RgI-closed Sets in Ideal Topological Spaces

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ABSTRACT
The paper introduces a new generalized closed set via semi local function. Its relationship with other existing generalized closed sets are established. It's basic properties are discussed. A new decomposition of continuity is derived.

General Terms
Ideal topological space, local function and semi-local function.

Keywords
rgI-closed, rgI-open and rgI-continuous function.

1. INTRODUCTION
Ideals play an important role in topology. Jankovic and Hamlet[2] introduced the notion of I-open sets in topological spaces. Kuratowski[5] has introduced local function of a set with respect to a topology τ and an ideal. Khan et al[3] have introduced semi-local function and derived its properties. Khan et al[4] have introduced gI-closed sets in ideal topology. Antony Rex et.al.[1] have introduced $\hat{g}$ -closed sets in ideal topological spaces. Navaneethakrishnan et.al.[8,7] have introduced regular generalized closed sets and g-closed sets in ideal topological spaces. In this paper generalized regular closed sets is introduced in ideal topological spaces using semi-local function. Its relationship with other existing sets are established. Its basic properties are studied. A new decomposition of continuity is derived in terms of rgI-closed sets. As an application maximal rgI-closed sets are defined and their properties are discussed.

2. PRILMINARIES
We list some definitions which are useful in the following sections. The interior and the closure of a subset A of (X, τ) are denoted by Int(A) and Cl(A), respectively. Throughout the present paper (X, τ) and (Y, σ)(or X and Y) represent non-empty topological spaces on which no separation axiom is defined, unless otherwise mentioned.

Definition 2.1 A subset A of a space X is called
(i) a semi-open set [3] if A $\subseteq$ Cl(Int(A))
(ii) an α-open set [9] if A $\subseteq$ Int(Cl(Int(A)))
The complement of a semi-open (α- open) set is called a semi-closed (α-closed) set.

Definition 2.2 (3) An ideal I on a topological space (X, τ) is a non empty collection of subsets of X which satisfies the following conditions. i)A $\in$ I and B $\subseteq$ A implies B $\in$ I.
ii)A $\in$ I and B $\in$ I implies A $\cup$ B $\in$ I.
An ideal topological space (X, τ) with an ideal I on X is denoted by (X, τ, I).

If Y is a subset of X then I_Y = { I_0 ∩ Y : I_0 $\in$ I} is an ideal on Y and (Y, τ/Y, I_Y) denote the ideal topological subspace.

Definition 2.3 (3) Let (X, τ, I) be an ideal topological space and A $\subseteq$ X. $A^*(I,\tau)$ = {x $\in$ X : A ∩ U $\notin$ I for every U $\in$ τ (X, x)} is called the local function of A with respect to I and τ For every ideal topological space (X, τ, I) there exists a topology τ^* finer than τ defined as τ^* = {U $\subseteq$ X :Cl* X −U) = X-U}generated by the base β(I, J) = {U −J :U $\in$ τ and J $\in$ I} and Cl*(A) = A $\cup$ A^*.

Definition 2.4 (3) Let (X, τ, I) be an ideal topological space and A $\subseteq$ X. $A_*(I,\tau)$ = {x $\in$ X : A ∩ U $\notin$ I for every U $\in$ SO(X, x)} is called the semi-local local function of A with respect to I and τ where SO(X, x) = {U $\in$ SO(X) : x $\in$ U} where SO(X) denotes the collection of all semi-open sets in X.

Theorem 2.5 (3) (i) $A_*(I,\tau) \subseteq A^*(I,\tau)$ for every subset A of X.
(ii) $A^*(I,\tau) = A_*(I,\tau)$ if SO(X, τ) = τ
(iii) If A $\in$ I then $A_*(I,\tau)$ = φ
(iv) $(A\cup B)_* = A_* \cup B_*$
(v) If U $\in$ τ then U ∩ $A_* \subseteq (U\cap A)_*$
(vi) $(A_*)_* \subseteq A_*$.
(vii) If A $\subseteq$ B then $A_* \subseteq B_*$.
(viii) $(A\cap B)_* \subseteq A_* \cap B_*$.

Theorem 2.6 (4) Let (X,τ, I) be an ideal space and A $\subseteq$ Y $\subseteq$ X where Y is α-open in X then
$A_*(I_Y,\tau_{\backslash Y}) = A_*(I,\tau)\cap Y$

Definition 2.7 A subset A of an ideal space (X, τ, I) is said to be
(i) *-closed if $A^* \subseteq$ A.[3]
(ii) *-dense itself if A $\subseteq$ A^*[3].
(iii) semi-*-closed if $A_* \subseteq$ A[4].

Definition 2.8 A subset A of an ideal topological space (X, τ, I) is said to be
(i) I_g - closed [7]if $A^* \subseteq$ A whenever A $\subseteq$ U and U is open in X.
(ii) $I_{\hat{g}}$ -closed[1] if $A^* \subseteq$ A whenever A $\subseteq$ U and U is semi-open in X.
(iii) I_{rg} -closed[8] if if $A^* \subseteq$ A whenever A $\subseteq$ U and U is regular open in X.

(iv) gI-closed [4] if $A_* \subseteq A$ whenever A $\subseteq$ U and U is open in X.
(v) rg-closed[8] if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

3. NEW CLOSURE OPERATOR

We define a new closure operator in terms of semi local function.

Definition 3.1 For a subset A of an ideal topological space (X, τ, I),we define $Cl_*(A) = A \cup A_*$.

Theorem 3.2 Cl_* satisfies Kuratowski's closure axioms.
Proof.
(i) $Cl_*(\varphi) = \varphi$, $A \subseteq Cl_*(A)$ $\forall A \subseteq X$
(ii) $Cl_*(A \cup B) = A \cup B \cup (A \cup B)_* = A \cup B \cup (A_* \cup B_*) = Cl_*(A) \cup Cl_*(B)$.
(iii) For any $A \subseteq X$, $Cl_*(Cl_*(A)) = Cl_*(A \cup A_*) = (A \cup A_*) \cup ((A \cup A_*)_* = A \cup A_* \cup (A_*)_* = A \cup A_* = Cl_*(A)$(since $(A_*)_* \subseteq A_*$).

Definition 3.3 The topology generated by Cl_* is denoted by $\tau_*(I)$ and is defined as $\tau_*(I) = \{U \subseteq X : Cl_*(X - U) = X - U\}$. Without ambiguity it will be denoted as τ_*.
(i)$\varphi \subseteq X$, $Cl_*(X - \varphi) = Cl_*(X) = X$ and $Cl_*(X - X) = Cl_*(\varphi) = \varphi$. Hence $\varphi, X \in \tau_*$.
(ii)Let $\{U_i\}_{i\in I} \in \tau_*$ then $Cl_*(X - U_i) = X - U_i$ $\forall$ i.
i.e $(X - U_i) \cup (X - U_i)_* = X - U_i$ $\forall$ i.
Therefore $(X - U_i)_* \subseteq X - U_i$ $\forall$ i.
Claim $Cl_*(X - \bigcup_i U_i) = Cl_*(\bigcap_i (X - U_i)) = X - \bigcup_i U_i = \bigcap_i (X - U_i)$. By definition $Cl_*(\bigcap_i (X - U_i)) = \bigcap_i (X - U_i) \cup \left(\bigcap_i (X - U_i)\right)_* \Rightarrow Cl_*(\bigcap_i (X - U_i)) \supseteq \bigcap_i (X - U_i)$. Also by hypothesis $\bigcap_i (X - U_i) \cup \left(\bigcap_i (X - U_i)\right)_* \subseteq \bigcap_i (X - U_i)$. Hence $Cl_*(\bigcap_i (X - U_i)) \subseteq \bigcap_i (X - U_i)$.
Thus $Cl_*(X - \bigcup_i U_i) = X - \bigcup_i U_i$

(iii) Let $U_1, U_2 \in \tau$ then $Cl_*(X - U_i) = X - U_i$ for i=1,2. $Cl_*(X-(U_1 \cap U_2)) = Cl_*(\bigcup_{i=1}^{2} (X - U_i)) = Cl_*(X-U_1) \cup Cl_*(X-U_2)(=X-U_1) \cup (X-U_2)$. Hence $U_1 \cap U_2 \in \tau_*$ Hence τ_* is a topology. Cl_* and Int_* will represent the closure and interior of A in (X, τ_*).

Proposition 3.4 Every τ_* is finer than τ^*.

Proof.Since every open set is semi-open. Therefore $\tau \subseteq \tau^* \subseteq \tau_*$.

Example 3.5 Let X = {a, b, c, d}, τ = {φ, {a}, {b}, {a, b},X}, I = {φ, {b}}.
τ^* = {φ, {a}, {b}, {a, b}, {a, c, d},X}.
τ_* = {φ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b,, c}, {b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

4. RGI-CLOSED SETS

A new generalized closed set called rgI-closed set is introduced and its relationship with other existing sets are established.

Definition 4.1 A subset A of an ideal topological space (X, τ,I) I) is said to be rgI-closed if $A_* \subseteq A$ whenever $A \subseteq U$ and U is regular open in X.

Proposition 4.2 Every I_{rg} -closed set is rgI-closed.

Proof. Let A be a I_{rg} -closed set and U be a regular open set containing A. Then $A_* \subseteq A^* \subseteq U$. Hence A is rgI-closed.

Remark 4.3 The converse is not true.

Example 4.4 Let X = {a, b, c, d}, τ = {φ,X, {a}, {b, c}, {a, b, c}}, I = {φ, {b}}.the rgI-closed sets are φ,X, {a}, {b}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d} and the I_{rg}- closed sets are φ,X, {b}, {d}, {a, b}, {a, c}, {a, d}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.The set {a} is rgI-closed but not I_{rg} -closed.

Proposition 4.5 Every closed set is rgI-closed.

Proof.Let A be a closed set and U be a regular open set containing A. Then $A_* \subseteq Cl(A) \subseteq U$. Hence A is rgI-closed.

Remark 4.6 The converse is not true.

Example 4.7 Let X = {a, b, c, d}, τ = {φ,X, {a}, {b, c}, {a, b, c}}, I = {φ, {b}}. The set {a} is rgI-closed but not closed.

Proposition 4.8 Every $I_{\hat{g}}$ closed set is rgI-closed.

Proof. Let A be a $I_{\hat{g}}$ -closed set and U be a regular open set containing A. Since every regular open set is semi open we have Then $A_* \subseteq A^* \subseteq U$. Hence A is rgI-closed.

Proposition 4.9 Every *-closed set is rgI-closed.

Proof.Let A be a *-closed set and U be a regular open set containing A. Then $A_* \subseteq A^* \subseteq A \subseteq U$. Hence A is rgI-closed.

Proposition 4.10 Every semi- *-closed set is rgI-closed.

Proof. Let A be a semi- *-closed set and U be a regular open set containing A. Then $A_* \subseteq A \subseteq U$. Hence A is rgI-closed.

Proposition 4.11 Every gI-closed set is rgI-closed.

Proof.Let A be a gI-closed set and U be a regular open set containing A. Since every regular open set is open then we have $A_* \subseteq U$. Hence A is rgI-closed.

Remark 4.12 The converse of the propositions 4.8,4.9,4.10,4.11 are not true. In Example 4.4 the *- closed sets are φ,X, {b}, {d}, {a, d}, {b, d}, {a, b, d}, {b, c, d} , the semi-*-closed sets are φ,X, {a}, {b}, {d}, {a, b}, {a, d}, {b, c}, {b, d}, {a, b, d}, {b, c, d}, gI-closed sets are φ,X, {a}, {b}, {d}, {a, b}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d} , $I_{\hat{g}}$ sets are φ,X, {b}, {d},{a, d}{b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}. The set {a} is rgI-closed but not $I_{\hat{g}}$ -closed. The set {a} is rgI-closed but not *-closed. The set {a, c} is rgI-closed but not semi-*-closed. The set {a, b, c} is rgI-closed but not gI-closed.

Proposition 4.13 Every rgI-closed and regular open set is semi-*-closed.

Proof. Let A be a rgI-closed and regular open set then $A_* \subseteq A$. Hence A is semi-*-closed.

Proposition 4.14 Let (X, τ, I) be an ideal space. Then either {x} is regular closed or $\{x\}^c$ is rgI-closed. for every x ∈ X.
Proof.Suppose {x} is not regular closed then $\{x\}^c$ is not regular open and the only regular open set containing $\{x\}^c$ is X and $(\{x\}^c)_* \subseteq$ X. Hence $\{x\}^c$ is rgI-closed.

Theorem 4.15 Every subset of an ideal topological space (X, τ, I) is rgI-closed if and only if every regular open set is semi-*-closed.
Proof.**Necessity**. Every subset A of X is rgI-closed. If A is regular open then $A_* \subseteq$ A which implies that A is semi-*-closed.
Sufficiency. If A ⊆ X and U is a regular open set containing A such that $A_* \subseteq U_* \subseteq$ U. Hence A is rgI-closed.

Theorem 4.16 Union of two rgI-closed sets is rgI-closed set.
Proof.Let A and B are two rgI-closed sets and U be a regular open set containing A ∪ B. Then $(A \cup B)_* = A_* \cup B_* \subseteq$ U. Hence A ∪ B is rgI-closed.

Remark 4.17 Intersection of two rgI-closed sets is need not be a rgI-closed set. In Example 4.4 the sets {a, c} and {b, c} are rgI-closed sets but their intersection {c} is not rgI-closed.

Theorem 4.18 If A is a rgI-closed set in an ideal topological space then $Cl_*(A)-A$ contains no non empty regular closed set.
Proof.Suppose A is rgI-closed and F be a regular closed subset of Cl_* (A) − A. Then F ⊆ X − A. Since A is rgI-closed $A_* \subseteq$ X − F. F ⊆ X − Cl_* (A). Therefore F ⊆ Cl_* (A) ∩ X − Cl_* (A) = φ. Hence Cl_* (A) − A contains no non empty regular closed set.

Remark 4.19 The converse of the theorem 4.18 is not true. In Example 4.4 Cl_* ({c})-{c}= {b, d} does not contain any non empty regular closed set. But {c} is not rgI-closed.

Theorem 4.20 Let A be a rgI-closed set of an ideal topological space (X, τ, I). The following are equivalent.
(i) A is a semi-*-closed set.
(ii) Cl_* (A) − A is a regular closed set.
(iii) A_* − A is a regular closed set.
Proof.(i) ⇒ (ii) If A is semi-*-closed then Cl_* (A) = A. Therefore Cl_* (A) − A = φ which is regular closed.
(ii) ⇒ (i) Let Cl_* (A)− A be regular closed. By the Theorem 4.18 Cl_* (A) − A contains no non empty regular closed set. Therefore Cl_* (A) − A = φ.i.e $(A \cup A_*) \cap A^C = \varphi$ ⇒ $A_* \cap A^c = \varphi$ ⇒ $A_* \subseteq$ A. Hence A is semi-*-closed.
(ii) ⇔ (iii). Let Cl_* (A) − A be regular closed.
Cl_* (A) − A = A ∪ A_* − A = A_* − A.

Theorem 4.21 Let A be a rgI-closed set in an ideal topological space (X, τ, I) such that A ⊆ B ⊆ A_*. Then B is also a rgI-closed set.
Proof.Let U be a regular open set containing B.
Then A ⊆ B ⊆ U ⇒ $B_* \subseteq A_* \subseteq$ U. Hence B is rgI-closed.

Theorem 4.22 Let A be a rgI-closed set in an ideal topological space (X, τ, I). Then A ∪ X − A_* is also rgI-closed.
Proof. Let U be a regular open set containing A ∪ X − A_*. X − U ⊆ (X − A) ∩ A_*. Since X-U is regular closed and A is rgI-closed by theorem 4.18 X − U = φ and so X = U. Thus X is the only regular open set containing A ∪ X − A_*. Hence A ∪ X − A_* is also rgI-closed.

Theorem 4.23 Let (X, τ, I) be an ideal space and A ⊆ Y ⊆ X where Y is regular open. A is rgI-closed in $(Y, \tau/Y, I_Y)$ if and only if A is rgI-closed in X.
Proof.Let A be rgI-closed in X. Let U be a regular open subset of (Y, τ/Y) such that A ⊆ U. Since Y is regular open in X then U is regular open in X. $A_*(I, \tau) \subseteq$ U. By theorem 2.6 $(A_*(I_Y, \tau/Y) = A_*(I, \tau) \cap Y \subseteq U \cap Y = U$. Hence A is rgI-closed in $(Y, \tau/Y, I_Y)$. Conversely let A be rgI-closed in $(Y, \tau/Y, I_Y)$. Then $A_*(I_Y, \tau/Y) = A_*(I, \tau) \cap Y \subseteq U \cap Y = U$ Therefore $A_*(I, \tau) \subseteq$ U. Hence A is rgI-closed in X.

Remark 4.24 If I = {φ} and SO(X, τ) = τ then $A^* = A_* =$ Cl(A) and rgI-closed sets coincides with rg-closed sets.

Theorem 4.25 In an ideal topological space (X, τ, I) where SO(X, τ) = τ , A is a *-dense itself and rgI-closed then A is rg-closed.
Proof. If A is *-dense in itself and rgI-closed, U is any regular open set containing A, then Cl_* (A) ⊆ U. By the Lemma 2.9 Cl(A) ⊆ U. Hence A is rg-closed.

Remark 4.26 The following table shows the relationship of rgI-closed sets with other existing sets. The symbol "1" in a cell means that a set implies the other and the symbol "0" means that a set does not imply the other set.

sets	closed	*-closed	Semi-*-closed	$I_{\hat{g}}$	I_{rg}	gI	I_g	rgI
closed	1	1	1	1	1	1	1	1
*-closed	0	1	1	1	1	1	1	1
Semi-*-closed	0	0	1	0	0	1	0	1
$I_{\hat{g}}$	0	0	0	1	1	1	1	1
I_{rg}	0	0	0	0	1	0	0	1
gI	0	0	0	0	0	1	0	1
I_g	0	0	0	0	1	1	1	1
rgI	0	0	0	0	0	0	0	1

5 RGI-OPEN SETS

Definition 5.1 The complement of a rgI-closed set is said to be rgI-open.

Theorem 5.2 Let A be a subset of an ideal topological space (X, τ, I). Then A ∪ X −A_* is rgI-closed if and only if A_* − A is rgI-open.
Proof. It follows from the fact that A ∪ X − A_* = X − (A_* −A).

Theorem 5.3 A subset A of an ideal topological space (X, τ, I) is rgI-open if and only if F ⊆ Int_* (A) whenever F is regular closed and F ⊆ A.
Proof.**Necessity**. Suppose that A is rgI-open and F is a regular closed set contained in A.X − A ⊆ X − F ⇒ Cl_*(X − A) ⊆ X − F. Then F ⊆ X − Cl_*(X − A) = Int_*(A).
Sufficiency. Suppose X −A ⊆ U where U is regular open. X −U ⊆ A and X −U is regular closed. X − U ⊆ Int_*(A). i.e Cl_*(X − A) ⊆ U. X-A is rgI-closed and hence A is rgI-open.

Theorem 5.4 If A is a rgI-open subset of an ideal space (X, τ, I) and $Int_*(A) \subseteq B \subseteq A$ then B is also a rgI-open subset of X.

Proof. Let F be a regular closed set contained in B. Then $F \subseteq B \subseteq A$. Since A is rgI-open and $F \subseteq Int_*(A)$ Since $Int_*(A) \subseteq Int_*(B)$) $\Rightarrow F \subseteq Int_*(B)$. By the theorem 5.3 B is rgI-open.

Theorem 5.5 In an ideal space (X, τ, I) if A is a rgI-open set then G = X whenever G is regular open and $Int_*(A) \cup (X - A) \subseteq G$.

Proof.Let A be a rgI-open set. If G is a regular open set such that $Int_*(A) \cup X - A) \subseteq G$. Then $X-G \subseteq (X-Int_*(A)) \cap A = (X-Int_*(A))-(X-A) = Cl_*(X-A)-(X-A)$.Since X-A is rgI-closed by theorem 4.18 X − G = φ and so X = G.

Remark 5.6 The converse of the theorem 5.5 is not true.

Example 5.7 Let X = {a, b, c, d}, τ = {φ,X, {a}, {b, c}, {a, b, c}}, I = {φ, {a}, {c}, {a, c}} The set A = {a, c, d} satisfies the conditions of the theorem but A is not rgI-open where τ_* = {φ,X, {a}, {b}, {a, b}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}}

Theorem 5.8 If A is a rgI-closed set in an ideal space (X, τ, I) then $Cl_*(A)-A$ is rgI-open.

Proof.Since A is rgI-closed by the theorem 4.18 φ is the only regular closed set contained in $Cl_*(A) - A$ and by the theorem 5.3 $Cl_*(A) - A$ is rgI-open.

Remark 5.9 The converse of the theorem 5.8 is not true. In Example 4.4 for set A = {c},
$Cl_*(A) - A$ = {b, d} is rgI-open but A is not rgI-closed.

6 RGI-CONINUITY AND RGI-IRRESOLUTENESS

Definition 6.1 A function f : (X, τ, I) → (Y, σ) is said to be rgI-continuous if $f^{-1}(V)$ is rgI-closed in X for every closed set V in Y.

Example 6.2 Let X = {a, b, c} = Y, τ = {φ,X, {a}}, I = {φ, {a}}, σ = {φ, Y, {a}, {b, c}}. The function f is defined as f(a) = b, f(b) = c, f(c) = a is rgI-continuous. Here the rgI-closed sets are the power set of X.

Definition 6.3 A function f : (X, τ, I) → (Y, σ, J) is said to be rgI-irresolute if $f^{-1}(V)$ is rgI-closed in X for every rgI-closed set V in Y.

Example 6.4 Let X = {a, b, c} = Y, τ = {φ,X, {a, b}}, I = {φ, {b}}, σ = {φ, Y, {a, c}, {b, c}, {c}},J = {φ, {b}}. The identity function f is rgI-irresolute.

Theorem 6.5 Every rgI-irresolute function is rgI-continuous.
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Proof.Let V be a closed in Y which is rgI-closed then $f^{-1}(V)$ is rgI-closed in X. Hence f is rgI-continuous.

Remark 6.6 The converse of the theorem 6.5 is not true.

Example 6.7 Let X = {a, b, c} = Y, τ = {φ,X, {a}, {b}, {a, b}}, I = {φ, {c}}, σ = {φ, {a, b}}.J = {φ, {b}}. The identity function is rgI-continuous but not rgI-irresolute. Since the rgI- closed sets in Y are the power set of Y and the rgI-closed sets of X are φ,X,{c}, {a, b}, {a, c}, {b, c}}

Theorem 6.8 Let $f : (X, \tau, I_1) \rightarrow (Y, \sigma, I_2)$, $g : (Y, \sigma, I_2) \rightarrow (Z, \eta, I_3)$ be any two functions then the following hold.

(i) gof is rgI-continuous if f is rgI-continuous and g is continuous.

(ii)gof is rgI-continuous if f is rgI-irresolute and g is rgI-continuous.

(iii)gof is rgI-irresolute if f is rgI-irresolute and g is rgI-irresolute.

Proof.(i) Let V be a closed set in Z. Since g is continuous $g^{-1}(V)$ is closed in Y. Since f is rgI-continuous $f^{-1}(g^{-1}(V))$ is rgI-closed in X. Hence gof is rgI-continuous.

(ii) Let V be a closed set in Z. Since g is rgI-continuous $g^{-1}(V)$ is rgI-closed in Y. Since f is rgI-irresolute $f^{-1}(g^{-1}(V))$ is rgI-closed in X. Hence gof is rgI-continuous.

(iii) Let V be a rgI-closed set in Z. Since g is rgI-irresolute $g^{-1}(V)$) is rgI-closed in Y. Since f is rgI-irresolute $f^{-1}(g^{-1}(V))$) is rgI-closed in X. Hence gof is rgI-irresolute.

Remark 6.9 Composition of two rgI-continuous functions need not be rgI-continuous.

Example 6.10 Let X = Y = Z = {a, b, c}, τ = {φ,X, {a}, {b}, {a, b}}, I_1 = {φ, {c}},
σ = {φ, Y, {a, b}}, I_2 = {φ, {b}}, η = {φ, Z, {a, c}, {b, c}, {c}}. f is defined as f(c) =a, f(a) = b, f(b) = c and g is the identity map . The functions f and g are rgI-continuous but their composition is not rgI-continuous. Since the rgI-closed sets of are φ,X, {c}, {a, b}, {a, c}, {b, c} and the rgI-closed sets of Y is the power set of Y. For the closed set {b} in Z $(gof)^{-1}(\{b\}) = f^{-1}(\{b\})$ = {a} is not rgI-closed in X.

7 APPLICATIONS

Definition 7.1 A proper non empty rgI-closed subset U of an ideal space (X, τ, I) is said to be maximal rgI-closed if any rgI-closed set containing U is either X or U.

Example 7.2 In Example 4.4 the sets {a, b, c},{a, c, d},{a, b, d} and {b, c, d} are maximal rgI-closed sets.

Remark 7.3 Every maximal rgI-closed sets is rgI-closed set. But a rgI-closed set need not be a maximal rgI-closed set. In Example 4.4 {a} is rgI-closed but not maximal rgI-closed.

Theorem 7.4 The following statements hold true for any ideal space (X, τ, I).

(i) Let F be a maximal rgI-closed set and G be a rgI-closed set. Then $F \cup G = X$ or $G \subset F$.

(ii) If F and G are maximal rgI-closed sets then $F \cup G = X$ or F = G.

Proof.(i) Let F be a maximal rgI-closed set and G be a rgI-closed set. If $F \cup G = X$ then there is nothing to prove. Assume that $F \cup G \neq X$. $F \subseteq F \cup G$. $F \cup G$ is rgI-closed. Since F is a maximal rgI-closed set $F \cup G = X$ or $F \cup G = F$. Hence $F \cup G = X$ or $G \subset F$.

(ii) Let F and G are maximal rgI-closed sets. If $F \cup G = X$, then there is nothing to prove. Assume that $F \cup G \neq X$. Then by (i) $F \subset G, G \subset F$ which implies that F = G.

Definition 7.5 A function f : (X, τ, I) → (Y, σ) is said to be maximal rgI-continuous if $f^{-1}(V)$ is maximal rgI-closed in X for every closed set V in Y.

Theorem 7.6 Every surjective maximal rgI-continuous function is rgI-continuous.

Proof.Let f : (X, τ, I) → (Y, σ) be a surjective maximal rgI-continuous map. The inverse image of φ and Y are rgI-closed sets in X. Let V be a closed set in Y then $f^{-1}(V)$ is a maximal rgI-closed set in X which is a rgI-closed set in X. Hence f is rgI-continuous.

Remark 7.7 The converse of the theorem 7.6 is not true.

Example 7.8 Let X = {a, b, c} = Y, τ = {φ,X, {a}, {b}, {a, b}}, I = {φ, {c}}, σ = {φ, Y, {a, b}}.
The identity function from X to Y is rgI-continuous but not maximal rgI-continuous.

Remark 7.9 Composition of two maximal rgI-continuous functions need not be maximal rgI-continuous.

Example 7.10 Let X = {a, b, c, d}, Y = Z = {a, b, c}, τ = {φ,X, {a}, {b, c}, {a, b, c}},
I = {φ, {b}}, σ = {φ, Y, {a}}, J = {φ, {c}}. The function f is defined as f(a) = b =f(c), f(b) = a, f(d) = c. Here f is maximal rgI-continuous where f : (X, τ, I) → (Y, σ).
The function g : (Y, σ, J) → (Z, η) with η = {φ,Z, {b}} is the identity function. g is also maximal rgI-continuous. But their composition g0f is not rgI-continuous. Since for the

closed set {a, c} in Z $(gof)^{-1}$ ({a, c}) = f^{-1} ({a, c}) = {b, d} is not maximal rgI-closed in X. Hence gof is not rgI-continuous.

Theorem 7.11 Let f : (X, τ, I) → (Y, σ) be a maximal rgI-continuous function and f :(Y, σ, I) → (Z, η) be surjective continuous function then gof : (X, τ, I) → (Z, η) is a maximal rgI-continuous function.

Proof.Let V be a nonempty proper closed set in Z. Since g is continuous g^{-1}(V) is a proper nonempty closed set in Y. Since f is maximal rgI-continuous $f^{-1}(g^{-1}(V))$ is a maximal rgI-closed set in X.

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