Takagi-Sugeno Fuzzy System based Stable Direct Adaptive Control of Nonlinear Systems

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ABSTRACT
This paper proposes a novel idea for stable direct adaptive control of nonlinear systems using Lyapunov function with fuzzy approach. Stable direct adaptive control law consists of an ideal control, and a sliding mode control. Sliding mode controller is used to ensure the stability of Lyapunov function. Stability of direct adaptive control law is tested on two nonlinear systems. Non linear systems analyzed are ball beam system and cart pole system. A computer simulation is performed on nonlinear systems by using MATLAB.

Keywords
Fuzzy system, Lyapunov function, adaptive control, ball beam system and cart pole system.

1. INTRODUCTION
A Fuzzy based mathematical model of a controller was described and designed with incorporation of Fuzzy Logic Controller (FLC) [1]. To test the performance of controller, a nonlinear system considered which was described by the least square identification method [2] and simulation studies were performed. It was observed that the performance of controller was robust for a wide range of operation and improved the system stability. Self tuning features [3] for both the data and the rule base can be designed by using FLC. The design of proportional derivative [4], linear quadratic and nonlinear controllers are based on differential geometric notions. Adaptive tracking control architecture for a class of continuous time nonlinear systems is performed for an explicit linear parameterization of the uncertainty in the dynamics is either unknown or impossible. The architecture of fuzzy systems, [5] expressed as a series expansion of basis functions. The controller output [6] has provided the result of applying fuzzy logic theory to manipulate the given set of control laws for nonlinear systems. The parameters of the membership functions in the fuzzy rule base [7] are changed according to adaptive algorithm for the purpose of controlling the system state in a user-defined sliding surface. The fuzzy sliding mode controller [8] efficiently controls most of the complex systems even though their mathematical models are unknown. The dynamic behavior of the controlled system can be approximately dominated by a fuzzified sliding surface. Fuzzy logic control and sliding mode control techniques have been integrated to develop a fuzzy sliding mode controller. A decentralized fuzzy logic controller [9] has been designed for large-scale nonlinear systems. An approximate method [10] is formulated for analyzing the performance of a broad class of linear and nonlinear systems controlled by using fuzzy logic. Decision rules can be automatically [11] generated for FLC to provide a stable closed loop system using Lyapunov function. Fuzzy logic system [12] that uses adaptive sliding mode control can approximate the unknown function of nonlinear systems. The FLC[13] which produces desirable transient performance for nonlinear systems which promises closed loop stability. An adaptive fuzzy control scheme [14] employs a fuzzy controller and a compensation controller for a class of nonlinear continuous systems. A stable fuzzy controller [15] has been synthesized in terms of Mamdani model to stabilize nonlinear systems. FLC [16] can control a cart balancing flexible pole under its first mode of vibration. Adaptive fuzzy logic controller [17] uses the uniform ultimate boundedness of the closed-loop signals for a class of discrete-time nonlinear systems. The overall system stability [18] governing the control of the plants has been put together into a rule base for the FLC. Design of FLC [19] for nonlinear systems with assured closed loop stability and its application on combining controller is based on heuristic fuzzy rules. Adaptive sliding mode schemes [20] along with fuzzy approximators are used to approximate the unknown function of nonlinear systems. The hybrid FLC [21] which is proportional plus conventional integral derivative controller is more effective in comparison with the conventional PID controller, when the controlled object operates under uncertainty or in the presence of a disturbance. A direct adaptive [22] FLC has been used for tracking a class of nonlinear dynamic systems. A nonlinear system [23] can be represented by Takagi-Sugeno FLC and it can be constructed by blending all local state feedback controllers with a sliding mode controller. A robust adaptive fuzzy controller [24] has been used for a class of nonlinear systems in the presence of dominant uncertainties. Fuzzy logic system can be used to compensate [25] the parametric uncertainties that has the capability to approximate any nonlinear function with the compact input space. Limit cycle of a system [26] can be controlled by a fuzzy logic controller via some of the classical control techniques used to analyze nonlinear systems in the frequency domain. An adaptive control scheme [27] with fuzzy logic control has been used for robot manipulator with parametric uncertainties. A model based fuzzy controller [28] can be used for a class of uncertain nonlinear systems to achieve a common observability. Exact fuzzy modeling and optimal control [29] has been used on inverted pendulum. The nonlinear fuzzy PID [30] controller has been applied successfully in control systems with various nonlinearities. The uncertain nonlinear system [31] has been represented by uncertain Takagi-Sugeno fuzzy model structure. Most of real word systems [32] have dynamic features, and these are also known as autoregressive dynamic fuzzy systems. A fuzzy variable structure controller [33] based on the principle of sliding mode variable control can be used both for the dynamic as well as static control properties of the system. Adaptive fuzzy sliding mode control scheme [34] incorporates the fuzzy logic into sliding mode control. The control algorithm [35] of robust controller for a nonlinear system is based on sliding mode incorporates a fuzzy tuning technique. Based on the Lyapunov approach [36], the adaptive laws and stability analysis can be used for a class of nonlinear uncertain systems. A neuro-fuzzy learning algorithm [37] has been

applied to design a Takagi-Sugeno type FLC for a biped robot walking problem. In this paper we discussed the fuzzy logic control system in Section 2, the proposed stability design approach using Lyapunov Function in Section 3, dynamics of nonlinear systems and simulation results in Section 4 are presented. Conclusion is given in Section 5 followed by References in Section 6.

2. TAKAGI SUGENO FUZZY SYSTEMS

For the functional fuzzy system singleton fuzzification is used and the premise is defined as same for the rule of the standard fuzzy system .The consequents of the rules are different, however instead of a linguistic term with an associated membership function, in the consequent a function $b_i = g_i(.)$ have been used that does not have an associated membership function. The argument of $g_i$contains the fuzzy system inputs which are used in the premise of the rule. For the functional fuzzy system an appropriate operation for representing the premise and defuzzification is obtained as

$$y = \frac{\sum_{i=1}^{R} b_i \mu_i(z)}{\sum_{i=1}^{R} \mu_i(z)} \qquad (1)$$

Where $\mu_i(z)$and R represents premise membership function and number of rules, respectively

$$b_i = g_i(.) = a_{i,0} + a_{i,1}u_1 + \cdots + a_{i,n}u_n \qquad (2)$$

where $a_{i,n}$ fixed real numbers . The functional fuzzy system is referred to as a"Takagi-Sugeno fuzzy system". Consider a Takagi-Sugeno fuzzy system that is given by

$$y = F_{ts}(x,\theta) = \frac{\sum_{i=1}^{R} g_i(x)\mu_i(x)}{\sum_{i=1}^{R} \mu_i(x)} \qquad (3)$$

Where if $a_{i,n}$ are constants

$$g_{i(x)} = a_{i,0} + a_{i,1}x_1 + \cdots . a_{i,n}x_n \qquad (4)$$

Also for $i = 1,2, \dots, R$.

$$\mu_i(x) = \prod_{j=1}^{n} exp\left(-\frac{1}{2}\left(\frac{x_j - c_j^i}{\sigma_j^i}\right)^2\right) \qquad (5)$$

Where c_j^i is the point in the j[th] input universe of discourse where the membership function for the i[th] rule achieves a maximum, and $\sigma_j^i > 0$ is the relative width of membership function for the j[th] input and the i[th] rule. First term is the standard fuzzy system.

Let

$$\phi = \begin{bmatrix} \xi_1(x), \xi_2(x), , .. \xi_R(x), x_1\xi_1(x), x_1\xi_2(x), \dots, \\ x_1\xi_R(x), \dots, x_n\xi_1(x), x_n\xi_2(x), \dots x_n\xi_R(x) \end{bmatrix}^T \qquad (6)$$

and

$$\theta = \left[a_{1,0}, a_{2,0}, \dots, a_{R,0}, a_{1,1}, a_{2,1}, \dots a_{R,1}, \dots, a_{R,n}\right]^T \qquad (7)$$

And also

$$\xi_j = \frac{\mu_j(x)}{\sum_{i=1}^{R} \mu_i(x)} \qquad (8)$$

$j = 1,2, \dots, R$, so that

$$y = F_{ts}(x,\theta) = \theta^T \phi(x) \qquad (9)$$

which represents the Takagi-Sugeno fuzzy system

3. STABLE DIRECT ADAPTIVE CONTROL USING LYAPUNOV STABILITY APPROACH

A. Adaptive control

For adaptive control aim is that reference model, trajectory to be tracked be $y_m(t)$ and its derivatives are $\dot{y}_m(t),\dots,y_m^{(d)}(t)$ such that output $y(t)$ and its derivatives $\dot{y}(t),\dots,y^{(d)}(t)$ follow the reference trajectory. Assume that $y_m(t)$ and its derivatives $\dot{y}_m(t),\dots,y_m^{(d)}(t)$ are bounded. For a reference input $r(t)$ and reference trajectory $y_m(t)$

$$\frac{Y_m(s)}{R(s)} = \frac{q(s)}{p(s)} = \frac{q_0}{s^d + p_{d-1}s^{d-1} + \cdots + p_0} \qquad (10)$$

For $r(t) = 0, t \geq 0,\ y(t) \to 0$ as $t \to \infty$. (11)

Hence choose

$$y_m(t) = \dot{y}_m(t) = \cdots = y_m^{(d)}(t) = 0. \qquad (12)$$

For $R(s) = 0$,

$$p(s)Y_m(s) = 0 \qquad (13)$$

Or

$$(s^d + p_{d-1}s^{d-1} + \cdots + p_0)Y_m(s) = 0 \qquad (14)$$

Or

$$y_m{}^{(d)}(t) + p_{d-1}y_m{}^{d-1}(t) + \cdots + p_0 y_m(t) = 0 \qquad (15)$$

Parameters $p_{d-1},\dots, p_0$ specify the dynamics of how $y_m(t)$ evolves over time and how $y(t)$ and its derivatives evolve over time. Figure 1 shows the block diagram for adaptive control using fuzzy logic system.

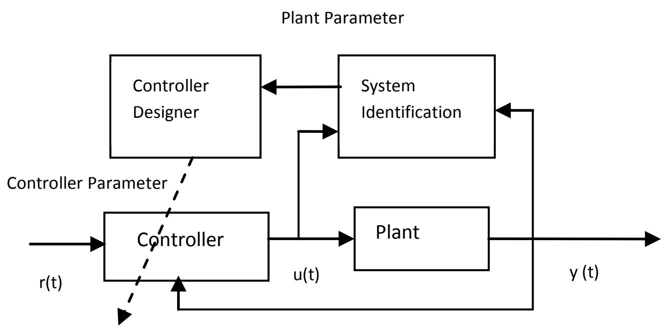

Fig.1 Adaptive Control using Takagi-Sugeno Fuzzy System

B. Online Approximator

Consider the plant

$$\dot{x} = f(x) + g(x)u \qquad (16)$$

$$y(k+d) = \alpha(x(k)) + \beta(x(k))u(k) \qquad (17)$$

$$y^{(d)} = (\alpha_k(t) + \alpha(x)) + (\beta_k(t) + \beta(x))u \qquad (18)$$

Here $d = n$ is the order of the plant. $y^{(d)}$ denotes the d- th derivative of y. $\alpha_k(t)$ and $\beta_k(t)$ are the known components of the plant dynamics, $\alpha(x)$ and $\beta(x)$ represent the non-linear dynamics of the plant that are unknown. $\alpha(x)$ and $\beta(x)$ are approximated with $\theta_\alpha{}^T\phi_\alpha(x)$ and $\theta_\beta{}^T\phi_\beta(x)$ by adjusting the θ_α and θ_β. The parameters θ_α and θ_β are defined within the compact parameter sets Ω_α and Ω_β respectively. $S_x \subseteq R^n$ be defined within the space through which the trajectory will travel under closed loop control.

$$\alpha(x) = \theta_\alpha^{*T}\phi_\alpha(x) + f_\alpha(x) \quad (19)$$

$$\beta(x) = \theta_\beta^{*T}\phi_\beta(x) + f_\beta(x) \quad (20)$$

$$\theta_\alpha^* = \arg\ \min_{\theta_\alpha \epsilon \Omega_\alpha}(\ sup_{x\epsilon S_x}\left|\theta_\alpha{}^T\phi_\alpha(x) - \alpha(x)\right|) \quad (21)$$

$$\theta_\beta^* = \arg\ \min_{\theta_\beta \epsilon \Omega_\beta}(\ sup_{x\epsilon S_x}\left|\theta_\beta{}^T\phi_\beta(x) - \beta(x)\right|) \quad (22)$$

so that $f_\alpha(x)$ and $f_\beta(x)$ are approximation errors, which arise when $\alpha(x)$ and $\beta(x)$ are represented by finite size approximators. The approximations of $\alpha(x)$ and $\beta(x)$ of the actual system are

$$\hat{\alpha}(x) = \theta_\alpha{}^T(t)\phi_\alpha(x) \quad (23)$$

$$\hat{\beta}(x) = \theta_\beta{}^T(t)\phi_\beta(x) \quad (24)$$

where the vectors $\theta_\alpha(t)$ and θ_β are updated online. The parameter errors are

$$\widetilde{\theta_\alpha}(t) = \theta_\alpha(t) - \theta_\alpha^* \quad (25)$$

$$\widetilde{\theta_\beta}(t) = \theta_\beta(t) - \theta_\beta^* \quad (26)$$

In addition to the assumptions made in the indirect adaptive control case, also required is

$$\beta_k(t) = \alpha_k(t) = 0 \quad (27)$$

for all $t \geq 0$, and that there exist positive constants $\beta_0\ and\ \beta_1$ such that

$$0 < \beta_0 \leq \beta(x) \leq \beta_1 \quad (28)$$

Also it is assumed here a function $B(x) \geq 0$ such that

$$\dot{\beta}(x) = \left|\left(\frac{\partial\beta}{\partial x}\right)^T \dot{x}\right| \leq B(x) \quad (29)$$

for all $x\epsilon S_x$. This requirement is often met in practice since $\dot{\beta}(x)$ is considered as the rate of change of the "gain" on the input term, it is often the case that this will be bounded . For example, notice that if $\beta(x)$ is constant that we know lies in a fixed interval, then all these coefficients are satisfied with $B(x) = 0$, for all x.

C. Controller Approximator

There exists some ideal controller

$$u^* = \frac{1}{\beta(x)}\left(-\alpha(x) + v(t)\right) \quad (30)$$

where $v(t)$ is defined the same as in indirect adaptive control case. Let

$$u^* = \theta_u^{*T}\phi_u(x,v) + u_k(t) + w_u(x,v) \quad (31)$$

Where u_k is a known part of the controller and

$$\theta_u^* = \arg \min_{\theta_u \epsilon \Omega_u}\left(\ sup_{x\epsilon S_x, v\epsilon S_m}\ \ |\theta_u^T\phi_u(x,v) - (u^* - u_k)|\right) \quad (32)$$

So that $w_u(x,v)$ is the approximation error. It is assumed that $W_u(x,v) \geq |w_u(x,v)|$, where $W_u(x,v)$is a known bound on the error in representing the ideal controller .The approximation is

$$\hat{u} = \theta_u^T(t)\phi_u(x,v) + u_k \quad (33)$$

Where the matrix $\theta_u(t)$is the updated online .The parameter error is

$$\tilde{\theta}_u(t) = \theta_u(t) - \theta_u^* \quad (34)$$

Consider the control law

$$u = \hat{u} + u_{sd} \quad (35)$$

which is the sum of an approximation to an ideal control law, and a sliding mode control term. With this, the d^{th} derivative of the tracking error becomes

$$e^{(d)} = y_m^{(d)} - \alpha(x) - \beta(x)(\hat{u} + u_{sd}) \quad (36)$$

Adding and subtracting $\beta(x)u^*$ and then using definition of u^*, we get

$$e^{(d)} = y_m^{(d)} - \alpha(x) - \beta(x)u^* - \beta(x)(\hat{u} - u^*) - \beta(x)u_{sd} \quad (37)$$

$$e^{(d)} = -\gamma e_s - \bar{e_s} - \beta(x)(\hat{u} - u^*) - \beta(x)u_{sd} \quad (38)$$

Or

in manner analogous to the indirect case,

$$\dot{e_s} + \gamma e_s = -\beta(x)(\hat{u} - u^*) - \beta(x)u_{sd} \quad (39)$$

D. Controller Parameter Updates

Consider the following Lyapunov function candidate

$$V_d = \frac{1}{2\beta(x)}e_s^2 + \frac{1}{2\eta_u}\tilde{\theta}_u^T\tilde{\theta} \quad (40)$$

where $\eta_u > 0$.Since $0 < \beta_0 \leq \beta(x) \leq \beta_1$, V_d is radially unbounded. The Lyapunov candidate V_d is used to measure both the error in tracking and the error between the desired controller and the current controlle5r.Taking the time derivative of equation (14) yields

$$\dot{V_d} = \frac{e_s}{\beta(x)}\dot{e_s} - \frac{\dot{\beta}(x)e_s^2}{2\beta^2(x)} + \frac{1}{\eta_u}\tilde{\theta}_u^T\dot{\tilde{\theta}} \quad (41)$$

Substituting $\dot{e_s}$,as defined in equation (13) ,we find

$$\dot{V_d} = \frac{e_s}{\beta(x)}\left(-\gamma e_s - \beta(x)\right)(\hat{u} - u^*) - \beta(x)u_{sd} - \frac{\dot{\beta}(x)e_s^2}{2\beta^2(x)} + \frac{1}{\eta_u}\tilde{\theta}_u^T\dot{\tilde{\theta}} \quad (42)$$

Use the update law

$$\dot{\theta_u}(t) = \eta_u\phi_u(x,v)e_s(t) \quad (43)$$

So $\eta_u > 0$ is an adaption gain. Since $\dot{\widetilde{\theta_u}} = \ \ \theta_u$,

$$\dot{V_d} = \frac{-\gamma}{\beta(x)}\ e_s^2 - \left(\tilde{\theta}_u^T\phi_u(x,v) - \omega_u(x,v) + u_{sd}\right)e_s - \frac{\dot{\beta}(x)e_s^2}{2\beta^2(x)} + \tilde{\theta}_u^T\phi_u(x,v)e_s \quad (44)$$

and so

$$\dot{V_d} \leq \frac{-\gamma}{\beta(x)}\ e_s^2 - \left(\frac{\dot{\beta}(x)e_s^2}{2\beta^2(x)} - \omega_u(x,v)\right)\ e_s - e_s u_{sd} \quad (45)$$

We use a projection method to ensure that $\phi_u\epsilon\Omega_u$so in an analogous manner to the indirect case,

$$\dot{V_d} \leq \frac{-\gamma}{\beta(x)}\ e_s^2 - \left(\frac{\dot{\beta}(x)e_s^2}{2\beta^2(x)} - \omega_u(x,v)\right)\ e_s - e_s u_{sd} \quad (46)$$

E. Sliding Mode Control Term and Stability Properties

Once again a sliding mode control term is used to compensate for the approximation error in modeling u^* by a finite size approximator. In

$$\dot{V_d} \leq \frac{-\gamma}{\beta_1}\ e_s^2 + \left(\frac{|\dot{\beta}(x)||e_s|}{2\beta^2(x)} + |\omega_u(x,v)|\right)|e_s| - e_s u_{sd} \quad (47)$$

Now defining the sliding mode control term for the direct adaptive controller as

$$u_{sd} = \left(\frac{B(x)|e_s|}{2\beta_0^2} + W_u(x,v)\right) sgn(e_s) \qquad (48)$$

which ensures that

$$\dot{V}_d \le -\frac{\gamma e_s^2}{\beta_1} \qquad (49)$$

so that V_d is a non-increasing function of time.

All signals are bounded and $e(t) \to 0\ as\ t \to \infty$ so we get asymptotic tracking .There are practical application where u_k can be designed so that the resulting transient performance can be improved. In an analogous manner to the indirect case, it is possible to define a smoothed version of the sliding mode control term that will only result in e_s reducing to a neighborhood of zero.

4. SIMULATION RESULTS AND DISCUSSION

A. Dynamics of Ball Beam System

Consider a 4th order nonlinear system, ball and beam system shown in figure 2, for the simulation. The beam is made to rotate in a vertical plane by applying the torque at the centre of rotation and the ball is free to roll along the beam. The ball remains in contact with the beam. Let $x = [x_1\ x_2\ x_3\ x_4]$ $:= \left[r\ \frac{dr}{dt}\ \theta\ \frac{d\theta}{dt}\right]$ be the state of the system, $y = x_1$ be the output of the system and u be the input to the system. Then system can be represented by the state-space model

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \\ \frac{dx_4}{dt} \end{bmatrix} = \begin{bmatrix} x_2 \\ B(x_1 x_4^2 - G\sin x_3) \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \qquad (50)$$

Where u is the input to the system, and B,G are the system parameter

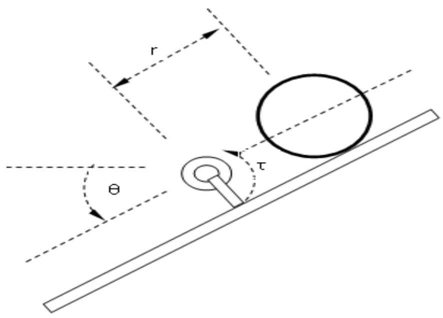

Fig.2 Ball Beam System

B. *Simulation Results and Discussion of Ball Bam System*

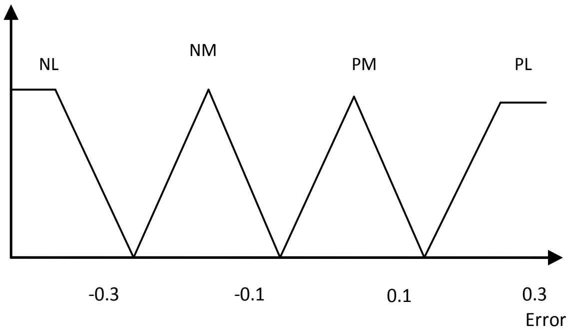

Fig.3 Membership Functions for input Error

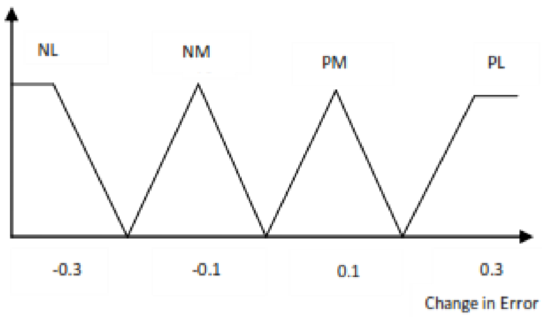

Fig. 4 Membership Functions for input change in error

Table 1 Tuned Design Parameters for Ball Beam System

Initial position	K_0	K_1	K_2	γ	ξ_α	ξ_β
[0.4 0.0 0.9 0.0]	2.71	1	1.521	5	4	4

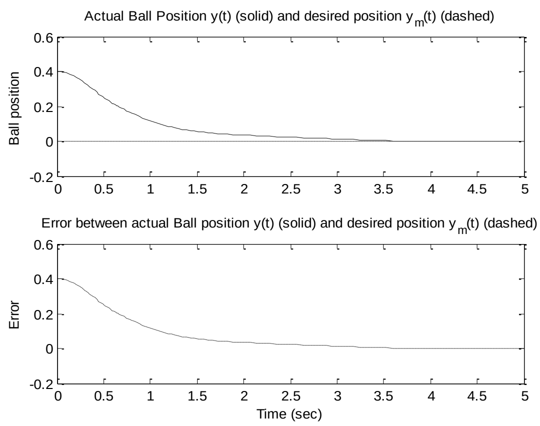

Fig.5 Actual Ball Position, Desired Ball Position and Error between Actual and Desired Ball Position

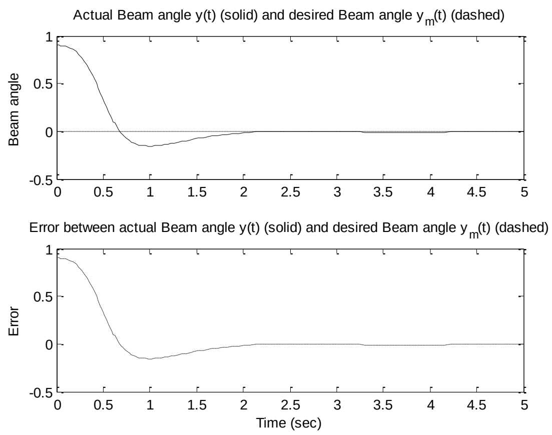

Fig.6 Actual Beam Angle, Desired Beam Angle and Error between Actual and Desired Beam Angle

Universe of discourse for membership function has been taken from -0.3 to 0.3 have been shown in figures 3 and 4. Total sixteen rules have been used. Design parameters k_0, k_1, k_2 , γ, ξ_α and ξ_β are tuned in order to get desired results as shown in Table 1. Figure 5 and 6 shows it has been observed that Ball Position converges to origin and Beam angle converges to zero such that at balance Ball position and Beam angle are at origin.

C. Dynamics of Inverted Pendulum

A single-input multi-output inverted pendulum shown in figure 7 can be represented by a fourth order non-linear equation.

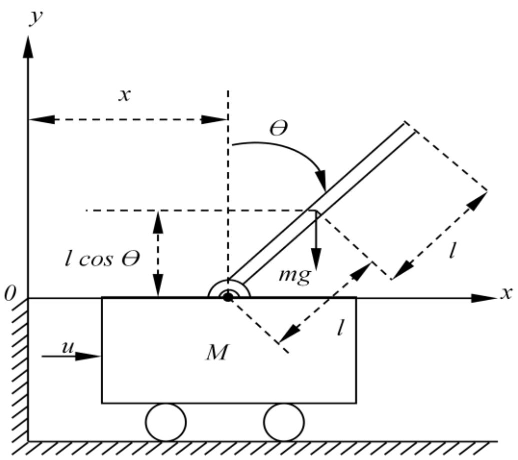

Fig.7 Inverted Pendulum Mounted on Cart

Let d be the distance of the cart from centre and α be the angle of the pendulum.
Defining the state vector $x = [d\ \dot{d}\ \alpha\ \dot{\alpha}]$,
The system equations can be written as follows:

$$\dot{x_1} = x_2 \tag{51}$$

$$\dot{x_2} = \frac{lx^3 \sin x_3 - g \sin x_3 \cos x_3 + \frac{1}{m}u}{\frac{M}{m} + (\sin x_3)^2} \tag{52}$$

$$\dot{x_3} = x_4 \tag{53}$$

$$\dot{x_4} = \frac{\frac{M+m}{m} g \sin x_3 - l x_3^2 \sin x_3 \cos x_3 - \frac{\cos x_3}{m} u}{l\left(\frac{M}{m} + (\sin x_3)^2\right)} \tag{54}$$

Mass of the cart, M = 0.455 kg
Mass of pendulum, m = 0.210 kg
Half length of pendulum rod, l = 0.305 m
Acceleration due to gravity, g = 9.81m/s^2
u= input force applied to the cart, N

D. Simulation Results and Discussion of Cart Pole System

Table 2 Tuned Design Parameters for Inverted Pendulum System

Initial position	K_0	K_1	K_2	γ	ξ_α	ξ_β
[0.4 -1.2 0.2 -0.2]	2600	620	110	4	2	2

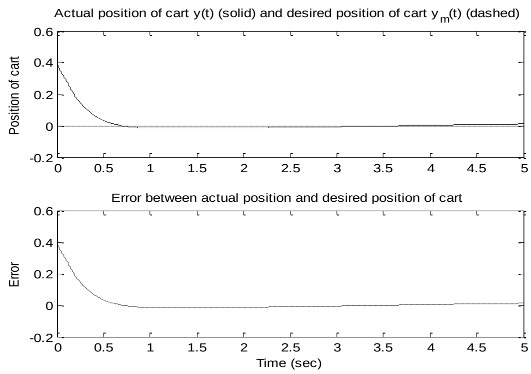

Fig. 8 Actual Cart Position, Desired Cart Position and Error between Actual Cart Position and Desired Cart Position

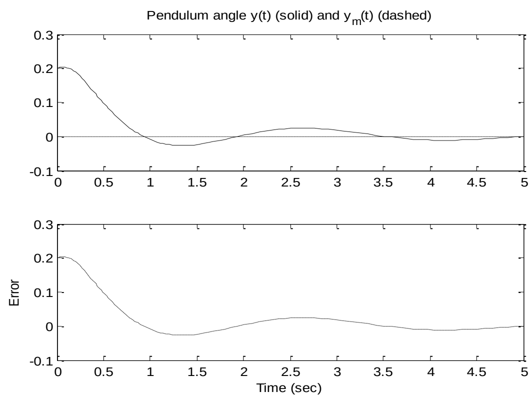

Fig. 9 Actual Pendulum Angle, Desired Pendulum Angle and Error between Actual and Desired Pendulum Angle.

Universe of discourse for membership function has been taken from -0.3 to 0.3 as shown in figures 3 and 4. Total sixteen rules have been used. Design parameters k_0, k_1, k_2 , γ, ξ_α and ξ_β are tuned in order to get desired results as shown in Table 2.
Figure 8 and 9 shows that Cart position converges to origin and Pendulum angle also converges at origin which is also the requirement for a balance of Inverted Pendulum on Cart. Also error reduces to zero for both Cart position as well as Pendulum angle.

5. CONCLUSION

A Stable adaptive control using Lyapunov function has been proposed. Takagi–Sugeno fuzzy system is used for identification and to control nonlinear systems. The proposed stable adaptive control law is tested on Ball Beam System and Inverted Pendulum mounted on a cart. It has been observed that Ball Position converges to origin and Beam angle converges to zero such that at balance Ball position and Beam angle are in equilibrium at origin. Inverted pendulum on a cart taken as, another nonlinear system and has been observed that inverted pendulums angle reaches to zero while cart Position converges at origin which is the requirement for a balance between inverted pendulum and position of cart. Above nonlinear systems have been tested for given initial positions and confirms the stability of proposed direct adaptive control law.

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