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On interval valued supra- openness

F. M. Sleim
Zagazig University, Faculty of Science

ABSTRACT

In [9]a new definition of a fuzzy set was given by K.Kandil et al by introducing the concept of an interval valued fuzzy set . In this paper a new definition of supra -fuzzy topological (resp.supra-fuzzy proximity ,supra - fuzzy uniform) space is given by introducing the concept of an interval valued supra-fuzzy topological (resp.fuzzy proximity ,fuzzy uniformity) space . The connection between interval valued supra-fuzzy topological ,interval valued supra-fuzzy proximity and interval valued supra-fuzzy uniformity space are investigated . Throughout this paper ,the class of all interval valued fuzzy sets on a non-empty set X will be denoted by Π^X and the class of all fuzzy sets on non-empty set X will be denoted by I^X

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Keywords:the supra-opennes ,supra-proximity ,supra-uniformity ,interval valued fuzzy set and interval valued fuzzy topology .

1. INTERVAL VALUED SUPRA-FUZZY TOPOLOGICAL SPACE

The concept of a supra- fuzzy topological space has been introduced as follows [11] A collection S⊂$I^X$is a supra- fuzzy topology on X if 0 , 1 ∈S and S is closed under arbitrary suprema . The pair (X, S)is supra- fuzzy topological space .

Definition1.1[2 ,9] An interval valued fuzzy set (Ivfs for short)is a set $\underline{\mathbf{A}}$**=**$(A_1, A_2)\in I^X\times I^X$**such that**$A_1\leq A_2$**.The family of all interval valued fuzzy sets on a given non empty set X will be dented by** Π^X

The Infs $\underline{1}$ **= (1, 1)is called the universal Ivfs and the Ivfs** $\underline{o}$ **=(0 ,0) is called the empty Ivfs .**

The operations on Π^X **are given by the following**

Let$\underline{\mathbf{A}}$,$\underline{\mathbf{B}}$$\in\Pi^X$

$[1]\underline{A} = \underline{B}\Leftrightarrow A_1 = B_1, A_2 = B_2$

$[2]\underline{A} \leq \underline{B}\Leftrightarrow A_1\leq B_1, A_2\leq B_2$

$[3]\underline{A} \vee \underline{B} = (A_1\vee B_1, A_2\vee B_2)$

$[4]\underline{A} \wedge \underline{B} = (A_1\wedge B_1, A_2\wedge B_2)$

$[5]\underline{A}^c=(A_2^c, A_1^c)$

Definition1.2[9]

The family $\eta \subseteq\Pi^X$ **is called an interval -valued fuzzy topology on X iff** η **contains** $\underline{1}$ **,** $\underline{o}$ **and it is closed under finite intersection and arbitrary union . The pair (X ,**η **)is called interval -valued fuzzy topological space(Ivfts for short). Any interval -valued fuzzy set** $\underline{\mathbf{A}}\in\eta$ **is called an open Ivfs and the complement of** $\underline{\mathbf{A}}$ **denoted by** $\underline{A}^c$ **is called a closed Ivfs . The family of all closed interval -valued fuzzy sets is denoted by** η^c

In the following we define the concepts of interval valued supra- fuzzy topology , interval valued supra- fuzzy closure operator ,and interval valued supra- fuzzy interior operator on a non empty set X and we study the relation between them

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Definition1.3 :The family $\eta^\star\subseteq\Pi^X$**is called an interval valued supra -fuzzy topology on X if** $\eta^\star$**contains** $\underline{1}$ **and** $\underline{o}$ **and it is closed under arbitrary union . The pair (X ,**$\eta^\star$**)is called an interval valued supra- fuzzy topological space (Ivsfts)**

Definition1.4: A mapping C^* **:**$\Pi^X\to\Pi^X$ **is called an interval -valued supra- fuzzy closure operator on X if it satisfies the following axioms:**

$[c1]C^*(\underline{\mathbf{1}})=\underline{1}$, $C^*(\underline{\mathbf{o}})=\underline{o}$

$[c2]C^*(\underline{A})\geq(\underline{A})$

$[c3]C^*(\underline{A} \vee \underline{B})\geq C^*(\underline{A})\vee C^*(\underline{B})$

$[c4]C^*C^*(\underline{A})=C^*(\underline{A})$

Definition1.5: A mapping Int^***:**$\Pi^X\to\Pi^X$ **is called an interval -valued supra- fuzzy interior operator on X if it satisfies the following axioms:**

$[i1]Int^*(\underline{\mathbf{1}})=\underline{1}$, $Int^*(\underline{\mathbf{o}})=\underline{o}$

$[i2]Int^*(\underline{A})\leq(\underline{A})$

$[i3]Int^*(\underline{A}\wedge\underline{(}B)\leq Int^*(\underline{A}) \wedge Int^*(\underline{B})$

$[i4]Int^*Int^*(\underline{A})=Int^*(\underline{A})$

Proposition 1.1: Let(X ,η**)be an Ivsfts . The operator cl(**$\underline{A}$**)=**$\wedge\{\underline{\mathbf{F}} \in\eta^c$**:**$\underline{F}\geq\underline{A}\}$**is an interval valued supra-fuzzy closure operator on X**

Proof

$[c1]$ **Since** $\underline{1}$,$\underline{\mathbf{o}}\in \eta^c$ **then cl(**$\underline{1}$**)=**$\underline{1}$ **cl(**$\underline{o}$**)=**$\underline{o}$

$[c2]$ **From the definition we have** $cl(\underline{\mathbf{A}})\geq\underline{A}$

$[c3]$ **cl(**$\underline{A} \vee \underline{B}$**)=**$\wedge\{\underline{\mathbf{F}}\in\eta^c$**:**$\underline{F} \geq \underline{A}\vee\underline{B}\}$

Let $\underline{\mathbf{F}} \in \Pi^X$ **such that** $\underline{F}\geq \underline{A}\vee\underline{B} \Rightarrow\underline{F}\geq\underline{A}$**and**$\underline{F}\geq\underline{B}\Rightarrow \{\underline{\mathbf{F}}\in\Pi^X$ **:**$\underline{F}\geq\underline{A}\vee\underline{B}\} \subseteq\{\underline{\mathbf{F}}\in\Pi^X$ **:**$\underline{F}\geq\underline{A}\}$**and**$\{\underline{\mathbf{F}}\in\Pi^X$ **:**$\underline{F}\geq\underline{A}\vee\underline{B}\}$ $\subseteq\{\underline{\mathbf{F}}\in\Pi^X$ **:**$\underline{F}\geq\underline{B}\}$

$\Rightarrow$ **cl(**$\underline{A}\vee\underline{B}$**)**$\geq$**cl(**$\underline{A}$**)**$\vee$**cl(**$\underline{B}$**)**

2. INTERVAL VALUED SUPRA-FUZZY PROXIMITY

The concept of a supra- fuzzy proximity space has been defined [7 ,10] as follows

Definition 2.1: A binary relation δ^***on** I^X**is called supra-fuzzy proximity on X if it satisfies**

$[sp1](A, B)\in\delta^* \Rightarrow$**(B ,A)**$\in\delta^*$

$[sp2](A, B)\in\delta^*$ **or(A ,C)** $\in\delta^*$ $\Rightarrow$**(A ,B**$\vee$**C))**$\in\delta^*$

$[sp3](o, 1)\notin\delta^*$

$[sp4](A, B)\notin\delta^*\Rightarrow\exists$**C**$\in I^X$**such that (A ,C)**$\notin\delta^* and (1 - C, B)\notin\delta^*$

$[sp5](A, B)\notin\delta^*\Rightarrow$**A**$\leq$**1-B**

In the following we generalize the concept of supra- fuzzy proximity space to the concept an interval valued supra-fuzzy proximity space (Ivsfps for short)

Definition 2.2: Let X $\neq \phi$,a binary relation$\delta^*\subset\Pi^X\times\Pi^X$ is called an interval valued supra-fuzzy proximity on X iff it satisfied the following

$[Ivsp1]$($\underline{\mathbf{A}}$,$\underline{\mathbf{B}}$)$\in\delta^*$ $\Rightarrow$($\underline{B}$,$\underline{A}$)$\in\delta^*$

$[Ivsp2]$($\underline{\mathbf{A}}$,$\underline{\mathbf{B}}$)$\in\delta^*$ **or(** $\underline{A}$,$\underline{C}$) $\in\delta^*$ $\Rightarrow$($\underline{A}$,$\underline{B}\vee\underline{C}$))$\in\delta^*$

$[Ivsp3]$($\underline{\mathbf{o}}$,$\underline{\mathbf{1}}$)$\notin\delta^*$

$[Ivsp4]$($\underline{\mathbf{A}}$,$\underline{\mathbf{B}}$)$\notin\delta^*\Rightarrow\exists\underline{C}\in\Pi^X$ **such that** ($\underline{A}$,$\underline{C}$)$\notin\delta^* and$ ($\underline{C}^c$,$\underline{B}$)$\notin\delta^*$

$[Ivsp5]$($\underline{\mathbf{A}}$,$\underline{\mathbf{B}}$)$\notin$ $\delta^*\Rightarrow$ $\underline{A}\leq\underline{B}^c$

The pair (X , δ^*)is called an interval valued supra-fuzzy proximity space .For each interval valued supra-fuzzy proximity space δ^*on X induces an interval valued supra-fuzzy topology on X as follows

Proposition 2.1: Let (X ,δ^*)be an Ivsfps and let $\underline{A}\in\Pi^X$ .Then the mapping $cl_\delta(\underline{A})$=$\wedge\{\underline{B}\in\Pi^X$: ($\underline{A}$,$\underline{B}^c$)$\notin\delta^*\}$ is supra-fuzzy closure operator .

Proof

$[Ivc1] Since$($\underline{\mathbf{o}}$,$\underline{\mathbf{1}}$)$\notin\delta$ **, then** $cl_\delta(\underline{o})$=$\underline{o}$ **and Since** ($\underline{1}$, $\underline{o}$) $\notin\delta$ **, then** $cl_\delta(\underline{1})$= $\underline{1}$

$[Ivc2] Since$ $cl_\delta(\underline{A})$=$\wedge\{\underline{B}\in\Pi^X$: ($\underline{A}$,$\underline{B}^c$)$\notin\delta^*\}$**and let** ($\underline{A}$,$\underline{B}^c$)$\notin\delta^*$ $\Rightarrow\underline{A}\leq\underline{B}\Rightarrow\wedge\{\underline{B}\in\Pi^X$: ($\underline{A}$,$\underline{B}^c$)$\notin\delta^*\}\geq$ $\wedge\{\underline{B}\in\Pi^X$: $\underline{A}\leq\underline{\mathbf{B}}\}$ **then**$cl_\delta(\underline{A})\geq\underline{A}$

$[Ivc3] Since(\underline{A}\vee\underline{B}$,$\underline{C}^c$)$\notin\delta^*$ $\Leftrightarrow$ ($\underline{C}^c$,$\underline{A}\vee\underline{B}$)$\notin\delta^*\Rightarrow$ ($\underline{C}^c$,$\underline{A}$)$\notin\delta^*$ **or** ($\underline{C}^c$,$\underline{B}$)$\notin\delta^*$ **. Then** $cl_{\delta^\star}(\underline{A})\vee cl_{\delta^\star}(\underline{B})$ $\leq cl_{\delta^\star}(\underline{A}\vee\underline{B})$

Proposition 2.2 Let (X ,$\delta^\star$) be an Ivsfps .The family $\tau_{\delta^\star}$ = $\{\underline{A}\in\Pi^X$:$cl_{\delta^\star}(\underline{A}^c)$= $\underline{A}^c\}$ is an interval valued supra-fuzzy topology on X , and it is called the interval valued supra-fuzzy topology induced by $\delta^\star$

Proof : It is obvious

3. AN INTERVAL VALUED SUPRA-FUZZY UNIFORM SPACE

In [1 ,8] the crisp supra- uniform space was defined as follows : Let D be the set of maps u:$I^X\to$ I^X such that the following conditions : u(ϕ) = ϕ , u(A)$\geq$ A and u ($\vee_i$ A_i)=$\vee_i(\cup A_i)$ .A crisp supra- fuzzy uniformity on X is a subset $U^\star$ of D which satisfies

$[u1]$ $U^\star\neq\phi$

$[u2]u\in U^\star\Rightarrow$ $u^{-1}\in U^\star$

$[u3]u\in U^\star$ **, u $\leq$ v $\Rightarrow$ v$\in U^\star$**

$[u4]u\in U^\star\Rightarrow\exists$ **v$\in U^\star$such that v$\circ$v $\leq$ u**

where u^{-1} **:**$I^X\to$ I^X **such that**u^{-1}**(A) =** $\wedge\{$**B** $\in I^X$**: u** (B^c)$\leq A^c$ $\}$

In the following we can define an interval valued supra-fuzzy uniformity on X as follows : Let D be the set of maps U :$\Pi^X\to\Pi^X$ such that U($\underline{o}$)=$\underline{o}$, U($\underline{A}$)$\geq\underline{A}$,U ($\vee_i$ A_i)=$\vee_i$(U(A_i))

An interval valued supra-fuzzy uniformity on X (for short Ivsfu)is a subset $U^\star\subseteq$ D such that

$[Ivsfu1]$ $U^\star\neq\phi$

$[Ivsfu2]u\in U^\star\Rightarrow$ $u^{-1}\in U^\star$

$[Ivsfu3]u\in U^\star$ **, u $\leq$ v $\Rightarrow$ v$\in U^\star$**

$[Ivsfu4]u\in U^\star\Rightarrow\exists$ **v$\in U^\star$such that v$\circ$v $\leq$ u**

where U^{-1}:$\Pi^X\to\Pi^X$such that U^{-1}($\underline{A}$)=$\wedge\{\underline{B}\in$ Π^X : U ($\underline{B}^c$) $\leq$ $\underline{A}^c\}$ and $\underline{B}^c$ = (B_2^c ,B_1^c) , $B_1\leq B_2$

The pair(X , $U^\star$) is called an interval valued supra-fuzzy uniform space

Proposition 3.1 : Let (X ,$U^\star$) be an Ivsfus.Then the family $\tau_{U^\star}$ define by $\underline{A}\in\tau_{U^\star}$ $\Leftrightarrow\underline{A}$=$\vee\{\underline{B}\in$ Π^X :u($\underline{B}$)$\leq\underline{A}$ for some u $\in$U$\}$is an interval valued supra-fuzzy topology on X

Proof

$[Ivst1]$ **Since u($\underline{B}$)$\leq\underline{1}\forall$ u$\in$U ,then$\vee\{\underline{B}\in$ Π^X :u($\underline{B}$)$\leq\underline{1}$ for some u $\in$U$\}$=$\underline{1}$. thus $\underline{1}\in\tau_{U^\star}$**

Since u($\underline{o}$)=$\underline{o}$ $\forall$u$\in$D ,then$\underline{o}\in\tau_{U^\star}$

$[Ivst2]$ **Let$\underline{A_\alpha}$ $\in\tau_{U^\star}$ for all α , then$\underline{A_\alpha}$ =$\vee\{\underline{B}\in$ Π^X :u($\underline{B}$)$\leq\underline{A_\alpha}$ for some u $\in$U$\}$. Thus $\vee_\alpha\underline{A_\alpha}$ = $\vee_\alpha$ ($\vee\{\underline{B}\in$ Π^X :u($\underline{B}$)$\leq\underline{A_\alpha}$ for some u $\in$U$\}$). This implies that $\vee_\alpha\underline{A_\alpha}\in\tau_{U^\star}$**

Hence$\tau_{U^\star}$ is an interval valued supra-fuzzy topology on X and it is called an interval valued supra-fuzzy topology induced by $U^\star$

Proposition 3.2 : Let (X ,$U^\star$) be an Ivsfus.Then the family $\delta_{U^\star}$ $\in\Pi^X$ $\times\Pi^X$ define by ($\underline{A}$,$\underline{B}$)$\in\delta_{U^\star}\Leftrightarrow$ u ($\underline{A}$)$\not\leq\underline{B}^c$ $\forall$u$\in U^\star$ is an interval valued supra-fuzzy proximity on X associated with the uniformity$U^\star$

proof :

$[Ivsp1]$ **Let ($\underline{A}$,$\underline{B}$)$\notin\delta_{U^\star}\Leftrightarrow$u($\underline{A}$)$\leq$ $\underline{B}^c$ for some u$\in U^\star$ $\Rightarrow\exists u^{-1}\in U^\star$ s .t u^{-1}($\underline{B}$)$\leq\underline{A}^c\Rightarrow$($\underline{B}$,$\underline{A}$)$\notin\delta_{U^\star}$**

$[Ivsp2]$**Let ($\underline{A}$,$\underline{B}\vee\underline{C}$)$\notin\delta_{U^\star}\Rightarrow$u($\underline{A}$)$\leq$ ($\underline{\mathbf{B}}\vee\underline{\mathbf{C}}$)^c for some u$\in U^\star$ $\Rightarrow$u($\underline{A}$)$\leq\underline{B}^c$ $\wedge\underline{C}^c$for some u$\in U^\star$ $\Rightarrow$($\underline{A}$,$\underline{B}$)$\notin\delta_{U^\star}$ and($\underline{A}$,$\underline{C}$)$\notin\delta_{U^\star}$**

$[Ivsp3]$ **Since $\forall$u$\in U^\star$, u($\underline{o}$)=$\underline{o}$ then$\delta_{U^\star}$($\underline{o}$, $\underline{1}$)=0**

$[Ivsp4]$($\underline{A}$,$\underline{B}$)$\notin\delta_{U^\star}$ **$\Rightarrow$u($\underline{A}$)$\leq$ $\underline{B}^c$ for some u$\in U^\star$**

$\Rightarrow\exists$v$\in U^\star$ s . t v$\circ$v$\leq$ u ,u($\underline{A}$)$\leq$ $\underline{B}^c$

$\Rightarrow\exists$v$\in U^\star$ s . t . v(v($\underline{A}$))$\leq$u($\underline{A}$),u($\underline{A}$)$\leq$ $\underline{B}^c$

$\Rightarrow\exists$v$\in U^\star$ s . t .v($\underline{C}$)$\leq$ $\underline{B}^c$ where$\underline{C}$ = u($\underline{A}$)

$\Rightarrow$ ($\underline{A}$,$\underline{C}^c$) $\notin\delta_{U^\star}$ and($\underline{C}$,$\underline{B}$)$\notin\delta_{U^\star}$

$\Rightarrow\exists\underline{E}\in$ Π^X s . t.($\underline{A}$,$\underline{E}$) $\notin\delta_{U^\star}$ and ($\underline{E}^c$,$\underline{B}$)$\notin\delta_{U^\star}$

$[Ivsp5]$**Let($\underline{A}$,$\underline{B}$) $\notin\delta_{U^\star}$ $\Rightarrow\exists$u$\in U^\star$ s . t. u($\underline{A}$)$\leq$ $\underline{B}^c$ $\Rightarrow\underline{A}\leq\underline{B}^c$**

Hence$\delta_{U^\star}$ is an interval valued supra-fuzzy proximity on X and it is called an interval valued supra-fuzzy proximity induced by $U^\star$

Theorem 3.1 : For any interval valued supra- fuzzy uniformity $U^\star$ on X then$\tau_{U^\star}$=$\tau_{\delta_{U^\star}}$

proof :$\underline{A}\in\tau_{U^\star}\Leftrightarrow\underline{A}$=$\vee\{\underline{B}\in$ Π^X :u($\underline{B}$)$\leq\underline{A}$ for some u $\in$U$\}$

$\Leftrightarrow\underline{A}^c$=$\wedge\{\underline{B}^c\in$ Π^X :($\underline{B}$,$\underline{A}^c$)$\notin\delta_{U^\star}\}$

$\Leftrightarrow\underline{A}^c$=$\wedge\{\underline{B}^c\in$ Π^X :($\underline{A}^c$,$\underline{B}$)$\notin\delta_{U^\star}\}$

Hence$\underline{A}^c$ = $cl_{\delta_{U^\star}}$($\underline{A}^c$)

Thus $\underline{A}\in\tau_{U^\star}\Leftrightarrow\underline{A}\in\tau_{\delta_{U^\star}}$.

4. CONCLUSION

Today ,the applications of interval -valued fuzzy sets are taken into account with more and more experts and scholars . In this paper , interval valued supra-fuzzy interior operator , interval valued supra-fuzzy closure operator are generalized supra-fuzzy interior operator , supra-fuzzy closure operator and study the relation between them . Also , the concepts of the interval -valued supra fuzzy topological spaces , the interval -valued supra fuzzy proximity spaces , the interval -valued supra fuzzy uniform spaces are generalized to supra fuzzy topological spaces , supra fuzzy proximity spaces , supra fuzzy uniform spaces ,and we study the relation between them .

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