Reliability Measures for Fuel System in Diesel Engine

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ABSTRACT
The authors have considered here helps to transfer the fuel from the fuel tank to the engine and regulate its flow into the cylinder depending on speed and local requirement. Here, a multi-component fuel system in diesel engine, comprised two subsystems *A* and *B* in series has been considered. In this system the fuel is injected into the cylinder using an injection pump and injectors. The block diagram depicts the fuel system in an *M*-cylinder diesel engine. The diesel fuel from the tank is pump to the fuel injection device. This device consists of small plunger pumps operating to push the fuel through the injectors (nozzles) into the cylinders one can operated one pump from the fuel injection pumps at the appropriate time i.e., towards the end of the compression stroke the fuel is pushed through the small hole in the injector and is atomized as it enters into the cylinder.

Keywords
Supplementary variable techniques, Laplace transform, Abel's Lemma, Cost function, availability function and reliability function.

1. INTRODUCTION
Suppose a multi-component fuel system in diesel engine, comprised two subsystems *A* and *B* in series has been considered. In this system the fuel is injected into the cylinder using an injection pump and injectors. The block diagram depicts the fuel system in an *M*-cylinder diesel engine. The diesel fuel from the tank is pump to the fuel injection device. This device consists of small plunger pumps operating to push the fuel through the injectors (nozzles) into the cylinders one can operated one pump from the fuel injection pumps at the appropriate time i.e., towards the end of the compression stroke the fuel is pushed through the small hole in the injector and is atomized as it enters into the cylinder.

Now *M*-cylinders is *j*-out-of-M: *D* system) more than M-*j*+1 cylinders s must fail for *A* to fail), while sub-system *B* may fail due to failure of anyone of its units. Also the whole system can fail due to critical human error and environmental pressure from the normal efficiency and degraded state.

The lifetime of the active units depend on each other in having simultaneous failure of all the operating units and repair times are distributed quite generally. *A* failed unit is repaired at a single service channel. [1], [2], [3] Earlier researchers have studied the model for *j* =1, 2; under the assumption that the repair of a unit is possible only when the system stops operating and the life times of the active units are mutually independent, and have studied the model with exponential failure distribution under the different repair policies. [2] The concept of human error and environmental pressure has been examined. The availability and reliability function for the system are obtained simultaneously by using supplementary variable technique.

2. ASSUMPTIONS
(*i*). Initially, all the units are operating.

(*ii*). Failure rates are exponential.

(*iii*). The system goes to complete breakdown if more than *M*-*j* units in A are simultaneously failed or if any failure in B occurs, critical human error or under environmental disasters.

(*iv*). *A* failed unit is repaired at a single service channel.

(*v*). *A* repaired unit is put into the operation immediately.

(*vi*). *A* repaired unit is assumed to behave like new after repair.

(*vii*). Repair times are distributed quite generally. The system has three modes; good, degraded and failed state.

(viii). The system has three modes; good, degraded and failed state.

3. NOTATIONS

$\int$	:	$\int_0^\infty$ Otherwise stated
M, N	:	Number of units in subsystem *A* and *B*
$\lambda / \lambda' / \lambda'' / e_1 / e_2 / h_1 / h_2$	:	Constant failure rates from state 0 to *Q* 0 to 3/0 to1/1to 2/0 to 5/ 1 to 5/0 to 6/1 to 6.
$\phi_i(x), S_i(x)$	:	Transition repair rate of the subsystem *B*/pdf of repair rate
$\phi(r)$	:	Transition repair rate of subsystem *A* *pdf* of repair rate

$\phi_{//}(z)$: Transition repair rate for critical human error

$\phi_e(k)$: Transition repair rate for environmental Pressure

$\bar{f}(s)$: Laplace transform of $f(t)$

$P_{M.N}(t)$: The probability that at time t, the system is operating in the state of normal efficiency.

$P_{M-j.N}(y,t)\Delta$: The probability that at time t, the system is in degraded state due to the failure units of subsystem A and elapsed repair time lies in the interval (y, y + Δ)

$P_{M-j+1.N}(r,t)\Delta$: The probability that at time t, the system is in the failed state due to the failure of $M-j+1$ units of subsystem A. The elapsed repair time lies in the (r, r + Δ) .

$P_{M.i}(x,t)$: The probability that at time t, the system is in failed state due to the failure of i^{th}-unit of subsystem B. The elapsed repair time lies in the interval (x, x + Δ).

$P_{M-j.i}(x,t)\Delta$: The probability that at time t the system is in failed state. The elapsed repair time lies in the interval (x, x + Δ).

$P_H(z,t)$: The probability that at time t the system is in failed state due to critical human error. The elapsed repair time lies in interval (z, z + Δ).

$P_e(k,t)$: The probability that at time t the system is in failed state due to environmental disorders. The elapsed repair time lies in the interval (k, k + Δ).

4. FORMULATION OF THE MATHEMATICAL MODEL

The analysis crucially depends on the method of supplementary variables, and the supplementary variable × denotes the elapsed time that a unit has been undergoing repair. Viewing the nature of the problem, the following set of difference-differential equations is obtained as follows:

$$\left[\frac{\partial}{\partial t}+\lambda+\lambda'+e_1+h_1\right]P_{M,N}(t)=\sum_{p=1}^{j}\int P_{M-j.N}(y,t)\phi_p(y)dy+\sum_{i=1}^{M}\int P_{M,i}(x,t)\phi_i(x)dx+\int P_{M-j+1,N}(r,t)\phi(r)dr+\int P_e(k,t)\phi_e(k)dk+\int P_{//}(z,t)\phi_{//}(z)dz \quad \ldots(1)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\lambda+\lambda''+e_2+h_2+\phi_p(y)\right]P_{M-j,N}(y,t)=0 \quad \ldots(2)$$

$$\left[\frac{\partial}{\partial r}+\frac{\partial}{\partial r}+\phi(r)\right]P_{M-j+1,N}(r,t)=0 \quad \ldots(3)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\phi_i(x)\right]P_{M,1}(x,t)=0 \quad \ldots(4)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\phi_i(x)\right]P_{M-j,i}(x,t)=0 \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial k}+\frac{\partial}{\partial t}+\phi_e(k)\right]P_e(k,t)=0 \quad \ldots(6)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\phi_{//}(z)\right]P_{//}(z,t)=0 \quad \ldots(7)$$

These equations can be solved under following boundary and initial conditions:

4.1 Boundary Conditions

$$P_{M-j,N}(0,t)=\lambda' P_{M,N}(t)\sum_{i=1}^{M}\int P_{M-j,i}(x,t)\phi_i(x)dx+\int P_e(k,t)\phi_e(k)dk+\int P_H(z,t)\phi_H(z)dz \quad \ldots(8)$$

$$P_{M-j+1,N}(0,t)=\lambda'' P_{M-j,N}(t) \quad \ldots(9)$$

$$P_{M,t}(0,t)=\lambda P_{M,N}(t) \quad \ldots(10)$$

$$P_{M-j,t}(0,t)=\lambda P_{M-i,N}(t) \quad \ldots(11)$$

$$P_e(0,t)=e_1P_{M,N}(t)+e_2P_{M-j,N}(t) \quad \ldots(12)$$

$$P_H(0,t)=h_1P_{M,N}(t)+h_2P_{M-j,N}(t) \quad \ldots(13)$$

4.2 Initial Condition

$P_k(0)=1$, as $k = M, N$ otherwise zero ... (14)

5. CYLINDRICAL FUEL SYSTEM AND TRANSITION STATE DIAGRAM

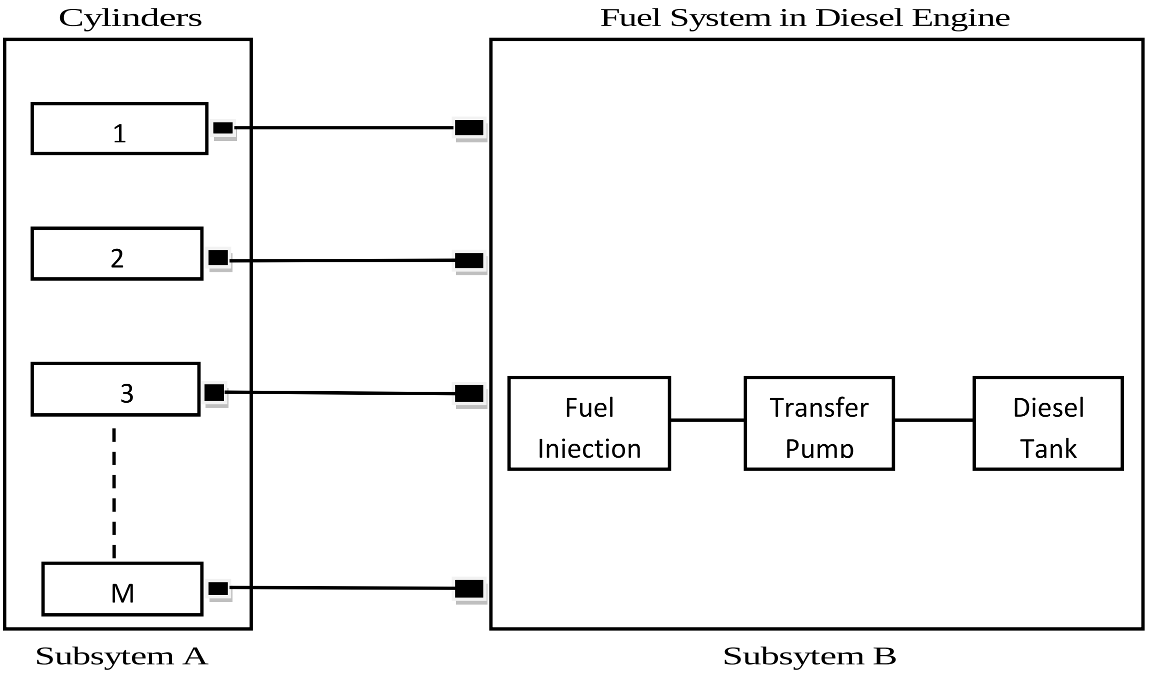
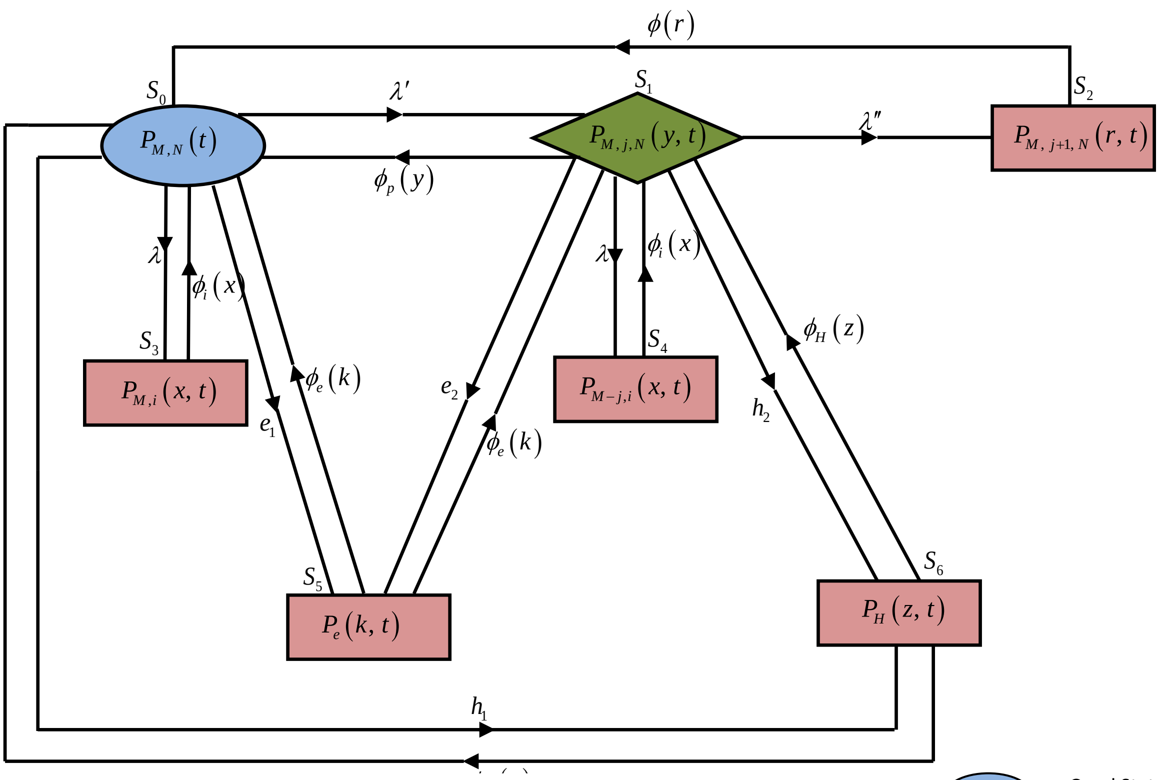


Figure 1: Block Diagram of Cylindrical Fuel System



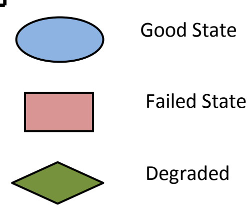


Figure 2: Transition State Diagram

6. SOLUTIONS OF MATHEMATICAL MODEL

Taking Laplace transforms of the equations (1) through (13) and using initial conditions we may get:

$$[s+\lambda+\lambda'+e_1+h_1]\bar{P}_{M,N}(s)=1+\sum_{p=1}^{j}\int \bar{P}_{M-j,N}(y,s)\phi_p(y)dy$$

$$+\sum_{i=1}^{M}\int \bar{P}_{M,i}(x,s)\phi_i(x)dx+\int \bar{P}_{M-j+1,N}(r,s)\phi(r)dr$$

$$+\int \bar{P}_e(k,s)\phi_e(k)dk+\int \bar{P}_H(z,s)\phi_H(z)dz \quad \dots (15)$$

$$\left[\frac{\partial}{\partial y}+s+\lambda+\lambda'+e_2+h_2+\phi_p(y)\right]\bar{P}_{M-j,N}(y,s)=0 \quad \dots (16)$$

$$\left[\frac{\partial}{\partial r}+s+\phi(r)\right]\bar{P}_{M-j+1,N}(r,s)=0 \quad \dots (17)$$

$$\left[\frac{\partial}{\partial x}+s+\phi_i(x)\right]\bar{P}_{M,1}(x,s)=0 \quad \dots (18)$$

$$\left[\frac{\partial}{\partial x}+s+\phi_i(x)\right]\bar{P}_{M,j,i}(x,s)=0 \quad \dots (19)$$

$$\left[\frac{\partial}{\partial k}+s+\phi_e(k)\right]\bar{P}_e(k,s)=0 \quad \dots (20)$$

$$\left[\frac{\partial}{\partial z}+s+\phi_H(z)\right]\bar{P}_H(z,s)=0 \quad \dots (21)$$

$$\bar{P}_{M-j,N}(0,s)=\lambda'\bar{P}_{M,N}(s)+\sum_{i=1}^{M}\int \bar{P}_{M-j,i,s}(x,s)\phi_i(x)dx$$

$$+\int \bar{P}_e(k,s)\phi_e(k)dk+\int \bar{P}_H(z,s)\phi_H(z)dz \quad \dots (22)$$

$$\bar{P}_{M-j+1,N}(0,s)=\lambda''\bar{P}_{M-j,N}(s) \quad \dots (23)$$

$$\bar{P}_{M,i}(0,s)=\lambda\bar{P}_{M,N}(s) \quad \dots (24)$$

$$\bar{P}_{M-j,i}(0,s)=\lambda\bar{P}_{M-j,N}(s) \quad \dots (25)$$

$$\bar{P}_e(0,s)=e_1\bar{P}_{M,N}(s)+e_2\bar{P}_{M-j,N}(s) \quad \dots (26)$$

$$P_H(0,s)=h_1\bar{P}_{M,N}(s)+h_2\bar{P}_{M-j,N}(s) \quad \dots (27)$$

Integrating (18) and using (24) one obtain:

$$\bar{P}_{M,j}(x,s)=\lambda\bar{P}_{M,N}(s)\exp.[-sx-\int_0^x\phi_i(x)dx]$$

$$\bar{P}_{M,i}(s)=\lambda\bar{P}_{M,N}(s)D_{\phi_i}(s) \quad \dots (28)$$

Integrating equation (18) by making use of (24) we get:

$$\bar{P}_{M-j,i}(x,s)=\lambda\bar{P}_{M-j,N}(s)\exp.\left[-sx-\int_0^x\phi_i(x)dx\right]$$

$$\bar{P}_{M-j,i}(s)=\lambda\bar{P}_{M-j,N}(s)D_{\phi_i}(s) \quad \dots (29)$$

Integrating (17) using (23) one may get

$$\bar{P}_{M-j+1,i}(r,s)=\lambda''\bar{P}_{M-j,N}(s)\exp.\left[-sx-\int_0^r\phi(r)dr\right]$$

$$\bar{P}_{M-j+1,i}(s)=\lambda''\bar{P}_{M-j,N}(s)D_{\phi}(s) \quad \dots (30)$$

Integrating (20) and using (26) one has

$$\bar{P}_e(k,s)=\left[e_1\bar{P}_{M,N}(s)+e_2\bar{P}_{M-j.N}(s)\right]\exp.\left[-sk-\int_0^k\phi_e(k)dk\right]$$

$$\bar{P}_e(s)=\left[e_1\bar{P}_{M,N}(s)+e_2\bar{P}_{M-j.N}(s)\right]D_{\phi_e}(s) \quad \dots (31)$$

Integrating (21) and using (27) one may have

$$\bar{P}_H(z,s)=\left[h_1\bar{P}_{M,N}(s)+h_2\bar{P}_{M-j.N}(s)\right]\exp.\left[-sz-\int_0^z\phi_H(z)dz\right]$$

$$\bar{P}_H(s)=\left[h_1\bar{P}_{M,N}(s)+h_2\bar{P}_{M-j.N}(s)\right]D_{\phi_H}(s) \quad \dots (32)$$

Integrating (16) by using relevant relations one may obtain:

$$\bar{P}_{M-j,N}(y,s)=G(s)\bar{P}_{M,N}(s)$$

$$G(s)=\left[\lambda'+e_1\bar{S}_{\phi_e}(s)+h_1\bar{S}_{\phi_H}(s)\right]$$

$$\times D_{\phi_r}\left(s+\lambda'+\lambda''+e_2+h_2-\lambda\bar{S}_{\phi_i}(s)-e_2\bar{S}_{\phi_e}(s)-h_2\bar{S}_{\phi_H}(s)\right) \quad \dots (33)$$

Where, $D_k(s)=\dfrac{1-\bar{S}_k(s)}{s}, \forall k$

Lastly, equation (15) gives by using relevant relations

$$\bar{P}_{M,N}(s)=\frac{1}{A(s)}$$

Where,

$$A(s)=s+\lambda+\lambda'+e_1\left[1-\bar{S}_{\phi_e}(s)\right]+h_1\left[1-\bar{S}_{\phi_H}(s)\right]$$

$$-\left(\lambda'+e_1\bar{S}_{\phi_e}(s)+h_1\bar{S}_{\phi_H}(s)\right)\bar{S}_p(s)[s+\lambda'+\lambda''+e_2$$

$$+h_2-\lambda\bar{S}_{\phi_i}(s)-e_2\bar{S}_{\phi_e}(s)-h_2\bar{S}_{\phi_H}(s)\Big]$$

$$\times\left[s+\lambda'+\lambda''+e_2+h_2-\lambda\bar{S}_{\phi_i}(s)-e_2\bar{S}_{\phi_e}(s)-h_2\bar{S}_{\phi_H}(s)\right]$$

$$\left[e_2\bar{S}_{\phi_e}(s)+h_2\bar{S}_{\phi_H}(s)+\lambda''\bar{S}_{\phi}(s)\right]G(s) \quad \dots (34)$$

Thus finally we have

$$\bar{P}_{M,N}(s)=\frac{1}{A(s)} \quad \dots (35)$$

$$\bar{P}_{M-j,N}(s)=\frac{G(s)}{A(s)} \quad \dots (36)$$

$$\bar{P}_{M-j+1,N}(s)=\frac{\lambda''G(s)}{A(s)}D_{\phi}(s) \quad \dots (37)$$

$$\bar{P}_{M,j}(s)=\frac{\lambda}{A(s)}D_{\phi}(s) \quad \dots (38)$$

$$\bar{P}_{M-j,i}(s)=\frac{\lambda G(s)}{A(s)}D_{\phi}(s) \quad \dots (39)$$

$$\bar{P}_e(s)=\frac{1}{A(s)}\left[e_1+e_2G(S)\right]D_{\phi}(s) \quad \dots (40)$$

$$\bar{P}_H(s)=\frac{1}{A(s)}\left[h_1+h_2G(s)\right]D_{\phi}(s) \quad \dots (41)$$

7. ERGODIC BEHAVIOR OF THE SYSTEM

Using Abel's Lemma in L.T. *viz.*

$\lim_{s\to 0} s\bar{F}(s)=\lim_{t\to\infty}F(t)=F\ (Say)$ Provided the limit on R.H.S.

exists, the time independent probabilities are obtained as follows:

$$P_{M,N} = \frac{1}{A'(0)} \quad \dots (42)$$

$$P_{M-j,N} = \frac{G(0)}{A'(0)} \quad \dots (43)$$

$$P_{M-j+1,N} = \frac{\lambda'' G(0)}{A'(0)} M_\phi(s) \quad \dots (44)$$

$$P_{M,t} = \frac{\lambda}{A'(0)} M_\phi \quad \dots (45)$$

$$P_{M-j,t} = \frac{\lambda G(0)}{A'(0)} M_\phi \quad \dots (46)$$

$$P_e = \frac{1}{A'(s)}\left[e_1 + e_2 G(0)\right] M_\phi \quad \dots (47)$$

$$P_e = \frac{1}{A'(0)}\left[h_1 + h_2 G(0)\right] M_\phi \quad \dots (48)$$

Where,

$M_k = -\bar{S}_k'(0) =$ Mean time to repair k^{th} unit

8. SOME PARTICULAR CASES

When the repair follow exponential time distribution

Setting $\bar{S}_{\phi_i}(s) = \frac{\phi_i}{s+\phi_i}, \bar{S}_{\phi_e}(s) = \frac{\phi_e}{s+\phi_e} etc.$ in equation (35) through (41) one obtains:

$$\bar{P}_{M,N}(s) = \frac{1}{E(s)} \quad \dots (49)$$

$$\bar{P}_{M-j,N}(s) = \frac{g(s)}{E(s)} \quad \dots (50)$$

$$\bar{P}_{M-j+1,N}(s) = \frac{\lambda'' g(s)}{E(s)} \frac{1}{s+\phi} \quad \dots (51)$$

$$\bar{P}_{M,i}(s) = \frac{\lambda}{E(s)} \frac{1}{s+\phi_i} \quad \dots (52)$$

$$\bar{P}_{M-j,i}(s) = \frac{\lambda g(s)}{E(s)} \frac{1}{s+\phi_i} \quad \dots (53)$$

$$\bar{P}_e(s) = \frac{1}{E(s)}\left[e_1 + e_2 g(s)\right]\frac{1}{s+\phi_i} \quad \dots (54)$$

$$\bar{P}_H(s) = \frac{1}{E(s)} \frac{1}{s+\phi_H}\left[h_1 + h_2 g(s)\right] \quad \dots (55)$$

Where, $E(s) = \left[A(s)\right]_{\bar{S}k(s)=\frac{k}{s+k}}$, $g(s) = \left[G(s)\right]_{\bar{S}k(s)=\frac{k}{s+k}}$

9. EVALUATION OF UP AND DOWN STATE PROBABILITIES

$$\bar{P}_{UP}(s) = \bar{P}_{M,N}(s) + \bar{P}_{M-j,N}(s)$$

$$= \frac{1}{s+\lambda+\lambda'+e_1+h_1}\left[1 + \frac{\lambda'}{s+\lambda'+\lambda''+e_2+h_2}\right]$$

Inverting this we have

$$P_{up}(t) = (1+A)\exp\left[-(\lambda+\lambda'+e_1+h_1)t\right] + B\exp\left[-(\lambda'+\lambda''+e_2+h_2)t\right] \quad \dots (A)$$

And $P_{down}(t) = 1 - P_{up}(t)$ … (B)

10. COST ANALYSIS FUNCTION

The cost analysis function is defined as

$$G(t) = C_1 \int_0^t P_{up}(t)\,dt - C_2 t$$

Where, C_1 = revenue cost per unit up time

C_2 = repair cost per unit time

After integration we get,

$$G(t) = C_1(1+A)\left[\frac{1-\exp.-(\lambda+\lambda'+e_1+h_1)t}{\lambda+\lambda'+e_1+h_1}\right] + C_1 B\left[\frac{1-\exp.-(\lambda'+\lambda''+e_2+h_2)t}{\lambda'+\lambda''+e_2+h_2}\right] - C_2 t \quad \dots (C)$$

Where, $A = \frac{\lambda'}{\lambda''+e_2+h_2-\lambda-e_1-h_1}$ and

$$B = \frac{\lambda'}{\lambda+e_1+h_1-\lambda''-e_2-h_2} \quad \dots (D)$$

11. NUMERICAL COMPUTATION

Suppose

$\lambda = h_1 = 0.01, \lambda' = h_2 = 0.02, \lambda'' = 0.03, e_1 = 0.04, e_2 = 0.05$ and $C_1 = 2, C_2 = 1$

Putting these values in equations (D), (B), (C) and (A), we get

$$A = \frac{\lambda'}{\lambda''+e_2+h_2-\lambda-e_1-h_1} = \frac{0.02}{0.03+0.05+0.02-0.01-0.04-0.01} = 0.5$$

$$B = \frac{\lambda'}{\lambda+e_1+h_1-\lambda''-e_2-h_2} = \frac{0.02}{0.01+0.04+0.01-0.03-0.05-0.02} = -0.5$$

$$P_{up}(t) = 1.5\exp(-0.08)t - 0.5\exp(-0.12)t$$

and $G(t) = 3\left[\frac{1-\exp(-0.08)t}{0.08}\right] - \left[\frac{1-\exp(-0.12)t}{0.12}\right] - t$

12. INTERPRETATION

Table 3.1 computes the availability of the system decreases as the time increases and Figure 3 shows the graph for the same.

Table 3.2 & Figure 3.4 shows that cost function increases with time *t*.

Table 1. Availability function with respect to time

S.No.	t	Pup(t)
1	0	1
2	1	0.9412143
3	2	0.8849018
4	3	0.8311036
5	4	0.7798319
6	5	0.7310743
7	6	0.684799
8	7	0.6409583
9	8	0.5994922
10	9	0.5603306
11	10	0.5233963
12	11	0.4886067
13	12	0.4558754
14	13	0.425114
15	14	0.3962327
16	15	0.3691419

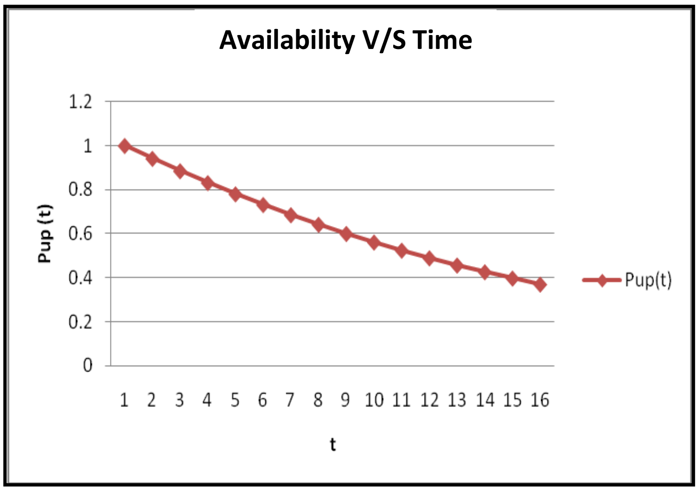


Figure 3: Availability as function of time

Table 2. Cost function with respect to time

S.No.	t	G(t)
1	0	0
2	1	0.9408073
3	2	1.7665068
4	3	2.4820913
5	4	3.092606
6	5	3.6030952
7	6	4.0185583
8	7	4.3439145
9	8	4.5839748
10	9	4.7434198
11	10	4.8267823
12	11	4.838435
13	12	4.7825814
14	13	4.66325
15	14	4.4842908
16	15	4.2493745

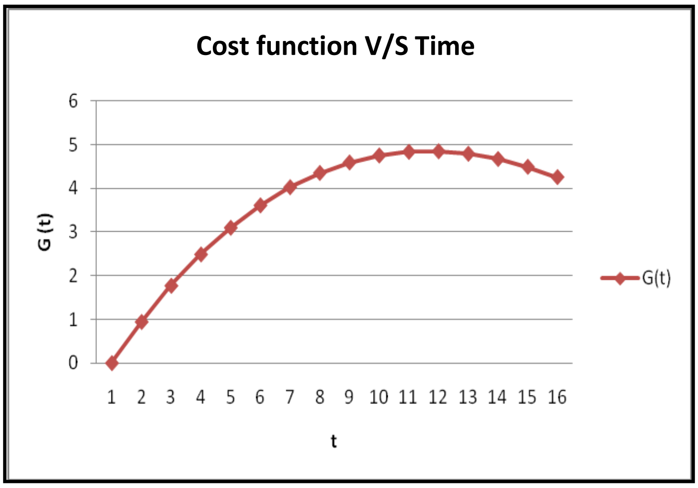


Figure 4 : Cost Profit as function of time

13. CONCLUSION

Table 1 and Figure 3 provide information how availability of the complex engineering repairable system changes with respect to the time when failure rate increases availability of the system decreases.

Table 2 and Figure 4 when revenue cost per unit time C_1 and C_2 are fixed, then one can conclude by observing this graph that as cost increases, when time increases.

The further research area is widely open, where one may think of the application of MTTF and sensitivity analysis.

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