On the Reliability of Multi-State *m*-consecutive-at least-*k*-out-of-*n*: *F* Systems

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ABSTRACT
Reliability importance of a component is a quantitative measure of the importance of the individual component in contributing to system reliability. In this paper, an appropriate Markov chain imbedding technique is employed to obtain the reliability of an multi-state *m*-consecutive-at least-*k*-out-of-*n*: *F* systems when the system components are independently functioning with not necessarily equal reliability. Finally, an illustrative is given example.

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Keywords
Reliability; Multi-state; Markov chain imbedding; Consecutive systems.

1. INTRODUCTION
In the binary system: the system and its components are allowed to have only two possible states (completed failure and perfect functioning). In the multi-state (M.S)system: both the system and its components may experience more than two states, for example, completely failed, partially functioning and perfect functioning. There are numerous examples of M.S systems, with more than 2 ordered or unordered states at the system level, or the component level. As water distribution, a power plant which has states 0,1,2,3,4 that correspond to generating electricity of 0%, 25%, 50%, 75%, 100% of its full capacity is an example of a multi-state system that has ordered multiple states by[1]. A nuclear reactor system or a pumping system, telecommunications, a light-emission diode which emits red, green, and yellow lights under different inputs. Furthermore, a state in a system may take a continuous range of quantitative measurement instead of discrete levels, for example, a branking system might produce an output branking force ranging from 250 to 300 kilograms.

One of the most important measures of the performance of a system is its reliability. The reliability of a system is defined to be the probability that the system will perform its functions satisfactorily for a certain time period under specified conditions. A consecutive-*k*-out-of-*n*: *F* system ($C(k, n : F)$), consists of an ordered sequence of *n* components for which the existence of *k* (or more) consecutive failed components causes the system's failure, $1 \le k \le n$. $C(1, n: F)$ and $C(n, n:F)$ are series and parallel systems with *n* components, respectively. Since Kontoleon [2] first introduced and studied these systems in 1980, a series of articles have been published studying their reliability properties under various assumptions because of their wide applicability; e.g. they have been used to model telecommunication systems, oil pipeline systems, vacuum accelerators, etc. A generalization of $C(k, n : F)$ system was formulated by Griffth [3], who considered a system of n ($n \ge m\ k$) components ordered on a line, for which m (≥ 2) non-overlapping strings of *k* consecutive failed components are needed for system failure. For such a system, named *m*-consecutive-*k*-out-of-*n* : *F* system ($C_m(k, n : F)$), in [4] the failure probability of $C_m(k, n : F)$ having independent components was obtained while in [5]the Stein-Chen method was employed to obtain Poisson approximations for the reliability. Agarwal et al.[6] obtained the reliability of both types of systems *m*-consecutive-*k*-out-of-*n*: *F* system with Block-k dependence ($C^b{}_m (k, n :F)$) and *m*-consecutive-*k*-out-of-*n*: *F* system having (k-1)-step Markov dependence ($C_m{}^{k-1} (k, n : F)$) by using Graphical Evaluation and Review Technique(GERT) when its components are iid. The Finite Markov Chain Imbedding approach (FMCIA) was first employed by Fu [7], Fu, Hu [8], and formally named by Fu and Koutras [9]. After that, Koutras and Alexandrou [10] refined the method by providing a general recursive scheme for the probability distribution of a Markov chain imbedding random variable of binomial type (MVB). The concept of MVB was extended later by Han and Aki [11] who introduced a Markov chain imbedding random variable of returnable (MVR) and also gave a general recursive scheme for its probability distribution.

Spiros et al.[12]employed an appropriate FMCIA to obtain the reliability of a binary *m*-consecutive-at least-*k*-out-of-*n*:F system ($C^+{}_m (k, n : F)$) which fails if there are at least *m* non-overlapping runs of at least *k*. This system, for $m = 1$ reduces to $C (k, n : F)$. It is mentioned that for $C^+{}_m (k, n :F)$ a run of failures of length rk, $r \ge 1$, is treated as one run of length at least *k* whereas it is treated as *r* runs of length *k* for a $C_m(k, n : F)$.

Recently, researchers have partially extended the definitions of the binary consecutive-*k*-out-of-*n* system to the multi-state (M.S) case by allowing the system to remain binary and its components to have more than two possible states, for example, see Zuo and Liang [13] and Malinowski and Preuss [14,15]. Koutras [16] extends the binary consecutive-*k*-out-of-*n*: *F* system to the dual failure mode environment whereas the system and each component may experience one working state and two different failure states. Haim and Porat [17] provide a Bays reliability model of the consecutive-*k*-out-of- *n* system, in which both the system and its components are assumed to have more than two possible states while *k* is assumed to be constant. When *k* is constant, the system has the same reliability structure at all system state levels.

A definition of the generalized multi-state *k*-out-of-*n*: *G* system, in which *k* could take different values for different system state levels, has been proposed and also two algorithms have been provided for evaluating system state distribution of decreasing multi-state consecutive-*k*-out-of-*n*:*F* systems and to bound system state distribution of M.S consecutive-*k*-out-of-*n*: *F* and *G* systems by Huang et al.[18,19]. An efficient recursive algorithm based on minimal cut vectors has been developed to evaluate the state distributions of a generalized multi-state *k*-out-of-*n*: *F* and *G* systems by Zuo et al.[20].

The linear consecutive *k*-out-of-*r*-from-*n*: *F*(*G*) system is another consecutive-*k* system which consists of *n* components ordered in a line or a circle. It fails if there exists a window consisting of consecutive-*r* components in which at least *k* components fail [21]. Based on an extended universal moment generating function, algorithms for obtaining the reliability of the linear consecutive sliding window system, and linear M.S sliding window system are proposed by Levitin [22] and Levitin and Haim [23] respectively. Habib et al.[24] found the exact reliability of decreasing M.S consecutive-*k*-out-of-*r* -from-*n*: *F* systems. Radwan et al.[25]suggested bounds for the increasing M.S consecutive-*k*-out-of-*r*-from-*n*: *F* system by using second order Boole-Bonferroni bounds, and Hunter-Worsley upper bound.

FMCIA was improved to accelerated scan FMCIA by Zhao and Cui [26]. Furthermore, it was first used to obtain the state distribution of multi-state *k*-out-of-*n*: *F* systems, and the reliability of two-dimension systems by Zhao and Cui [27], and Zhao et al.[28]respectively. Zhao et al.[29] presented a unified formula for obtaining state distributions of the six M.S consecutive-*k*-systems by means of the FMCIA: M.S consecutive-*k*-out-of-*n*: *G* systems (including increasing, decreasing and non-monotonic cases), and M.S consecutive *k*-out-of-*r*-from-*n* systems (including linear sliding window systems, consecutive sliding window systems, and M.S consecutive *k*-out-of-*r*-from-*n*: *F* systems).

In this paper, by using an appropriate MCIA the reliability of an M.S *m-c*onsecutive-at least-*k*-out-of-*n*: *F* *s*ystem is obtained when the system components are independently functioning with not necessarily equal reliability.

2. The M.S -*m*-consecutive-at least-*k*-out-of-*n*: *F* System

A M.S consecutive *k* -out of *n*: *F* system is a system with *n* linearly arranged components, which are labeled 1, 2,…., *n*. Each component has states 0, 1, 2,......c and their corresponding probabilities $P_{i,0}, P_{i,1}, \ldots\ldots, P_{i,c}$ of occurring; i=1,2,..,*n*.. Let the random variables $Z_{1\,;}Z_2, \ldots\ldots, Z_n$, represent the states of the system components, i.e. $Z_i = j$ if component *i* works and in state j; j=1,2,...,c and $Z_i = 0$ if component *i* fails. The system fails or it is in state 0 if there are at least m ($m \geq 1$) non-overlapping runs of at least *k* consecutive failures. Throughout this section, we denote the random variable $X_n, (X_n = \phi(Z_1, Z_2, \ldots, Z_n)$by the number of failure runs of length at least *k* in a sequence of *n* multi-state trials.

Definition 1.

The non-negative integer random variable X_n is finite Markov chain embeddable of binomial type (MVB) if

(a) There exists a finite Markov chain $\{Y_t\,, t \geq 0\}$ defined on a discrete state space Ω which can be partitioned as

$$\Omega = \cup_{x\geq 0} C_x, \quad C_x = \{c_{x,0}, c_{x,1}, \ldots\ldots, c_{x,s-1}\} \tag{2.1}$$

(b)For every $x = 0, 1, \ldots\ldots, l_n,\ l_n = \max\{x: P(X_n = x) > 0\}$ upper end $point\ and\ n\ \geq\ 0$.

$$P(X_n = x) = P(Y_n \in C_x) \tag{2.2}$$

(c) For all $y \neq x, x+1, t \geq 1$

$$P(Y_t = c_{y,j} \mid Y_{t-1} = c_{x,i}) = 0 \tag{2.3}$$

For any MVB, two types of transitions give birth to the next ***s*× *s*** transition probability matrices

$$A_t(x) = P(Y_t = c_{x,j} \mid Y_{t-1} = c_{x,i}), \tag{2.4}$$

$$B_t(x) = P(Y_t = c_{x+1,j} \mid Y_{t-1} = c_{x,i}), \tag{2.5}$$

and the probability (row) vectors for $0 \leq t \leq n$

$$f_t(x) = \Big(P(Y_t = c_{x,0}), P(Y_t = c_{x,1}), \ldots., P(Y_t = c_{x,s-1})\Big); \tag{2.6}$$

Consider a sequence of *n* i.d. multi-state trials $Z_{1\,;}Z_2, \ldots\ldots, Z_n$. The object now to find the exact distribution of the random variable $X_n, (X_n = \phi(Z_1, Z_2, \ldots, Z_n))$, the number of non-overlapping failure runs of length at least *k* (i.e specified patter $\xi_k = \underbrace{00\ldots.0}_{\mathrm{k1\geq k}}$) in a sequence of *n* multi-state trials. In the following the forward and backward principle for the finite Markov chain imbedding technique is introduced to find the exact distribution of a given simple pattern ξ_k .

(i) Decompose the pattern ξ_k forward into k+1 sub-patterns labeled 0 when there is not any failure run, 1 when there is failure run of length 1,....,k-1, when there exist failure runs of length k-1 and -1 when there exist exactly failure runs of length at least *k*. We shall refer to these k+1 sub-patterns 0, 1, 2, ...,k-1 and -1 as ending blocks.

(ii) Let $w = (z_{1\,;}z_2, \ldots\ldots, z_n)$ be a realization of a sequence of *n* multi-state trials where z_i is the outcome of the *i* th trials. We define the Markov chain $\{\,Y_t : t = 1, \ldots., n\}$ operating on *w* as $Y_t(w) = (u, v)$ for every $t = 1, \ldots., n$, where u = the total number of non-overlapping patterns ξ_k that occurred in the first *t* trials (counting forward from the first trial to the *t* th trial, v = the sub-pattern (ending block), counting backward from *t*.

Based on (i) and (ii), the state space Ω associated with the imbedded Markov chain is defined by

$$\Omega = \{(u, v): u = 0, 1, \ldots, l_n\ and\ v = 0, 1, \ldots, k-1, -1\} \tag{2.7}$$

where $l_n = \left[\frac{n+1}{k+1}\right]$ is the maximum number of ξ_k possible in the sequence of *n* multi-state trials.

(iii) For t = 1,....,*n*, the transition probabilities of the transition matrix M_t is determined by the following two equations: for $u = 0, 1, \ldots, l_n - 1$,

$$P(Y_t = (u+1, -1)\,|Y_{t-1} = (u, k-1)) = p_{t,0} \tag{2.8}$$

where $p_{t,0}$ is the probability of the last symbol (0) of the specified pattern ξ_k , for $u = 0, 1, ., \mathrm{l_n}$ and $v, \acute{v} = 0, 1, ., k-1, -1$

$$P(\,Y_t = (u, \acute{v}) \;\mid Y_{t-1}) = (u, v)\,) = \sum_{v \to \acute{v}} p_{t,j} \tag{2.9}$$

where $\sum_{v \to \acute{v}}$ sums over all $p_{t,j}$ corresponding to the symbol of which the ending block v is changed to the ending block $\acute{v}$, and zero otherwise.

(iv) The partition $\{C_x = [(x, v): (x, v) \in \Omega, v = 0, 1, \dots \dots, k-1, -1], for\ x = 0, 1, \dots \dots, l_n\}\ on\ \Omega$ is one − to − one corresponding to the random variable X_n in the sense that $P(X_n) = P(Y_n \in C_n)\ for\ every\ x = 0, 1, 2, \dots \dots, l_n$. It follows from our construction that the transition probabilities given by (2.8) and (2.9) depend only on $Y_{t-1} = (u, v)$; hence the sequence $\{Y_t\}$ is a Markov chain. The four steps (i), (ii), (iii), and (iv) of construction mentioned above establish that X_n is MVB. For the MVB associated with the pattern ξ_k has a state space Ω and transition probability matrices for $t = 1, 2, \dots, n$

$$M_t = \begin{pmatrix} A_t & B_t & 0 & 0 & 0 \\ 0 & A_t & B_t & 0 & 0 \\ 0 & 0 & A_t & B_t & 0 \\ 0 & 0 & 0 & A_t & B_t \\ 0 & 0 & 0 & 0 & A_t^* \end{pmatrix}, \ where \qquad (2.10)$$

$$A_t = \begin{array}{c} \begin{matrix} (.,0) & (.,1) & (.,2) & . & (.,k-1) & (.,-1) \end{matrix} \\ \begin{pmatrix} \sum_j p_{t,j} & p_{t,0} & 0 & . & 0 & 0 \\ \sum_j p_{t,j} & 0 & p_{t,0} & . & 0 & 0 \\ . & . & . & . & . & . \\ \sum_j p_{t,j} & 0 & 0 & . & 0 & 0 \\ \sum_j p_{t,j} & 0 & 0 & . & 0 & p_{t,0} \end{pmatrix} \end{array},$$

$$B_t = \begin{array}{c} \begin{matrix} (.,0) & (.,1) & (.,2) & . & (.,k-1) & (.,-1) \end{matrix} \\ \begin{pmatrix} 0 & 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & p_{t,0} \\ 0 & 0 & 0 & . & 0 & 0 \end{pmatrix} \end{array},$$

$$A_t^* = \begin{array}{c} \begin{matrix} (.,0) & (.,1) & (.,2) & . & (.,k-1) & (.,-1) \end{matrix} \\ \begin{pmatrix} \sum_j p_{t,j} & p_{t,0} & 0 & . & 0 & 0 \\ \sum_j p_{t,j} & 0 & p_{t,0} & . & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & 1 & 0 \\ 0 & 0 & 0 & . & 0 & 1 \end{pmatrix} \end{array}$$

Theorem 1. [**10**]
For an MVB the double sequence of vectors $f_t(x), 0 \le x \le l_t, 1 \le t \le n$ satisfies the following recurrence relations:

$$f_t(0) = f_{t-1}(0)\ A_t(0) \qquad (2.11)$$

$$f_t(0) = f_{t-1}(0)\ A_t(0) + f_{t-1}(x-1)\ B_t(x-1) \qquad (2.12)$$

Theorem 2.
For positive integers ***k, m*** and ***n*** the reliability $R_m^+(k, n{:}P)$ of the M.S $C_m^+(k, n{:}F)$ with independent components and reliability of the *i*-th component in the state j, $p_{i,j}$, $i = 1, 2, \dots, n$, $j = 0, 1, \dots \dots, c$ is given by

$$R_m^+(k, n{:}P) = 1 - \prod_{i=1}^{n}(1 - \sum_{j=1}^{c} p_{i,j}), \quad n < mk + m - 1 \qquad (2.13)$$

$$R_1^+(k, n{:}P) = f_{n-1}(0) A_n\, \mathbf{\acute{1}}, \quad n \ge k, \qquad \mathbf{1} = (1, 1, , 1) \in R^s \qquad (2.14)$$

$$R_m^+(k, n{:}P) = \sum_{x=0}^{m-2} f_{n-1}(x)(A_n + B_n)\, \mathbf{\acute{1}} + f_{n-1}(m - -1) A_n\, \mathbf{\acute{1}} - \prod_{i=1}^{n}(1 - \sum_{j=1}^{c} p_{i,j})) \qquad (2.15)$$

Proof. Let $Z_1, Z_2, \dots, Z_n$ be a sequence of n independent multi random variables with $P(Z_i = j) = p_{i,j}$,

$i = 1, 2, \dots \dots, n, j = 0,$ 1,........,c and $\Gamma_n = \{Z_1 = Z_2 = \dots = Z_n = 0\}$.

For $n < mk + m - 1$

$$R_m^+(k, n{:}P) = (1 - P(\Gamma_n) = 1 - \prod_{i=1}^{n} p_{i,0} \Rightarrow eq. (2.13)$$

Q.E.D.

For $n \ge k$, $R_1^+(k, n{:}P) = P(X_n(\xi_k) < 1) = P(X_n(\xi_k) = 0) = P(Y_n \in C_0) = f_n(0)\acute{1} \Rightarrow eq. (2.14)$ $\quad Q.E.D$

For $n \ge mk + m - 1, m \ge 2,$

$$R_m^+(k, n{:}P) = P\big((X_n(\xi) < m) \cap \Gamma_n^c\big)$$

$$= \sum_{x=0}^{m-1} P(X_n(\xi) = x) - P((X_n(\xi) < m) \cap \Gamma_n)$$

$$= \sum_{x=0}^{m-1} P(Y_n \in C_x) - \prod_{i=1}^{n} p_{i,0}$$

$$= \sum_{x=0}^{m-1} f_n(x)\acute{1} - \prod_{i=1}^{n}(1 - \sum_{j=1}^{c} p_{i,j})$$

$$= \sum_{x=0}^{m-2} f_n(x)\acute{1} + f_n(m-1)\,\acute{1} - \prod_{i=1}^{n}(1 - \sum_{j=1}^{c} p_{i,j})$$

$$= \sum_{x=0}^{m-2} [\, f_{n-1}(x) A_n + f_{n-1}(x-1) B_n\,]\acute{1} + f_{n-1}(m-1) A_n\, \acute{1}$$

$$+ f_{n-1}(m-2) B_n\, \acute{1} - \prod_{i=1}^{n}(1 - \sum_{j=1}^{c} p_{i,j})$$

$$= \sum_{x=0}^{m-2} [\, f_{n-1}(x) A_n + f_{n-1}(x) B_n\,]\acute{1} - \sum_{x=0}^{m-2} [\, f_{n-1}(x) B_n\, \acute{1}$$

$$+ \sum_{x=0}^{m-1} [\, f_{n-1}(x-1) B_n\, \acute{1} + f_{n-1}(m-1) A_n\, \acute{1}$$

$$- \prod_{i=1}^{n}(1 - \sum_{j=1}^{c} p_{i,j}) \Rightarrow eq. (2.15)$$

3. Example

Let an alarm system of an accelerator consists of (***n*** = 20) detectors (feelers) posed along the surface of an accelerator. The feelers (i.e. the system components) might measure the temperature or the radioactivity level of the accelerator. Their failures are likely to occur independently. Both the system and its components may experience three states denoted by 0, 1 and 2. The reliability of the detectors may be different because of the holding conditions and the operational procedures among the individual feelers or they may be identical due to economic reasons and maintenance policy. We consider that such a system fails if there are at least (***m*** = 3) clusters of feelers that each has at least ***k***= 4 consecutive feelers failed. The search space consists of ***n*** =20 components;

Table 1 gives state probability distributions $p_{i,j}; j = 0,1,2, i = 1,2, \dots\dots\dots,20$. By using the state probability distributions $p_{i,j}$ for each component in Table (1), equations 2.10, 2.11, 2.12 and 2.15 we have :

$f_{19}(0) = 0.643156874 \quad 0.3117225680 \quad 0.0259806826$

$0.0079676205 \quad 0)$,

$f_{19}(1) = (0.0068363900 \quad 0.0032755333 \quad 0.0002691976$

$0.0000823449 \quad 0.0006928739)$,

$f_{19}(2) = \exp(-5)(0.4735837853 \;\; 0.1898612964 \;\; 0.0119805397 \;\; 0.0036637626 \;\; 0.7123436384\;)$,

and the reliability of $C_3^+(4,20\colon F)$ system

$R_3^+(4,20\colon P) = 0.9999999902$

3.1 The M.S consecutive-*k*-out-of-*n*:*F* System *C*(*k*, *n* : *F*)

$C(k,n\colon F)$ System is a special case from $C_m^+(k,n\colon F)$ system at *m*=1. By using the state probability distributions $p_{i,j}$ for each component in Table (1), equations 2.10, 2.11, and 2.14 we have the reliability of $C(4,20\colon F)$ system $R(4,20\colon P) = 0.9873138969$.

Table.1
Probability distribution of the states of components

$P_{i,j}$	1 2 3 4 5 6 7 8 9 10 11
$P_{i,0}$ $P_{i,1}$ $P_{i,2}$	.12 .24 .40 .21 .26 .15 .04 .05 .07 .13 .12 .10 .13 .37 .24 .01 .52 .66 .26 .73 .21 .23 .78 .63 .23 .55 .73 .33 .30 .69 .20 .66 .65
$P_{i,j}$	12 13 14 15 16 1 7 1 8 19 20
$P_{i,0}$ $P_{i,1}$ $P_{i,2}$	.14 .05 .11 .07 .08 .25 .10 .35 .19 .02 .85 .01 .30 .43 .23 .31 .14 .10 .84 .10 .88 .63 .49 .52 .59 .51 .71

4. Conclusions

In this paper, two important tasks are finished by using the finite Markov chain imbedding approach. one task is constructing a unified formula for obtaining the state distributions of random variable denoting the number of occurrences of the pattern ξ_k in a sequence of *n* multi-state trials. The other task is proposing formula for obtaining the reliability of M.S $C(k,n\colon F)$ system and its generalized M.S $C_m^+(k,n\colon F)$ system ($m \geq 2$). M.S $C_m^+(k,n\colon F)$ is more reliable than M.S $C(k,n\colon F)$ with the same parameters k and *n*. This is so because the set of possible state configurations causing M.S $C_m^+(k,n\colon F)$ failure is a subset of the respective set of a M.S $C(k,n\colon F)$.

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