most important branches of operations research and system engineering. Any system is analyzed in order to be complete, must give due consideration to system reliability. With remarkable advances made in electronics engineering, military and communication systems have become more sophisticated and when such systems fail, very serious situations aries. Thus in the present day context, high system reliability has become very important from the viewpoint of both makers and users.

A system designer is faced often with problems of determining the various system measures like reliability, availability $\cdots$ etc. He also has to suggest ways that improve the efficiency of a given system. Due to the nature of the subject, the methods of probability theory and mathematical statistics are necessary to study and solve the problems that arise in reliability theory.

Many papers [1, 9–12, 14, 18] studied the reliability of the two-unit standby system from various points of view. In all these papers, the authors did not take into their account the human error failures in spite of $20-30\%$ of failures are due to human error (see Mister [17]).

In addition, many papers [2–8], [13], [15, 16] dealt with the reliability of some standby systems are subject to the human error failures.

As we know, there is no papers in the literature deal with $(n+1)$-unit standby system in which n units are initially in operation and the other unit is kept as a standby unit. Thus the aim of this paper is to bridge a gab of analyzing $(n + 1)$-unit warm standby system based on imperfect repair facility and two types of failures. Various measures of the system reliability such as mean time to system failure, steady-state availability, steady-state busy period and cost benefit analysis, are derived based on regenerative point technique.

Finally, numerical example is given to illustrate the theoretical results.

2. ASSUMPTIONS

The following assumptions are associated with the proposed model.

1- The system consists of $n + 1$ identical units.
2- Initially n units begin operation and the other unit is kept as warm standby.
3- The switch is perfect and instantaneous. (i.e. it is not fail and the time spend to put the standby in operating is negligible).
4- The operative units suffer two types of failures namely, hardware and human error failures, while standby unit suffers only one type of failure.
5- There is one repair facility one serverman which it is available with probability P.
6- After repair the unit is as good as new.
7- All random variables are independent and exponentially distributed
8- The system fails when there are $(n-1)$ operating only.

3. NOTATIONS

$\lambda_i,\ i = 1, 2$: failure rates of hardware and human error respectively
$\nu_i,\ i = 1, 2$: repair rates of repair for hardware and human error failures respectively
γ : failure rate of the standby unit
η : repair rate of the standby unit
$1 - \exp(\lambda_i t), i = 1, 2$: CDF of hardware and human error failures respectively

$1-\exp(-\nu_i t), i = 1, 2$: CDF of repair due to hardware and human error respectively
$1-\exp(-\gamma t)$: CDF of standby failure time
$1-\exp(-\eta t)$: CDF of repair time of the standby unit
$1-\exp(-\xi t)$: CDF of waiting time for repair facility to be active
E_0 : state of the system at $t = 0$
$\underline{E}$: set of regenerative states
$\overline{E}$: set of non-regenerative states
P : probability that facility the repair is available
$q_{i,j}(t)$: PDF of time for the system transition from regenerative state S_i to S_j
$Q_{ij}(t)$: CDF of time for the system transition from regenerative state S_i to S_j
$q_{ij}^{(k)}(t)$: PDF of time for the system transition from regenerative state S_i to S_j via state S_k
$q_{ij}^{(k,I)}(t)$: PDF of time for the system transition from regenerative state S_i to S_j via two non-regenerative state S_k and S_I
μ_i : $\int P$ (System sojourns in states S_i of the set E for at least time t) dt
$M_i(t)$: P (System is up initially in state S_i of the set E is up at time T without passing through any other regenerative state or returning to itself through on or more states of E)
$AV_i(t)$: P (system is up to time $t|E_0 = S_i$ of E)
$B_i^1(t)$: P (the serverman is busy with repair due to hardware failure)
$B_i^2(t)$: P (the serverman is busy with repair due to haman error failure)
$B_i^3(t)$: P (the serverman is busy with standby repair)
$\Pi_i(t)$: CDF of time to system failure starting from state S_i
u : dummy variable in Laplase transform (LT)
$*$: symbol for LT
© : symbol for convolution
St : unit is in standby case
r_i, $(i = 1, 2)$: unit is under repair due to hardware failure and human error failure respectivey
R : repair continued from earlier state
Sr : the standby unit is under repair
SR : the repair of standby unit continued from earlier state
Wr : the unit is waiting for repair
SWr : the standby unit is waiting for repair
rfg : the repair facility is available
rfb : the repair facility is not available
WR : the unit is waiting for repair from earlier state
SWR : the standby unit is waiting for repair from earlier state
O_n : n units are operating.
The proposed system can be in one of the following states: $S_0 \equiv (O_n, St)$, $S_1 \equiv (O_n, r_1, rfg)$, $S_2 \equiv (O_n, r_2, rfg)$, $S_3 \equiv (O_n, Sr, rfg)$, $S_4 \equiv (O_0, Wr_1, rfb)$, $S_5 \equiv (O_n, Wr_2, rfb)$, $S_6 \equiv (O_n, SWr, rfb)$, $S_7 \equiv (O_{n-1}, R_1, wr_1)$, $S_8 \equiv (O_{n-1}, R_1, wr_2)$, $S_9 \equiv (O_{n-1}, R_2, wr_1)$, $S_{10} \equiv (O_{n-1}, R_2, wr_2)$, $S_{11} \equiv (O_{n-1}, WR_1, wr_1)$, $S_{12} \equiv (O_{n-1}, WR_1, wr_2)$, $S_{13} \equiv (O_{n-1}, WR_2, wr_1)$, $S_{14} \equiv (O_{n-1}, WR_2, wr_2)$, $S_{15} \equiv (O_{n-1}, SR, wr_1)$, $S_{16} \equiv (O_{n-1}, SR, wr_2)$, $S_{17} \equiv (O_{n-1}, SWR, wr_1)$, $S_{18} \equiv (O_{n-1}, SWR, wr_2)$.

REMARK 1. *1. The states $S_0 - S_6$ are regenerative states.*

2. The states $S_7 - S_{18}$ are non-regenerative states.

4. THE TRANSITION PROBABILITIES

It can be observed that the points of entry into any of the states S_i of the set E are regenerative points. Let $T_0(\equiv 0), T_1, T_2, \cdots$ denote the epochs at which the system enters any state S_i of E and let X_n denote the state visited at epoch T_n+ (i.e. just after transition at T_n). It is easy to see that $\{X_n, T_n\}$ is a Markov renewal process with state space E and

$$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = t],$$

is the semi Markov kernel over E.
The matrix of transition probabilities s given by

$$P = (P_{ij}) = (Q_{ij}(\infty)) = Q(\infty),$$

with nonzero elements. It is easy to obtain

$$P_{01} = \frac{np\lambda_1}{n\lambda_1 + n\lambda_2 + \gamma}.$$

Similarly

$$P_{02} = \frac{np\lambda_2}{n\lambda_1+n\lambda_2+\gamma},\ P_{03} = \frac{p\gamma}{n\lambda_1+n\lambda_2+\gamma},\ P_{04} = \frac{nq\lambda_1}{n\lambda_1+n\lambda_2+\gamma},$$
$$P_{05} = \frac{nq\lambda_2}{n\lambda_1+n\lambda_2+\gamma},\ P_{06} = \frac{q\gamma}{n\lambda_1+n\lambda_2+\gamma},\ P_{17} = \frac{n\lambda_1}{n\lambda_1+n\lambda_2+\gamma_1}.$$

$$\begin{array}{ll}
P_{18} = \frac{n\lambda_2}{n\lambda_1+n\lambda_2+\gamma_1}, & P_{10} = \frac{\gamma_1}{n\lambda_1+n\lambda_2+\gamma_1}, \\
P_{11}^{(7)} = \frac{n\lambda_1}{n\lambda_1+n\lambda_2+\gamma_1} = P_{17}, & P_{12}^{(8)} = \frac{n\lambda_2}{n\lambda_1+n\lambda_2+\gamma_1} = P_{18}, \\
P_{20} = \frac{\gamma_2}{n\lambda_1+n\lambda_2+\gamma_2}, & P_{29} = \frac{n\lambda_1}{n\lambda_1+n\lambda_2+\gamma_2}, \\
P_{2,10} = \frac{n\lambda_2}{n\lambda_1+n\lambda_2+\gamma_2}, & P_{21}^{(9)} = \frac{n\lambda_1}{n\lambda_1+n\lambda_2+\gamma_2} = P_{29}, \\
P_{22}^{(10)} = \frac{n\lambda_2}{n\lambda_1+n\lambda_2+\gamma_2} = P_{2,10}, & P_{3,15} = \frac{n\lambda_1}{n\lambda_1+n\lambda_2+\eta}, \\
P_{3,16} = \frac{n\lambda_2}{n\lambda_1+n\lambda_2+\eta}, & P_{3,0} = \frac{\eta}{n\lambda_1+n\lambda_2+\eta}, \\
P_{31}^{(15)} = \frac{n\lambda_1}{n\lambda_1+n\lambda_2+\eta} = P_{3,15}, & P_{32}^{(16)} = \frac{n\lambda_2}{n\lambda_1+n\lambda_2+\eta} = P_{3,16}, \\
P_{41} = \frac{\xi}{n\lambda_1+n\lambda_2+\xi}, & P_{4,11} = \frac{n\lambda_1}{n\lambda_1+n\lambda_2+\xi}, \\
P_{4,12} = \frac{n\lambda_2}{n\lambda_1+n\lambda_2+\xi}, & P_{41}^{(11,7)} = \frac{n\lambda_1}{n\lambda_1+n\lambda_2+\xi} = P_{4,11}, \\
P_{42}^{(12,8)} = \frac{n\lambda_2}{n\lambda_1+n\lambda_2+\xi} = P_{4,12}, & P_{52} = \frac{\xi}{n\lambda_1+n\lambda_2+\xi} = P_{41}, \\
P_{5,13} = P_{4,11}, & P_{5,14} = P_{4,12}, \\
P_{51}^{(13,9)} = P_{42}^{(12,8)}, & P_{6,3} = P_{4,1}, \\
P_{6,17} = P_{4,11}, & P_{6,18} = P_{4,12}, \\
P_{61}^{(17,15)} = P_{6,17} = P_{4,11}, & P_{62}^{(18,16)} = P_{6,18} = P_{4,12}.
\end{array}$$

5. THE MEAN SOJOURN TIMES

One can show that

$$\mu_0 = \frac{1}{n\lambda_1 + n\lambda_2 + \gamma}, \quad (1)$$

$$\mu_1 = \frac{1}{n\lambda_1 + n\lambda_2 + \mu_1}, \quad (2)$$

$$\mu_2 = \frac{1}{n\lambda_1 + n\lambda_2 + \mu_2}, \quad (3)$$

$$\mu_3 = \frac{1}{n\lambda_1 + n\lambda_2 + \eta}, \quad (4)$$

$$\mu_4 = \mu_5 = \mu_6 = \frac{1}{n\lambda_1 + n\lambda_2 + \xi}. \quad (5)$$

6. THE MEAN TIME TO SYSTEM FAILURE

In this section, we derive the mean time to failure MTTF. Upon using the probability theory we get

$$\overline{\Pi}_0(t) = e^{-(n\lambda_1+n\lambda_2+\gamma)t} + \sum_{i=1}^{6} q_{0i}(t)©\overline{\Pi}_1(t), \quad (6)$$

$$\overline{\Pi}_1(t) = e^{-(n\lambda_1+n\lambda_2+\nu_1)t} + q_{10}(t)©\overline{\Pi}_0(t), \quad (7)$$

$$\overline{\Pi}_2(t) = e^{-(n\lambda_1+n\lambda_2+\nu_2)t} + q_{20}(t)©\overline{\Pi}_0(t), \quad (8)$$

$$\overline{\Pi}_3(t) = e^{-(n\lambda_1+n\lambda_2+\eta)t} + q_{30}(t)©\overline{\Pi}_0(t), \tag{9}$$

$$\overline{\Pi}_4(t) = e^{-(n\lambda_1+n\lambda_2+\xi)t} + q_{41}(t)©\overline{\Pi}_1(t), \tag{10}$$

$$\overline{\Pi}_5(t) = e^{-(n\lambda_1+n\lambda_2+\xi)t} + q_{52}(t)©\overline{\Pi}_2(t), \tag{11}$$

$$\overline{\Pi}_6(t) = e^{-(n\lambda_1+n\lambda_2+\xi)t} + q_{63}(t)©\overline{\Pi}_3(t). \tag{12}$$

Taking Laplace transform of both sides of the system (6)-(12), setting $u = 0$ and solving for $\overline{\Pi}_0^*(0)$ yields

$$\overline{\Pi}_0^*(0) = MTTF = \frac{L_2}{L_1}, \tag{13}$$

$$\begin{aligned} L_1 =& 1 - P_{01}P_{10} - P_{02}P_{20} - P_{03}P_{30} - P_{04}P_{41}P_{10} \\ & - P_{05}P_{52}P_{20} - P_{06}P_{63}P_{30}, \end{aligned}$$

$$L_2 = \mu_0 + a_1\mu_1 + a_2\mu_2 + a_3\mu_3 + P_{04}\mu_4 + P_{05}\mu_5 + P_{06}\mu_6,$$

where

$$a_1 = P_{01} + P_{04}P_{41}, \; a_2 = P_{02} + P_{05}P_{52}, \; a_3 = P_{03} + P_{06}P_{63}.$$

7. SYSTEM AVAILABILITY

Using probabilistic arguments, gives

$$AV_0(t) = M_0(t) + \sum_{i=1}^{6} q_{0i}(t)©AV_i(t), \tag{14}$$

$$\begin{aligned} AV_1(t) =& M_1(t) + q_{10}(t)AV_0(t) + q_{11}^{(7)}(t)AV_1(t) \\ & + q_{12}^{(8)}(t)AV_2(t), \end{aligned} \tag{15}$$

$$\begin{aligned} AV_2(t) =& M_2(t) + q_{20}(t)AV_0(t) + q_{21}^{(9)}(t)AV_1(t) \\ & + q_{22}^{(10)}(t)AV_2(t), \end{aligned} \tag{16}$$

$$\begin{aligned} AV_3(t) =& M_3(t) + q_{30}(t)AV_0(t) + q_{31}^{(15)}(t)AV_1(t) \\ & + q_{32}^{(16)}(t)AV_2(t), \end{aligned} \tag{17}$$

$$\begin{aligned} AV_4(t) =& M_4(t) + q_{41}(t)AV_1(t) + q_{41}^{(11,7)}(t)AV_1(t) \\ & + q_{42}^{(12,8)}(t)AV_2(t), \end{aligned} \tag{18}$$

$$\begin{aligned} AV_5(t) =& M_5(t) + q_{52}(t)AV_2(t) + q_{51}^{(13,9)}(t)AV_1(t) \\ & + q_{52}^{(14,10)}(t)AV_2(t), \end{aligned} \tag{19}$$

$$\begin{aligned} AV_6(t) =& M_6(t) + q_{63}(t)AV_3(t) + q_{61}^{(17,15)}(t)AV_1(t) \\ & + q_{62}^{(18,16)}(t)AV_2(t), \end{aligned} \tag{20}$$

where

$$\begin{gathered} M_0(t) = \exp[-(n\lambda_1 + n\lambda_2 + \gamma)t], \\ M_1(t) = \exp[-(n\lambda_1 + n\lambda_2 + \mu_1)t], \\ M_2(t) = \exp[-(n\lambda_1 + n\lambda_2 + \mu_2)t], \\ M_3(t) = \exp[-(n\lambda_1 + n\lambda_2 + \eta)t], \\ M_4(t) = \exp[-(n\lambda_1 + n\lambda_2 + \xi)t] = M_5(t) = M_6(t). \end{gathered}$$

Taking Laplace transform of both sides of Eqs. (14)-(20) and solving for $AV_0^*(u)$, the steady-state availability of the system AV_0 can be given by

$$AV_0 = \lim_{t\to\infty} AV_0(t) = \lim_{u\to 0} uAV_0^*(u) = \frac{N_0}{D_0},$$

where

$$\begin{aligned} N_0 =& \mu_0 + \mu_1P_{01} + \mu_2P_{02} + \mu_3P_{03} + \mu_4P_{04} + \mu_5P_{05} + \mu_6P_{06} \\ & - \mu_0P_{17} - \mu_2P_{02}P_{17} - \mu_3P_{03}P_{17} - \mu_4P_{04}P_{17} - \mu_5P_{05}P_{17} \\ & - \mu_6P_{06}P_{17} + \mu_2P_{01}P_{18} - \mu_0P_{2,10} - mu_1P_{01}P_{2,10} \\ & - \mu_3P_{03}P_{2,10} - \mu_4P_{04}P_{2,10} - \mu_5P_{05}P_{2,10} - \mu_6P_{06}P_{2,10} \\ & + \mu_0P_{17}P_{2,10} + \mu_3P_{03}P_{17}P_{2,10} + \mu_4P_{04}P_{17}P_{2,10} \\ & + \mu_5P_{05}P_{17}P_{2,10} + \mu_6P_{06}P_{17}P_{2,10} + \mu_1P_{02}P_{29} \\ & - \mu_0P_{18}P_{29} - \mu_3P_{03}P_{18}P_{29} - \mu_4P_{04}P_{18}P_{29} \\ & + \mu_5P_{05}P_{18}P_{29} + \mu_6P_{06}P_{18}P_{29} + \mu_1P_{03}P_{3,15} \\ & + \mu_2P_{03}P_{3,16} - \mu_2P_{03}P_{17}P_{3,16} + \mu_1P_{03}P_{29}P_{3,16} \\ & + \mu_1P_{04}P_{41} + \mu_2P_{04}P_{18}P_{41} - \mu_1P_{04}P_{2,10}P_{41} \\ & - \mu_1P_{04}P_{4,11} + \mu_2P_{04}P_{18}P_{4,11} - \mu_1P_{04}P_{2,10}P_{4,11} \\ & + \mu_2P_{04}P_{4,12} - \mu_2P_{04}P_{17}P_{4,12} + \mu_1P_{04}P_{29}P_{4,12} \\ & + \mu_1P_{05}P_{5,13} + \mu_2P_{05}P_{18}P_{5,13} - \mu_1P_{05}P_{2,10}P_{5,13} \\ & + \mu_2P_{05}P_{5,14} - \mu_2P_{05}P_{17}P_{5,14} + \mu_1P_{05}P_{29}P_{5,14} \\ & + \mu_2P_{05}P_{52} - \mu_2P_{05}P_{17}P_{52} + \mu_1P_{05}P_{29}P_{52} \\ & + \mu_1P_{06}P_{6,17} + \mu_2P_{06}P_{18}P_{6,17} - \mu_1P_{06}P_{2,10}P_{6,17} \\ & + \mu_2P_{06}P_{6,18} - \mu_2P_{06}P_{17}P_{6,18} + \mu_1P_{06}P_{29}P_{6,18} \\ & + \mu_3P_{06}P_{36} - \mu_3P_{06}P_{17}P_{36} - \mu_3P_{06}P_{2,10}P_{36} \\ & + \mu_3P_{06}P_{2,10}P_{17}P_{36} - \mu_3P_{06}P_{29}P_{18}P_{36} \\ & + \mu_1P_{06}P_{29}P_{3,15}P_{36} + \mu_2P_{06}P_{18}P_{3,15}P_{36} \\ & - \mu_1P_{06}P_{2,10}P_{3,15}P_{36} + \mu_2P_{06}P_{3,16}P_{36} \\ & - \mu_2P_{06}P_{17}P_{3,16}P_{36} + \mu_1P_{06}P_{29}P_{3,16}P_{36}, \end{aligned}$$

and

$$\begin{aligned} D_0 =& -\mu_0(-P_{10} - P_{18}P_{20} + P_{10}P_{2,10}) - \mu_1(-P_{01} + P_{01}P_{2,10} \\ & - P_{02}P_{29} - P_{03}P_{3,15} + P_{03}P_{2,10}P_{3,15} - P_{03}P_{29}P_{3,16} \\ & - P_{04}P_{41} + P_{04}P_{2,10}P_{41} - P_{04}P_{4,11} + P_{04}P_{2,10}P_{4,11} \\ & - P_{04}P_{2,29}P_{4,12} - P_{05}P_{5,13} + P_{05}P_{2,10}P_{5,13} - P_{05}P_{29}P_{5,14} \\ & - P_{05}P_{29}P_{52} - P_{06}P_{6,19} + P_{06}P_{2,10}P_{6,19} - P_{06}P_{29}P_{6,20} \\ & - P_{06}P_{3,15}P_{63} + P_{06}P_{2,10}P_{3,15}P_{63} - P_{06}P_{29}P_{3,16}P_{63}) \\ & - \mu_2(-P_{02}P_{10} - P_{18} + P_{03}P_{18}P_{30} - P_{03}P_{10}P_{3,16} \\ & - P_{04}P_{10}P_{4,12} - P_{05}P_{10}P_{5,14} - P_{05}P_{10}P_{52} \\ & - P_{06}P_{10}P_{6,20} + P_{06}P_{18}P_{30}P_{63} - P_{06}P_{10}P_{63}P_{3,16}) \\ & - \mu_3(-P_{03}P_{10} - P_{03}P_{18}P_{20} + P_{03}P_{10}P_{2,10} \\ & - P_{06}P_{10}P_{63} - P_{06}P_{18}P_{63}P_{20} + P_{06}P_{10}P_{63}P_{2,10}) \\ & - \mu_4(-P_{04}P_{10} - P_{04}P_{18}P_{20} + P_{04}P_{15}P_{2,10}) \\ & - \mu_5(-P_{05}P_{20} + P_{05}P_{17}P_{20} - P_{05}P_{10}P_{29}) \\ & - \mu_6(-P_{06}P_{10} - P_{06}P_{18}P_{20} + P_{04}P_{10}P_{2,10}). \end{aligned}$$

8. BUSY PERIOD ANALYSIS

8.1 Expected busy period with repair due to hardware failure

Upon using the probabilistic arguments, we obtain the following equations;

$$B_0^1(t) = \sum_{i=1}^{i=6} q_{0i}(t)©B_i^1(t), \tag{21}$$

$$B_1^1(t) = \overline{G}_1(t) + q_{10}(t)©B_0^1(t) + q_{11}^{(7)}(t)©B_1^1(t) + q_{12}^{(8)}(t)©B_2^1(t), \tag{22}$$

$$B_2^1(t) = q_{20}(t)©B_0^1(t) + q_{21}^{(9)}(t)©B_1^1(t) + q_{22}^{(10)}(t)©B_2^1(t), \tag{23}$$

$$B_3^1(t) = q_{30}(t) Ⓒ B_0^1(t) + q_{31}^{(15)}(t) Ⓒ B_1^1(t) + q_{32}^{(16)}(t) Ⓒ B_2^1(t), \tag{24}$$

$$B_4^1(t) = [q_{41}(t) + q_{41}^{(11,17)}(t)] Ⓒ B_1^1(t) + q_{42}^{(12,8)}(t) Ⓒ B_2^1(t), \tag{25}$$

$$B_5^1(t) = q_{51}(t) Ⓒ B_1^1(t) + [q_{52}(t) + q_{52}^{(14,10)}(t)] Ⓒ B_2^1(t), \tag{26}$$

$$B_6^1(t) = q_{61}^{(17,15)}(t) Ⓒ B_1^1(t) + q_{62}^{(18,16)}(t) Ⓒ B_2^1(t) + q_{63}(t) Ⓒ B_3^1(t). \tag{27}$$

Taking Laplace transform of Equations [(21)-(27)] and solving for $B_0^{1*}(s)$, we get

$$B_0^{1*}(u) = \frac{N_1(u)}{D_0(u)}. \tag{28}$$

The steady-state of the total fraction of time for which serverman is busy with hardware repair is given by

$$B_0^1 = \lim_{t\to\infty} B_0^1(t) = \lim_{u\to 0} uB_0^{1*}(u) = \frac{N_1}{D_0}, \tag{29}$$

where

$$\begin{aligned} N_1 =& \frac{1}{\nu_1}[P_{01} - P_{01}P_{29} + P_{03}P_{3,15} - P_{03}P_{2,10}P_{3,15} \\ &+ P_{03}P_{29}P_{3,16} + P_{04}P_{41} - P_{04}P_{2,10}P_{41} \\ &+ P_{04}P_{4,11} - P_{04}P_{4,11}P_{2,10} + P_{04}P_{4,10}P_{29} \\ &+ P_{05}P_{5,13} - P_{05}P_{2,10}P_{5,13} + P_{05}P_{29}P_{5,14} \\ &+ P_{05}P_{29}P_{52} + P_{06}P_{6,17} - P_{06}P_{2,10}P_{6,17} \\ &+ P_{06}P_{29}P_{6,18} + P_{06}P_{63}P_{3,15} \\ &- P_{06}P_{2,10}P_{63}P_{3,15} + P_{06}P_{29}P_{3,16}P_{63}]. \end{aligned}$$

8.2 Expected busy period with repair due to human failure

Let $B_k^2(t)$ be the probability that the server man is busy with repair due to human failure at time t starting form state $s_i \in E$. Upon using the probabilistic arguments, we obtain the following equations;

$$B_0^2(t) = \sum_{i=1}^{i=6} q_{0i}(t) Ⓒ B_i^2(t), \tag{30}$$

$$B_1^2(t) = q_{10}(t) Ⓒ B_0^2(t) + q_{11}^{(7)}(t) Ⓒ B_1^2(t) + q_{12}^{(8)}(t) Ⓒ B_2^2(t), \tag{31}$$

$$B_2^2(t) = q_{20}(t) Ⓒ B_0^2(t) + q_{21}^{(9)}(t) Ⓒ B_1^2(t) + q_{22}^{(10)}(t) Ⓒ B_2^2(t), \tag{32}$$

$$B_3^2(t) = \overline{V}(t) + q_{30}(t) Ⓒ B_0^2(t) + q_{31}^{(15)}(t) Ⓒ B_1^2(t) + q_{32}^{(16)}(t) Ⓒ B_2^2(t), \tag{33}$$

$$B_4^2(t) = [q_{41}(t) + q_{41}^{(11,17)}(t)] Ⓒ B_1^2(t) + q_{42}^{(12,8)}(t) Ⓒ B_2^2(t) \tag{34}$$

$$B_5^2(t) = q_{51}^{19,9}(t) Ⓒ B_1^2(t) + [q_{52}(t) + q_{52}^{(14,10)}(t)] Ⓒ B_2^2(t), \tag{35}$$

$$B_6^2(t) = q_{61}^{(17,15)}(t) Ⓒ B_1^2(t) + q_{62}^{(18,16)}(t) Ⓒ B_2^2(t) + q_{63}(t) Ⓒ B_3^2(t). \tag{36}$$

Once again takin Laplace transforms of Equations [(30)- (36)] and solving for $B_0^{2*}(s)$, we get

$$B_0^{2*}(u) = \frac{N_2(u)}{D_0(u)}, \tag{37}$$

The steady-state of expected busy period of the human error repair is given by

$$B_0^2 = \lim_{t\to\infty} B_0^2(t) = \lim_{u\to 0} uB_0^{2*}(u) = \frac{N_1}{D_0}. \tag{38}$$

Upon using Equations (38), we get

$$\begin{aligned} N_2 =& \frac{1}{\nu_2}[P_{02} - P_{02}P_{17} + P_{01}P_{18} + P_{03}P_{18}P_{3,15} \\ &+ P_{03}P_{3,16} - P_{03}P_{17}P_{3,16} + P_{04}P_{18}P_{41} \\ &+ P_{04}P_{18}P_{4,11} + P_{04}P_{4,12} - P_{04}P_{4,12}P_{17} \\ &+ P_{05}P_{18}P_{5,13} + P_{05}P_{5,14} - P_{05}P_{5,14}P_{17} \\ &+ P_{05}P_{52} - P_{05}P_{52}P_{17} + P_{06}P_{18}P_{6,17} + P_{06}P_{6,18} \\ &- P_{06}P_{17}P_{6,18} + P_{06}P_{18}P_{3,15}P_{63} + P_{06}P_{3,16}P_{63} \\ &- P_{06}P_{17}P_{3,16}P_{63}]. \end{aligned}$$

8.3 Expected Busy period with Standby Repair

Let $B_i^3(t)$ be the probability that the server man is busy with the standby repair at time t starting from state $S_i \in E$.
Using the probabilistic arguments, yields

$$B_0^3(t) = \sum_{i=1}^{6} q_{01}(t) Ⓒ B_i^3(t) \tag{39}$$

$$B_1^3(t) = q_{10}(t) Ⓒ B_0^3(t) + q_{11}^{(7)}(t) Ⓒ B_1^3(t) + q_{12}^{(8)}(t) Ⓒ B_2^3(t)$$

$$B_2^3(t) = q_{20}(t) Ⓒ B_0^3(t) + q_{21}^{(9)}(t) Ⓒ B_1^3(t) + q_{22}^{(10)}(t) Ⓒ B_2^3(t)$$

$$B_3^3(t) = q_{30}(t) Ⓒ B_0^3(t) + q_{31}^{(15)}(t) Ⓒ B_1^3(t) + q_{32}^{(16)}(t) Ⓒ B_2^3(t),$$

$$B_4^3(t) = [q_{41}(t) + q_{41}^{(11,17)}(t)] Ⓒ B_1^3(t) + q_{42}^{(12,8)}(t) Ⓒ B_2^3(t)$$

$$B_5^3(t) = q_{51}^{(19,9)}(t) Ⓒ B_1^3(t) + [q_{52}(t) + q_{52}^{(14,10)}(t)] Ⓒ B_2^3$$

$$B_6^3(t) = q_{61}^{(17,15)}(t) Ⓒ B_1^3 + q_{62}^{(18,16)}(t) Ⓒ B_2^3(t) + q_{63}(t) Ⓒ B_3^3(t).$$

After some calculations, the steady-state expected busy period with standby repair can be given as follows

$$B_0^3 = \lim_{t\to\infty} B_0^3(t) = \lim_{u\to 0} uB_1^{3*}(u) = \frac{N_3}{D_0},$$

where $B_1^{3*}(S)$ is LT of $B_0^3(t)$, and

$$\begin{aligned} N_3 =& \frac{1}{\eta}[P_{03}(1 - P_{17} - P_{2,10} + P_{P_17}P_{2,10} - P_{18}P_{29} - P_{06}P_{17}) \\ &- P_{06}P_{63}(P_{2,10} - P_{2,10}P_{17} + P_{29}P_{18})]. \end{aligned}$$

9. COST BENEFIT ANALYSIS

Let $C(t)$ be the net revenue of the system in $(0, t]$, then

$$C(t) = \alpha\mu_{up}(t) - \sum_{i=1}^{i=3} \beta_i \mu_R^i(t), \tag{40}$$

where

α is the revenue per unit of up time

$\beta_i; \quad i = 1, 2, 3$ are the cost per unit time of repair of the unit and the switch respectively.

$$\mu_{up} = \int_0^t AV_0(t)\,dt$$

$$\mu_R^i(t) = \int_0^t B_0^i(t)\,dt$$

Taking LT of (40), gives

$$C^*(u) = \alpha\mu_{up}^*(u) - \sum_{i=1}^{3} \beta_i \mu_R^{i*}(u).$$

Define C to represent the expected profit per unit of time in steady state.
So C can be written as follows

$$\begin{aligned} C = C(\infty) &= \lim_{t\to\infty} \frac{C(t)}{t} = \lim_{u\to 0} u^2 C^*(u) \\ &= \frac{\alpha N_0 - \sum_{i=1}^{3} \beta_i N_i}{D_0}. \end{aligned} \tag{41}$$

10. NUMERICAL EXAMPLE

Setting $\alpha = 300$, $\beta_1 = 20$, $\beta_2 = 10$ and $\beta_3 = 5$, $n = 2$, $\lambda_2 = 0.002, P = 0.8, q = 0.2, \nu_1 = 0.08, \nu_2 = 0.02, \eta = 0.06$ and $\xi = 0.04$.
Figures (1-4) represent the variation of MTTF, AV_0, B_0^2, B_0^3 and C versus λ_1 when $\nu_1 < \nu_2, \nu_1 = \nu_2$ and $\nu_1 > \nu_2$.
Note: To save the space, figures versus λ_2 are omitted because they have the same manner.

REMARK 2. *From Figures (1,2 and 4), we can see that*

1- The MTTF, AV_0 and C decreases as λ_1 increases

2- From Fig (3) the expected Basy period B_1^0, B_2^0 and B_3^0 increase as λ_1 increases.

11. SPECIAL CASES

1- Setting $n = 1$, we get the results for two-unit warm standby based on imperfect repair facility and two types of failures.

2- Setting $n = 1$ and $\gamma = 0$, the results for two-unit cold standby system based on imperfect repair facility and two types failures.

3- Setting $n = 1$, $\gamma = 0$ and $P = 1$, we obtain the result for two-unit cold standby system based on two types of failure.

12. CONCLUSION

The stochastic behaviour of $(n + 1)$-unit warm standby system based on imperfect repair facility and two types of failure are studied. Some measures of reliability for the system are derived in the steady state. Based on a numerical example, it has been showed that MTTF, AV_0 and C decrease as λ_1 or λ_2 increases while B_0^1, B_0^2 and B_0^3 are increasing as λ_1 or λ_2 increases.

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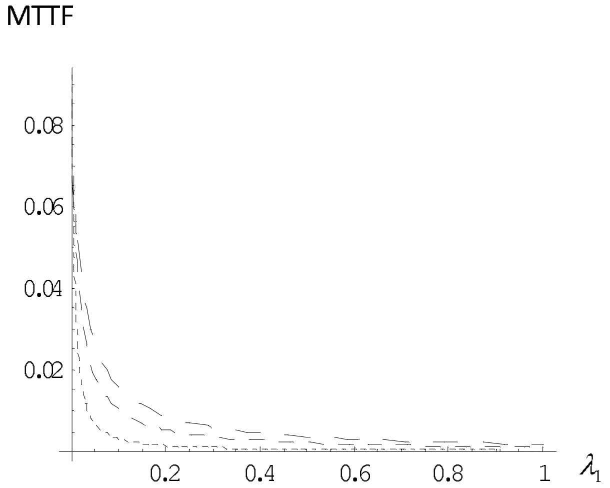

Fig (1): Variation of MTTF versus λ_1

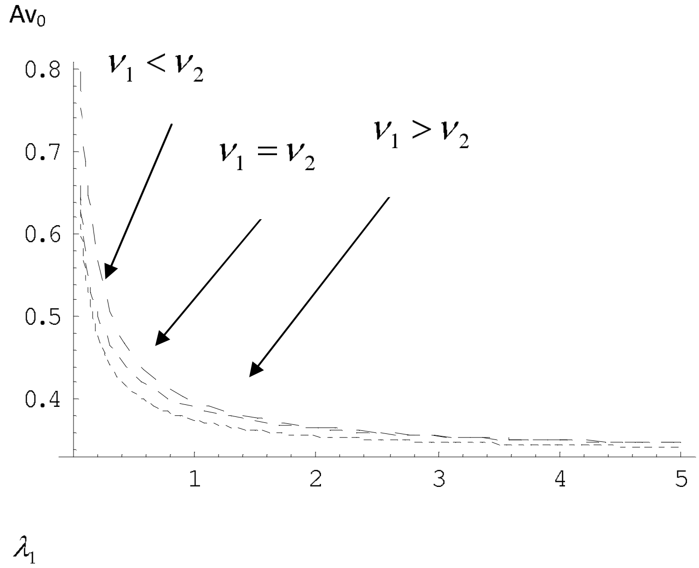

Fig (2): Variation of AV_0 versus λ_1

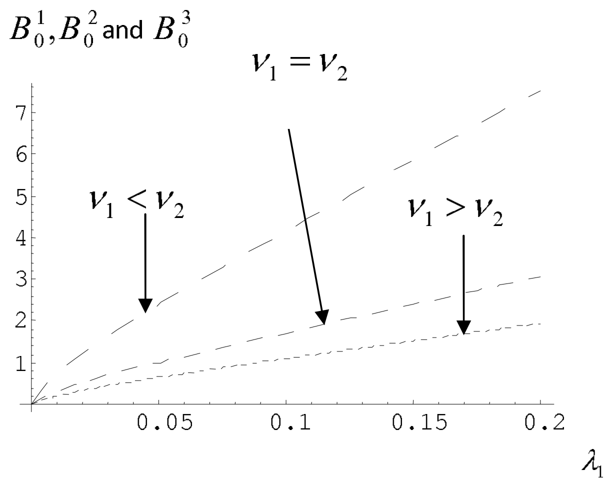

Fig (3): Variation of B_0^1, B_0^2 and B_0^3 versus λ_1

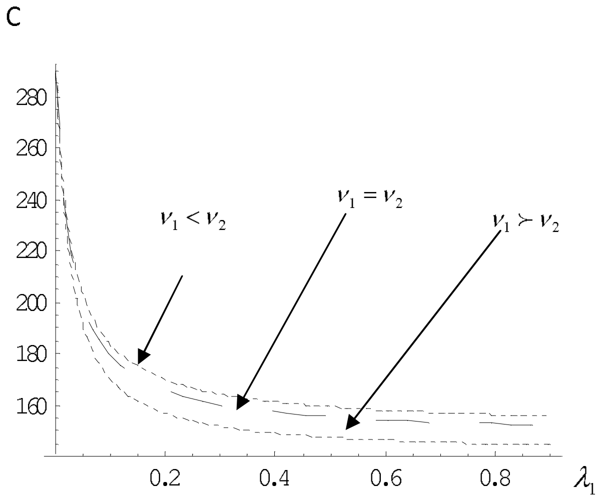

Fig (4): Variation of C versus λ_1